DESIGN OF FREQUENCY-INVARIANT BEAMFORMERS WITH SPARSE CONCENTRIC CIRCULAR ARRAYS

Yaakov Buchris, Israel Cohen, Alon Amar

Andrew and Erna Viterbi Faculty of Electrical and Computer Engineering
Technion, Israel Institute of Technology, Technion City, Haifa 3200003, Israel
{bucris@, icohen@ee, aamar@}.technion.ac.il

ABSTRACT

Frequency-invariant (FI) beamformers are used in audio and acoustic applications to enhance broadband signals with possibly varying angles of arrival. One of the main challenges is designing such beamformers using a small number of sensors while still performing well under adverse conditions. This paper introduces a new design approach for FI beamformers with sparse concentric circular arrays. We propose an iterative greedy design that optimizes both the number of required sensors and rings while preserving the predefined directivity pattern's properties for different frequencies and azimuthal steering directions. Experimental results demonstrate the benefits of the proposed sparse sensor design and FI beamformer in terms of array gain and rotationally invariant beampattern, under limited computational and hardware constraints.

Index Terms— Concentric circular array, frequency-invariant beamformer, rotation invariance, sparse array.

1. INTRODUCTION

Planar arrays are popular in practical applications that require a variable azimuthal mainlobe direction while preserving the performance of array gain and directivity pattern [1–3]. In particular, planar concentric circular arrays (CCAs) that contain several rings of sensors and provide similar and steerable beampattern for 360° azimuthal coverage are advantageous also for broadband beamformers as they enable robust and frequency-invariant (FI) beampatterns over a wide range of frequencies [4]. Among several extensively explored classical approaches for FI beamformer design [5–14], the sparse design class is of great interest since both the beamformer gains as well as the total number of sensors and their positions are optimized [15–19].

Previous work on sparse CCAs design includes genetic algorithms (GA) [20–22], modified particle swarm optimization (MPSO) [23], and biogeography-based optimization (BBO) [24]. However, these approaches focus mainly on the narrowband case and the optimization of the ring radii only, each with uniformly-spaced sensors. Broadband sparse designs of FI beamformers for rotationally symmetric sparse CCAs are presented in [25–30].

In this paper, we extend our recent work on a sparse design of FI-CCA beamformers [30], where we presented a greedy-based approach that optimizes the number of sensors and rings while taking into consideration the requirement regarding the rotation-invariance property. However, the design in [30] is based on a separate optimization framework for the sensors and rings. Herein, we propose a new generalized approach that simultaneously optimizes both numbers of sensors and rings while preserving FI and rotation invariance properties, while ensuring desired properties like array gain and robustness. We generalized the standard orthogonal matching pursuit (OMP) greedy search algorithm originally designed for the narrowband case to fit also to the joint-sparse case. Simulation results show that the proposed design yields a FI and rotationally invariant beamformer with high white noise gain (WNG) and directivity. In addition, it requires reasonable resource consumption leading to a practical design for applications involving large arrays with hundreds of candidate sensors.

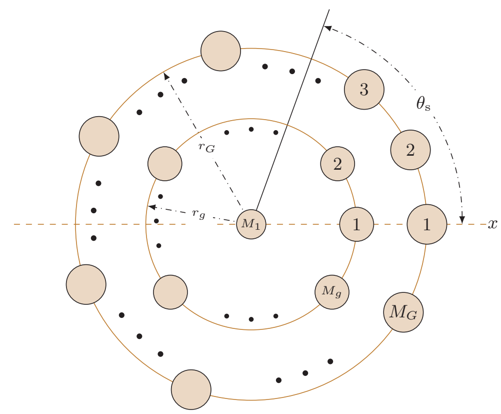


Figure 1: A concentric circular array with G rings. The gth ring, has a radius of r_g, and consists of M_g omnidirectional sensors.

2. PROBLEM FORMULATION

Consider a CCA composed of G rings, each characterized by its radius, r_g, and a number M_g which represents the maximum possible positions for locating sensors along the gth ring, where $g = 1, 2, \dots, G$ (see Fig. 1). It is assumed that $1 = M_1 \leq M_2 \leq \cdots \leq M_G$. The total number of candidate positions is $M = \sum_{g=1}^{G} M_g$, and the center of the CCA coincides with the first ring having $r_1 = 0$ and M_1=1. Let θ denote the azimuthal direction of arrival (DOA) of signals towards the array, measured anti-clockwise from the x axis. The azimuthal DOA of the desired source signal to the array is denoted by $\theta_{\rm ds}$, where the subscript 'ds' stands for 'desired.'

Let $\mathbf{H}(\omega)$ be an $M_G \times G$ weight matrix for a given frequency $\omega_j \in \Omega$, $j = 1, \dots, J$, where Ω is the frequency space, whose gth

This research was supported by the Pazy Research Foundation.

column contains the beamformer coefficients on the gth ring:

$$[\mathbf{H}(\omega_j)]_{\cdot,g} = [H_{1,g}(\omega_j), H_{2,g}(\omega_j), \ldots, H_{M_g,g}(\omega_j), \mathbf{0}^T_{M_G-M_g}]^T, \quad (1)$$

where $\mathbf{0}_n$ is an $n \times 1$ vector with all entries equal to zero. Similarly, let $\mathbf{D}(\omega_j, \theta_p)$ be an $M_G \times G$ steering matrix for a given $\theta_p \in \Theta$, $p = 1, \ldots, P$, whose gth column is given by

$$[\mathbf{D}(\omega_j, \theta_p)]_{\cdot,g} = [d_{g,1}(\omega_j, \theta_p), \ldots, d_{g,M_g}(\omega_j, \theta_p), \mathbf{0}^T_{M_G-M_g}]^T, \quad (2)$$

where Θ is the angle space, $d_{g,m}(\omega_j, \theta_p) = e^{\jmath \frac{\omega_j r_g}{c} \cos(\theta_p - \psi_{g,m})}$, and $\psi_{g,m} = \frac{2\pi(m-1)}{M_g}$, $m = 1, 2, \ldots, M_g$.

The beampattern of the sparse array for a given frequency ω_j and azimuth angle θ_p can be expressed as [30]:

$$\mathcal{B}(\omega_j, \theta_p) = \text{trace}(\mathbf{H}^H(\omega_j)\mathbf{D}(\omega_j, \theta_p)). \quad (3)$$

Let $\mathcal{B}_{\mathrm{d}}^{\theta_{\mathrm{ds}}}(\theta)$ denote the desired far-field FI beampattern in the bandwidth of interest Ω, with a mainlobe steered to a specified azimuth θ_{ds}, where $(\theta, \theta_{\mathrm{ds}}) \in \Theta$. Our goal is to design a sparse array with a FI beampattern which is approximately the same as $\mathcal{B}_{\mathrm{d}}^{\theta_{\mathrm{ds}}}(\theta)$, regardless of $\omega_j \in \Omega$, and regardless of the direction of the source, yielding a similar beampattern $\mathcal{B}_d^{\tilde{\theta}_{\mathrm{ds}}}(\theta)$ for each $\tilde{\theta}_{\mathrm{ds}}$ only by changing $\mathbf{H}(\omega_j)$, $\forall \omega_j \in \Omega$. Since designing such an array for any source direction is cumbersome, we focus on a more practical design. We partition the azimuth space into the Q sectors

$$\Theta^q : \{\theta : \frac{360^\circ}{Q}(q-1) \le \theta < \frac{360^\circ}{Q} q\}, q = 1, 2, \ldots, Q. \quad (4)$$

Each azimuth sector represents the possibility that $\theta_{\mathrm{ds}} \in \Theta^q$. We let θ^q_{ds} represent the azimuth angle at the center of the sector associated with the source direction assuming it arrives via the qth sector. Sector q is associated with the designed beampattern whose mainlobe is steered to the direction θ^q_{ds}:

$$\mathcal{B}^{(q)}(\omega_j, \theta_p) = \text{tr}((\mathbf{H}^q(\omega_j))^H \mathbf{D}(\omega_j, \theta_p)), q = 1, \ldots, Q, \quad (5)$$

where $\mathbf{H}^q(\omega_j)$ is the weight associated with the qth sector. We also set WNG and the distortionless response (DR) constraints (specified later) on these weights for a given sector q and frequency ω_j. We look for a sparse design such that the sensors are spread sparsely over a small set of rings out of the G rings, which means that $\|\mathbf{H}(\omega_j)\|_{0,2} = \|\|[\mathbf{H}]_{\cdot,1}(\omega_j)]\|_2, \ldots, \|\mathbf{H}]_{\cdot,G}(\omega_j)\|_2\|_0$ is small enough, i.e., less than a predefined threshold η. All requirements focus on minimizing the number of nonzero elements in the weight matrix defined by the zero-norm $\|\mathbf{H}(\omega_j)\|_0$ under the above constraints. Mathematically speaking, our optimization problem can be expressed as the following sparse optimization problem:

$$\min_{\mathbf{H}(\omega_j)} \|\mathbf{H}(\omega_j)\|_0 \text{ s.t. } \sum_{j=1}^{J}\sum_{p=1}^{P}\|\mathcal{B}_{\mathrm{d}}^{\theta^q_{\mathrm{ds}}}(\theta_p) - \mathcal{B}^{(q)}(\omega_j, \theta_p)\|^2 \le \epsilon$$
$$, \text{WNG}(\mathbf{H}(\omega_j)) \ge \gamma(\omega_j)^{-1}, \text{DR}(\mathbf{H}(\omega_j)) = 1, \|\mathbf{H}(\omega_j)\|_{0,2} \le \eta$$
$$, \forall \theta^q_{\mathrm{ds}} \in \Theta^q, q = 1, \ldots, Q, \quad (6)$$

where $\gamma(\omega_j)$ is a parameter expressing the maximal allowed white noise output power for frequency ω_j. Herein, we propose an approximated solution to this sparse optimization problem using a greedy-based algorithm.

3. A GREEDY BASED DESIGN

One of the popular tools for implementing greedy-based search algorithms is the OMP. However, applying the OMP algorithm directly to our problem is impractical. Hence, we derive a modified version of the OMP algorithm in the following.

Define the vector $\mathbf{d}(\omega_j, \theta_p)$ of length M containing the nonzero elements of the matrix $\mathbf{D}(\omega_j, \theta_p)$ arranged in a column vector. In general, the mainlobe and sidelobe regions may require different amount of resolution or accuracy. Thus we set $P' \le P$ directions that cover the mainlobe region Θ_{m}, where the subscript m stands for mainlobe, and the remaining $P - P'$ directions that cover the sidelobe region Θ_{s} where the subscript s stands for sidelobe. As a particular case, one can set $P' = P$ and treat the entire azimuthal space uniformly. Define the $M \times P'$ matrix

$$\mathbf{D}_{M,\Theta_{\mathrm{m}}}(\omega_j) = [\mathbf{d}(\omega_j, \theta_1), \mathbf{d}(\omega_j, \theta_2), \ldots, \mathbf{d}(\omega_j, \theta_{P'})], \quad (7)$$

and

$$\mathbf{b}_{\mathrm{d}}^{\mathrm{m}}(\theta_{\mathrm{ds}}) = \left[\mathcal{B}_{\mathrm{d}}^{\theta_{\mathrm{ds}}}(\theta_1), \mathcal{B}_{\mathrm{d}}^{\theta_{\mathrm{ds}}}(\theta_2), \ldots, \mathcal{B}_{\mathrm{d}}^{\theta_{\mathrm{ds}}}(\theta_{P'})\right]^T \quad (8)$$

to be a vector that contains samples of the desired beampattern in the directions covering the mainlobe. Similarly, define the matrix $\mathbf{D}_{M,\Theta_{\mathrm{s}}}(\omega_j)$, and the vector $\mathbf{b}_{\mathrm{d}}^{\mathrm{s}}(\theta_{\mathrm{ds}})$. Consider the following set of matrices $\mathbf{D}^q_{\omega_j}$ of size $P \times M$

$$\mathbf{D}^q_{\omega_j} = \begin{bmatrix} \mathbf{D}^H_{M,\Theta_{\mathrm{m}}+\theta^q_{\mathrm{ds}}}(\omega_j) \\ \mathbf{D}^H_{M,\Theta_{\mathrm{s}}+\theta^q_{\mathrm{ds}}}(\omega_j) \end{bmatrix}. \quad (9)$$

We treat the matrices $\{\mathbf{D}^q_{\omega_j}\}_{q=1}^{Q}$ as dictionaries that contain in each column one word of length P, also called an atom, which corresponds to one of the M candidate sensors. It is desired to express $\mathcal{B}_{\mathrm{d}}^{\theta_{\mathrm{ds}}+\theta^q_{\mathrm{ds}}}(\theta)$ with atoms from the dictionary $\mathbf{D}^q_{\omega_j}$ by finding vectors $\left\{\mathbf{h}^q(\omega_j) \in \mathbb{C}^M\right\}_{q=1}^{Q}$ with the same support that solve:

$$\mathbf{D}^q_{\omega_j}\mathbf{h}^q(\omega_j) = \mathbf{b}_{\mathrm{d}}(\theta_{\mathrm{ds}} + \theta^q_{\mathrm{ds}}), \quad (10)$$

where the desired beampattern is

$$\mathbf{b}_{\mathrm{d}}(\theta_{\mathrm{ds}} + \theta^q_{\mathrm{ds}}) = \begin{bmatrix} \mathbf{b}_{\mathrm{d}}^{\mathrm{m}}(\theta_{\mathrm{ds}} + \theta^q_{\mathrm{ds}}) \\ \mathbf{b}_{\mathrm{d}}^{\mathrm{s}}(\theta_{\mathrm{ds}} + \theta^q_{\mathrm{ds}}) \end{bmatrix}. \quad (11)$$

Since typically the linear system of equations that $\mathbf{D}^q_{\omega_j}$ represents is underdetermined, (10) has an infinite number of solutions. Among them, the class of sparse solutions with few nonzero elements is of great interest because it means that practically few sensors are required to construct the desired beampattern. Mathematically, the solution with the fewest nonzero elements can be found by solving the NP-hard ℓ_0-norm problem:

$$\min \|\mathbf{h}^q(\omega_j)\|_0 \text{ subject to } \mathbf{D}^q_{\omega_j}\mathbf{h}^q(\omega_j) = \mathbf{b}_{\mathrm{d}}(\theta_{\mathrm{ds}} + \theta^q_{\mathrm{ds}}), \quad (12)$$

where $\|\mathbf{x}\|_0$ is the number of nonzero elements in the vector $\mathbf{x}$.

The joint-sparse OMP algorithm contains the following steps. At first, $\forall \omega_j \in \Omega$, we initialize the vectors

$$\mathbf{r}_q^{(0)}(\omega_j) = \mathbf{b}_{\mathrm{d}}(\theta_{\mathrm{ds}} + \theta^q_{\mathrm{ds}}), \quad (13)$$

of length P to be a residual vector, which is supposed to converge to a zero vector iteratively, and we also initialize the sparse vectors

$$\mathbf{b}^{(0)}_{(\mathrm{g},q)}(\omega_j) = \mathbf{0}_P, \quad (14)$$

which store the designed desired beampattern after the iterative algorithm converges. Finally, we initialize the $M \times M$ diagonal weighting matrices $\mathbf{W}^{(0)} = \mathbf{I}_M$ and $\mathbf{R}^{(0)} = \mathbf{I}_M$ and set $q = 0$. In each iteration l, the matrix $\mathbf{W}^{(l)}$ is a weighting matrix used to assign low weights to sensors already chosen in the previous iteration and to their neighbors by multiplying the corresponding indices by a mask vector. The matrix $\mathbf{R}^{(l)}$ is also a weighting matrix used to assign priority to sensors belonging to rings already selected in the previous iterations. Both these matrices are defined later.

In each iteration of the basic standard OMP algorithm, it is desired to find the next atom which has the most contribution in the reconstruction of the desired vectors, $\mathbf{b}^{(l)}_{(\mathrm{g},q)}(\omega_j)$, $\omega_j \in \Omega$. It is done by projecting $\mathbf{r}^{(l)}_q(\omega_j)$, $\omega_j \in \Omega$, over all the remaining atoms in the current iteration and taking the maximal projection.

As we are interested in obtaining a sparse array layout that is joint to all $\omega_j \in \Omega$, we have to consider that in each iteration l when searching for the sensor with the largest contribution. Moreover, we want to obtain a rotationally invariant response and minimize the number of rings. Let the $(l-1) \times 1$ indication vector that contains the indices of the selected sensors at the end of iteration $l-1$ be $\mathbf{i}^{l-1}$. At the lth iteration we find the sensor out of $M \setminus \mathbf{i}^{l-1}$ that maximizes the function:

$$m^* = \underset{m \in \{1,\ldots,M\} \setminus \mathbf{i}^{l-1}}{\operatorname{argmax}} \mathbf{W}^{(l-1)}_{m,\cdot} \mathbf{R}^{(l-1)} \sum_{j=1}^{J} \left| (\mathbf{D}^q_{\omega_j})^H \mathbf{r}^{(l)}_q(\omega_j) \right|, \quad (15)$$

where $\mathbf{W}_{m,\cdot}$ is the mth row of the mask matrix $\mathbf{W}$. The $l \times 1$ indication vector is updated as:

$$\mathbf{i}^{(l)} = [(\mathbf{i}^{(l-1)})^T, m^*]^T. \quad (16)$$

Let $\{\mathbf{D}^q_{\omega_j}(\mathbf{i}^{(l)})\}_{q=1}^{Q}$ be matrices of size $l \times P$ containing only the rows of $\mathbf{D}^q_{\omega_j}$ whose indices are specified by $\mathbf{i}^{(l)}$, i.e.,

$$\mathbf{D}^q_{\omega_j}(\mathbf{i}^{(l)}) = \mathbf{T}_\mathrm{s}(\mathbf{i}^{(l)}) \mathbf{D}^q_{\omega_j}, \quad (17)$$

where $\mathbf{T}_\mathrm{s}(\mathbf{i}^{(l)})$ is an $l \times M$ selection matrix. We compute the $l \times 1$ projection vectors of the desired beampattern over the chosen atoms up to the lth iteration for all $\omega_j \in \Omega$ as

$$\mathbf{h}^q_l(\omega_j) = (\mathbf{D}^q_{\omega_j})^{\#} \mathbf{b}_\mathrm{d}(\theta_\mathrm{ds} + \theta^q_\mathrm{ds}), \quad (18)$$

where $\mathbf{X}^{\#} = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H$ is the pseudo-inverse of $\mathbf{X}$. Then (18) is used to update the following vectors $\forall \omega_j \in \Omega$:

$$\mathbf{b}^{(l)}_{(\mathrm{g},q)}(\omega_j) = (\mathbf{D}^q_{\omega_j}(\mathbf{i}^{(l)}))^H \mathbf{h}^q_l(\omega_j), \quad (19)$$

and

$$\mathbf{r}^{(l)}_q(\omega_j) = \mathbf{r}^{(l-1)}_q(\omega_j) - \mathbf{b}^{(l)}_{(\mathrm{g},q)}(\omega_j). \quad (20)$$

Regarding the weighting matrices $\mathbf{W}$ and $\mathbf{R}$, we proposed the following heuristic way. The masking matrix $\mathbf{W}^{(l)}$ is updated by

$$\mathbf{W}^{(l)} = \mathbf{W}^{(l-1)} \mathrm{diag}(\mathbf{m}^T_l), \quad (21)$$

where $\mathbf{m}_l$ contains attenuation factors in indices of the neighboring sensors to the current selected one, and a zero entry for the index of the selected sensor. Let $g(m)$ be the index of the ring associated with the mth sensor. The elements of the matrix $\mathbf{R}^{(l)}$ are

$$[\mathbf{R}^{(l)}]_m = 1 + \sqrt{\frac{\text{no. of sensors from } \mathbf{i}^{(l-1)} \text{ on ring } g(m)}{M_{g(m)}}}. \quad (22)$$

As a stopping criterion, we check whether the chosen sensors up to the lth iteration comply with the constraints specified in the previous section. To do so, we may solve the following constrained optimization problem for each $\omega_j \in \Omega$ separately:

$$\begin{aligned} &\underset{\mathbf{h}_l(\omega_j)}{\text{minimize}} \ \|\mathbf{h}_l(\omega_j)\|_2^2 \\ &\text{subject to} \\ &\quad \mathbf{h}^H_l(\omega_j) \mathbf{d}_l(\omega_j, \theta^q_\mathrm{ds}) = 1 \\ &\quad \left\| (\mathbf{b}^\mathrm{m}_\mathrm{d}(\theta_\mathrm{ds} + \theta^q_\mathrm{ds}))^T - \mathbf{h}^H_l(\omega_j) \mathbf{D}^q_{l,\Theta_\mathrm{m}}(\omega_j) \right\|_2^2 \le \epsilon_1(\omega_j) \\ &\quad \left\| (\mathbf{b}^\mathrm{s}_\mathrm{d}(\theta_\mathrm{ds} + \theta^q_\mathrm{ds}))^T - \mathbf{h}^H_l(\omega_j) \mathbf{D}^q_{l,\Theta_\mathrm{s}}(\omega_j) \right\|_2^2 \le \epsilon_2(\omega_j), \end{aligned} \quad (23)$$

where $\mathbf{d}_l(\omega_j, \theta^q_\mathrm{ds}) = \mathbf{T}_\mathrm{s}(\mathbf{i}^{(l)}) \mathbf{d}(\omega_j, \theta^q_\mathrm{ds})$, $\mathbf{D}^q_{l,\Theta_\mathrm{m}}(\omega_j) = \mathbf{T}_\mathrm{s}(\mathbf{i}^{(l)}) \mathbf{D}_{M,\Theta_\mathrm{m}+\theta^q_\mathrm{ds}}(\omega_j)$, $\mathbf{D}^q_{l,\Theta_\mathrm{s}}(\omega_j) = \mathbf{T}_\mathrm{s}(\mathbf{i}^{(l)}) \mathbf{D}_{M,\Theta_\mathrm{s}+\theta^q_\mathrm{ds}}(\omega_j)$. If we get a valid solution to (23) and also $\|\mathbf{h}_l(\omega_j)\|_2^2 \le \gamma(\omega_j)$, $\forall \omega_j \in \Omega$ then we stop, else, we set $q \to (q+1) \bmod Q$, and repeat (15).

The OMP algorithm adds only one atom in each iteration, i.e., in a forward selection way. Still, previously selected atoms may be less significant than the current ones. Thus, after selecting several atoms, we choose the atom with the minimal contribution to the design and remove it from the selected group of atoms. This process is sometimes termed a forward-backward selection. Finally, after obtaining a solution to (23) for all frequencies and directions, the selected sensors, which are spread across a smaller number of rings, are used to build the FI and rotationally invariant beampattern.

4. A DESIGN EXAMPLE

We consider a sparse design of concentric circular differential microphone arrays (CCDMAs) originally designed with a linear geometry, which combines closely spaced sensors to respond to the spatial derivatives of the acoustic pressure field. These small-size arrays yield nearly FI beampatterns and include the well-known superdirective beamformer as a particular case [14]. The general theoretical FI beampattern of an Nth-order DMA is [31]

$$\mathcal{B}^{\theta_\mathrm{ds}}_\mathrm{d}(\theta) = \mathcal{B}_N(\theta_\mathrm{ds} - \theta) = \sum_{n=0}^{N} a_{N,n} \cos^n(\theta_\mathrm{ds} - \theta), \quad (24)$$

where $\{a_{N,n}\}_{n=0}^N$ are real coefficients, and the desired signal arrives from the direction θ_ds. The element spacing δ is required to be small enough so that $\delta \ll \lambda/2 = \frac{\pi c}{\omega}$, that is, $\delta \ll \frac{\pi c}{\omega_\mathrm{max}}$ for all $\omega \in \Omega$, where ω_max is the angular frequency corresponding to the highest frequency in the bandwidth of interest Ω, and $c = 340$ m/sec. Therefore, $\delta = 1$ cm is satisfactory for our example.

The initial array geometry consists of $G = 15$ rings, where $r_g = (g-1)\delta$, $g = 1, 2, ..., G$, and M_g sensors are spread uniformly across it. The total number of candidate sensors is $M = \sum_{g=1}^{G} M_g = 234$. We design a FI broadband beampattern for the range between $f_\mathrm{low} = 200$ Hz and $f_\mathrm{high} = 8200$ Hz. Assuming a typical duration of $T = 25$ msec for the window analysis used for the corresponding time-domain received signal, the frequency resolution is $\Delta f = 1/T = 40$ Hz. Thus, the number of bins is given by $J = (f_\mathrm{high} - f_\mathrm{low}) / \Delta f = 202$.

We design a third-order hypercardioid pattern (i.e., $N = 3$) which maximizes the directivity factor (DF), whose theoretical beampattern for $\theta_\mathrm{ds} = 0^\circ$ is given according to (24) by [11]

$$\mathcal{B}^\mathrm{HC}_N(\theta) = -0.14 - 0.57 \cos\theta + 0.57 \cos^2\theta + 1.15 \cos^3\theta. \quad (25)$$

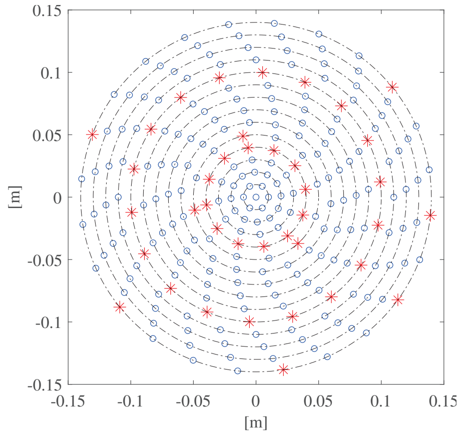
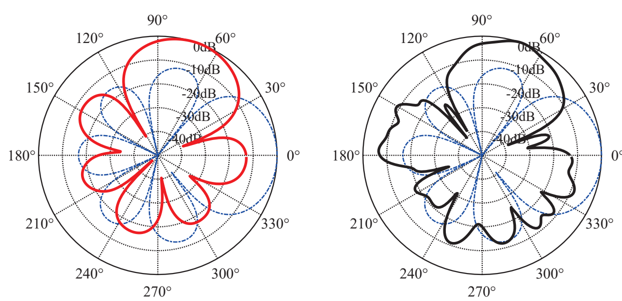


Figure 2: Array layout obtained by the proposed sparse design consisting of 39 sensors, spread over 4 rings. The blue circles are all the candidate sensors. The red stars are the selected sensors.

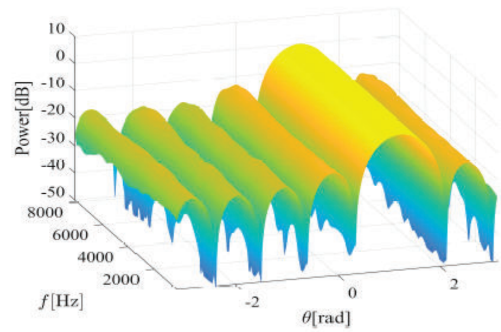


(a) greedy sparse design (b) uniform design

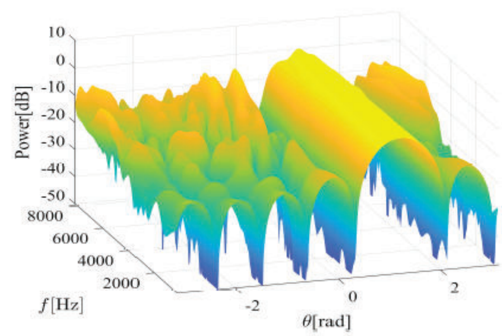
Figure 3: Beampatterns of a third-order hypercardioid for $f = 6280$ Hz, steered to $\theta_{\rm ds} = 70^\circ$ (red) obtained by the greedy sparse design. Also, we present the beampattern obtained by a uniform design (black line) where the 39 sensors are spread uniformly over the entire possible concentric aperture of the M candidate sensors, and the theoretical beampattern of a third-order hypercardioid (blue dotted line) steered to $\theta_{\rm ds} = 0^\circ$.

The angular axis is uniformly discretized with $\Delta\theta = 2^\circ$. For that case, we set $\theta_{P'} = 60^\circ$, i.e., the mainlobe region in the azimuthal axis is $-60^\circ \leq \theta \leq 60^\circ$. We set initial values for the tolerance parameters $\{\epsilon_1(\omega_j)\}_{j=1}^{J}$, $\{\epsilon_2(\omega_j)\}_{j=1}^{J}$, and $\{\gamma(\omega_j)\}_{j=1}^{J}$, by applying the parameters adjustment procedure introduced in [29].

We run the greedy algorithm proposed in Section 3. The resulting array layout, presented in Fig 2, consists of 39 sensors spread over 4 rings. One can see that the sparse design offers some compromise between high and low frequencies and rotationally invariant design. The FI beampatterns steered to $\theta_{\rm ds} = 0^\circ$(blue) and $\theta_{\rm ds} = 70^\circ$ (red), obtained by the proposed design are presented in Fig. 3. One can observe the rotation invariance property of the design. Also, we present a comparison to the case of a uniform design where the 39 sensors were spread uniformly over the entire possible concentric aperture of the M candidate sensors. Figure 4 shows that the beampattern versus frequency obtained by the sparse array is practically FI, especially in the mainlobe region. We also show the beampattern versus frequency obtained by the uniform design. It can be seen that the uniform design achieves a much higher sidelobes level, especially at higher frequencies.



(a) greedy sparse design



(b) uniform design

Figure 4: Beampatterns versus frequency for the examined design approaches.

Even though the proposed solution and the one presented in [30] are supposed to yield similar performance in terms of sparsity, robustness, FI, and rotation invariance, the proposed one is more efficient and elegant. Herein, determining the array layout involves a greedy search where only one additional sensor is selected in each iteration. In contrast, the solution in [30] consists of several steps, including solving iteratively an optimization problem with multiple constraints on the order of the number of frequency bins, J. This could be more exhaustive and resource-consuming, especially for a large number of candidate sensors, M.

5. CONCLUSION

We have presented a greedy sparse design approach for FI beamformers with CCAs, which perseveres almost the same directivity pattern for different frequencies and azimuthal steering directions using the same sparse CCA layout. The proposed design extends our recent work on the greedy sparse design of FI beamformers into a much simpler and elegant iterative greedy algorithm that solves a constrained optimization problem while incorporating several important desired properties for FI beamformers. Simulations show that the proposed design leads to a FI and rotationally invariant directivity pattern with reasonable computational complexity. Future research will focus on different array geometries and extensions to the volumetric arrays.

6. REFERENCES

[1] A. Frank and I. Cohen, "Constant-beamwidth kronecker product beamforming with nonuniform planar arrays," *Frontiers in Signal Processing*, vol. 2, no. 829463, pp. 1–17, May 2022.

[2] A. Kleiman, I. Cohen, and B. Berdugo, "Constant-beamwidth nonuniform concentric ring microphone arrays," *IEEE/ACM Trans. ASLP*, vol. 30, pp. 1952–1962, June 2022.

[3] G. Itzhak, J. Benesty, and I. Cohen, "Multistage approach for steerable differential beamforming with rectangular arrays," *Speech Communication*, vol. 142, pp. 61–76, July 2022.

[4] R. Sharma, I. Cohen, and B. Berdugo, "Controlling elevation and azimuth beamwidths with concentric circular microphone arrays," *IEEE/ACM Trans. ASLP*, vol. 29, pp. 1491–1502, 2021.

[5] S. Yan, "Optimal design of FIR beamformer with frequency invariant patterns," *Appl. Acoust.*, vol. 67, pp. 511–528, 2006.

[6] E. Mabande, A. Schad, and W. Kellermann, "Design of robust superdirective beamformer as a convex optimization problem," in *Proc. ICASSP*, 2009, pp. 77–80.

[7] S. C. Chan and H. H. Chen, "Uniform concentric circular arrays with frequency-invariant characteristics - theory, design, adaptive beamforming and doa estimation," *IEEE Trans. Signal Process.*, vol. 55, no. 1, pp. 165–177, Jan. 2007.

[8] M. Crocco and A. Trucco, "Design of robust superdirective arrays with a tunable tradeoff between directivity and frequency-invariance," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2169–2181, May 2011.

[9] W. Liu and S. Weiss, "Design of frequency invariant beamformers for broadband arrays," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 855–860, Feb. 2008.

[10] Y. Buchris, I. Cohen, and M. A. Doron, "Bayesian focusing for coherent wideband beamforming," *IEEE Trans. ASLP*, vol. 20, no. 4, pp. 1282–1295, May 2012.

[11] G. W. Elko, "Superdirectional microphone arrays," in *Acoustic Signal Processing for Telecommunication*, S. L. Gay and J. Benesty, Eds., chapter 10, pp. 181–237. Boston, MA: Kluwer Academic Publishers, 2000.

[12] M. Buck, "Aspects of first-order differential microphone arrays in the presence of sensor imperfections," *European Trans. Telecommunications*, vol. 13, pp. 115–122, Mar. 2002.

[13] E. De Sena, H. Hacihabibğlu, and Z. Cavetković, "On the design and implementation of higher order differential microphones," *IEEE Trans. ASLP*, vol. 20, no. 1, pp. 162–174, Jan. 2012.

[14] H. Cox, R. M. Zeskind, and T. Kooij, "Practical supergain," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, no. 3, pp. 393–397, June 1986.

[15] III J. H. Doles and F. D. Benedict, "Broad-band array design using the asymptotic theory of unequally spaced arrays," *IEEE Trans. Antennas Propagat.*, vol. 36, pp. 27–33, Jan. 1988.

[16] D. B. Ward, R. A. Kennedy, and R. C. Williamson, "Theory and design of broadband sensor arrays with frequency invariant farfield beam-patterns," *J. Acoust. Soc. Amer.*, vol. 97, no. 2, pp. 1023–1034, Feb. 1995.

[17] M. Crocco and A. Trucco, "Stochastic and analytic optimization of sparse aperiodic arrays and broadband beamformers with robust superdirective patterns," *IEEE Trans. ASLP*, vol. 20, no. 9, pp. 2433–2447, Nov. 2012.

[18] Z. Li, K. F. Cedric Yiu, and Z. Fen, "A hybrid descent method with genetic algorithm for microphone array placement design," *Appl. Soft Comput.*, vol. 13, pp. 1486–1490, 2013.

[19] B Preetham Kumar and GR Branner, "Generalized analytical technique for the synthesis of unequally spaced arrays with linear, planar, cylindrical or spherical geometry," *IEEE Trans. Antennas and Propagat.*, vol. 53, no. 2, pp. 621–634, 2005.

[20] Randy L Haupt, "Optimized element spacing for low sidelobe concentric ring arrays," *IEEE Trans. Antennas and Propagat.*, vol. 56, no. 1, pp. 266–268, 2008.

[21] Chunsheng Liu, Jin Cai, Zhongrui Huang, Xiaoping Wang, and Shiwa Chen, "Pattern synthesis of sparse concentric circular array based on the niche genetic algorithm," in *Proc. ICCSN*, 2017, pp. 906–911.

[22] Kesong Chen, Hui Chen, Ling Wang, and Honggang Wu, "Modified real ga for the synthesis of sparse planar circular arrays," *IEEE Antennas and Wireless Propagat. Lett.*, vol. 15, pp. 274–277, 2015.

[23] Narendra Nath Pathak, Gautam Kumar Mahanti, Shashank K Singh, Jitendra Kumar Mishra, and Ajay Chakraborty, "Synthesis of thinned planar circular array antennas using modified particle swarm optimization," *Progress In Electromagnetics Research*, vol. 12, pp. 87–97, 2009.

[24] U Singh and TS Kamal, "Synthesis of thinned planar concentric circular antenna arrays using biogeography-based optimisation," *IET Microwaves, Antennas & Propagat.*, vol. 6, no. 7, pp. 822–829, 2012.

[25] Li Zhang, Yong-Chang Jiao, Bo Chen, and Zi-Bin Weng, "Design of wideband concentric-ring arrays with three-dimensional beam scanning based on the optimization of array geometry," *Electromagnetics*, vol. 32, no. 6, pp. 305–320, 2012.

[26] Rui-Qi Wang and Yong-Chang Jiao, "Synthesis of wideband rotationally symmetric sparse circular arrays with multiple constraints," *IEEE Antennas and Wireless Propagat. Lett.*, vol. 18, no. 5, pp. 821–825, 2019.

[27] Micah D Gregory, Frank A Namin, and Douglas H Werner, "Exploiting rotational symmetry for the design of ultra-wideband planar phased array layouts," *IEEE Trans. Antennas and Propagat.*, vol. 61, no. 1, pp. 176–184, 2012.

[28] Yunhong Li, KC Ho, and Chiman Kwan, "3-d array pattern synthesis with frequency invariant property for concentric ring array," *IEEE Trans.signal processing*, vol. 54, no. 2, pp. 780–784, 2006.

[29] Y. Buchris, A. Amar, J. Benesty, and I. Cohen, "Incoherent synthesis of sparse arrays for frequency-invariant beamforming," *IEEE/ACM Trans. Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482–495, 2019.

[30] Y. Buchris, I. Cohen, J. Benesty, and A. Amar, "Joint sparse concentric array design for frequency and rotationally invariant beampattern," *IEEE Trans. ASLP*, vol. 28, pp. 1143–1158, April 2020.

[31] J. Benesty, J. Chen, and I. Cohen, *Design of Circular Differential Microphone Arrays*, Springer-Verlag, 2015.