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INCOHERENT SYNTHESIS OF SPARSE BROADBAND ARRAYS BASED ON A PARAMETER-FREE SUBSPACE CLUSTERING

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ABSTRACT

In this paper, we propose an incoherent design method of sparse broadband arrays that optimizes the number of sensors and their positions simultaneously. We introduce an iterative clustering procedure that merges different groups of sensors with a small distance, in terms of Bhattacharyya distance, between their angle distributions. The iterative clustering procedure is initialized with a large number of groups of sensors, and computes in each iteration a clustering score and a threshold. Then, near groups are merged into joint groups, yielding a new set of groups of sensors. We show that the optimal set of sensors is obtained when the clustering score is larger than the threshold, indicating that the remaining groups are distant. The proposed approach is demonstrated by a design of a superdirective beamformer, and its performance is compared with an existing incoherent approach. Experimental results show improved performance in terms of a more favorable tradeoff between directivity factor and white noise gain.

Index Terms— Sparse arrays, subspace clustering, frequency-invariant beamformers.

1. INTRODUCTION

Frequency-invariant (FI) broadband beamformers techniques are widely used in several real-world applications like audio, communications, and sonar systems [1–6]. Among several approaches for FI design, the sparse class comprises a promising concept since both the beamformer's coefficients and their locations are optimized. Consequently, sparse beamformers can maintain an adequate level of performance with fewer sensors, weight, size, and cost.

Previous works on FI sparse designs were based on analytical approaches [7,8], greedy algorithms such as genetic algorithms [9], and multidimensional searching algorithms [10] applied to find a global minimum of an appropriate cost function. Recently, a sparse design was proposed based on an ℓ_1 - norm constrained optimization [11, 12]. As the optimization is performed over all frequency bins simultaneously, we may refer to it as a coherent approach. Although exhibiting promising performance, the coherent approach carries a profound drawback of a high computational burden limiting its applicability in some scenarios. To overcome this problem, Buchris et al. [13, 14] introduced an incoherent approach, which solves separately an ℓ_1 constrained optimization problem for each frequency bin. Then, a fusion step is performed, yielding a joint-sparse selection of sensors. Finally, the selected sensors are used to synthesize the desired FI beamformer. In this way, a significant reduction in computation time is achieved. However, the optimal number of sensors in the sparse array's layout is determined empirically by trial and error, preventing such beamformers from being operated online.

This paper proposes a modified incoherent design that facilitates a built-in mechanism to retrieve the optimal number of sensors automatically. Specifically, we intervene in the fusion phase of the original approach by applying an iterative clustering procedure that merges different groups of sensors with a small distance, in terms of Bhattacharyya distance, between their angle distributions. The iterative clustering procedure is initialized with a large number of groups of sensors, and computes in each iteration a clustering score and a threshold. Then, near groups are merged into joint groups, yielding a new set of groups of sensors. The optimal set of sensors is obtained when the clustering score is larger than the threshold, indicating that the remaining groups are distant. The proposed approach is demonstrated by a design of a superdirective beamformer, and its performance is compared with the original incoherent method. Experimental results show that the proposed technique achieves the same number of optimal sensors as the former design. Still, the positions of the sensors are different, which yields a performance improvement in terms of a more favorable tradeoff between directivity factor (DF) and white noise gain (WNG).

The rest of this paper is organized as follows. In Section 2, we formulate the problem. The design constraints are given in Section 3. Section 4 describes our proposed modification in detail. A design example and simulations are provided in Section 5, and conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

Consider a linear array with M possible candidate sensor positions. We denote the position of the mth sensor by $\mathrm{p}_m, m = 1, 2, ..., M$, and by $\mathcal{B}_\mathrm{d}(\theta)$ the desired far-field FI beampattern in the bandwidth of interest Ω, and in azimuth θ. The beampattern of such an array for the angular frequency ω is generally defined as

$$\mathcal{B}\big(\mathbf{h}(\omega)\big) = \mathbf{h}^H(\omega)\mathbf{d}\big(k_\omega(\theta)\big), \tag{1}$$

where the superscript $(\ )^H$ denotes the conjugate-transpose operator,

$$\mathbf{h}(\omega) = [H_1(\omega), H_2(\omega), ..., H_M(\omega)]^T \tag{2}$$

is a vector containing the beamformer complex gains, and the superscript $(\ )^T$ stands for the transpose operator. The wavenumber at frequency ω and direction θ is

$$k_\omega(\theta) = -\frac{\omega}{c}\cos\theta, \tag{3}$$

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and is associated with the following $M \times 1$ steering vector

$$\mathbf{d}\big(k_\omega(\theta)\big) = \left[e^{-jk_\omega(\theta)\mathrm{p}_1}, e^{-jk_\omega(\theta)\mathrm{p}_2}, ..., e^{-jk_\omega(\theta)\mathrm{p}_M}\right]^T, \quad (4)$$

where $j = \sqrt{-1}$, and c is the waveform's speed.

Assume that we select a subset of $K \ll M$ positions $\{\mathrm{p}_{i_k}\}_{k=1}^K$, with indices $\{i_k\}_{k=1}^K \in [1, 2, ..., M]$. Then the beampattern becomes

$$\mathcal{B}\big(\mathbf{h}(\omega), \mathbf{i}_K\big) = \mathbf{h}^H(\omega)\mathbf{T}_\mathrm{s}(\mathbf{i}_K)^T\mathbf{T}_\mathrm{s}(\mathbf{i}_K)\mathbf{d}\big(k_\omega(\theta)\big), \quad (5)$$

where $\mathbf{i}_K = [i_1, i_2, ..., i_K]^T$, and $\mathbf{T}_\mathrm{s}(\mathbf{i}_K)$ is a $K \times M$ selection matrix, i.e., containing K rows, composed of $1 \times M$ unit vectors corresponding to the indices $\{i_k\}_{k=1}^K$. Our goal is to select the optimal number of sensors, K, and their positions, $\mathbf{i}_K$, out of the M candidate positions, such that the synthesized beampattern $\mathcal{B}\left(\mathbf{h}(\omega), \mathbf{i}_K\right)$ will be as close as possible to the desired beampattern $\mathcal{B}_\mathrm{d}(\theta)$, $\forall \omega \in \Omega$, but at the same time fulfilling several design constraints that will be discussed in the next section.

3. DESIGN CONSTRAINTS

Given a desired beampattern $\mathcal{B}_\mathrm{d}(\theta)$, let

$$\mathbf{b}_\mathrm{d}^\mathrm{m} = [\mathcal{B}_\mathrm{d}(\theta_1), \mathcal{B}_\mathrm{d}(\theta_2), ..., \mathcal{B}_\mathrm{d}(\theta_L)]^T \quad (6)$$

denote a vector containing the desired beampattern in L out of P directions that cover the mainlobe region, and let

$$\mathbf{b}_\mathrm{d}^\mathrm{s} = [B_\mathrm{d}(\theta_{L+1}), B_\mathrm{d}(\theta_{L+2}), ..., B_\mathrm{d}(\theta_P)]^T \quad (7)$$

denote a vector containing the desired beampattern in $P - L$ directions that cover the sidelobes region. Let

$$\begin{aligned}\mathbf{D}(\mathbf{k}_\omega^\mathrm{m}) &= [\mathbf{d}(k_\omega(\theta_1)), \mathbf{d}(k_\omega(\theta_2)), ..., \mathbf{d}(k_\omega(\theta_L)]\\ \mathbf{D}(\mathbf{k}_\omega^\mathrm{s}) &= [\mathbf{d}(k_\omega(\theta_{L+1})), \mathbf{d}(k_\omega(\theta_{L+2})), ..., \mathbf{d}(k_\omega(\theta_P)]\end{aligned} \quad (8)$$

be sets of steering vectors related to each of the directions that cover the mainlobe and the sidelobe regions, respectively, where

$$\begin{aligned}\mathbf{k}_\omega^\mathrm{m} &= \{k_\omega(\theta_1), k_\omega(\theta_2), ..., k_\omega(\theta_L)\}\\ \mathbf{k}_\omega^\mathrm{s} &= \{k_\omega(\theta_{L+1}), k_\omega(\theta_{L+2}), ..., k_\omega(\theta_P)\}.\end{aligned} \quad (9)$$

are their corresponding set of wavenumber values.

Then, several design constraints intended to ensure robust and FI broadband beampattern, can be formulated $\forall \omega \in \Omega$ by [13]

$$\begin{aligned}
\mathcal{C}_1 :&\quad ||(\mathbf{b}_\mathrm{d}^\mathrm{m})^T - \mathbf{h}^H(\omega)\mathbf{T}_\mathrm{s}^T(\mathbf{i}_K)\mathbf{T}_\mathrm{s}(\mathbf{i}_K)\mathbf{D}(\mathbf{k}_\omega^\mathrm{m})||_2^2 \leq \epsilon_1(\omega),\\
\mathcal{C}_2 :&\quad ||(\mathbf{b}_\mathrm{d}^\mathrm{s})^T - \mathbf{h}^H(\omega)\mathbf{T}_\mathrm{s}^T(\mathbf{i}_K)\mathbf{T}_\mathrm{s}(\mathbf{i}_K)\mathbf{D}(\mathbf{k}_\omega^\mathrm{s})||_2^2 \leq \epsilon_2(\omega),\\
\mathcal{C}_3 :&\quad \mathbf{h}^H(\omega)\mathbf{T}_\mathrm{s}^T(\mathbf{i}_K)\mathbf{T}_\mathrm{s}(\mathbf{i}_K)\mathbf{d}\big((k_\omega(\theta_s)\big) = 1,\\
\mathcal{C}_4 :&\quad \mathbf{h}^H(\omega)\mathbf{T}_\mathrm{s}^T(\mathbf{i}_K)\mathbf{T}_\mathrm{s}(\mathbf{i}_K)\mathbf{h}(\omega) \leq \gamma(\omega),
\end{aligned}$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are the mainlobe and sidelobes constraints which are used to obtain the desired FI beampatterns in each of the mainlobe and sidelobes regions in a least-square (LS) error sense. $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ are small positive tolerance parameters indicating the overall allowed error for $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively. $\mathcal{C}_3$ is the common distortionless response constraint, and $\mathcal{C}_4$ imposes a limitation on the maximal allowed white noise output power at frequency ω using the parameter $\gamma(\omega)$.

The optimization problem is formulated as follows:

$$\begin{aligned}&\text{minimize } \{\text{number of active sensors - } K\}\\ &\text{subject to } \mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4, \forall \omega \in \Omega\end{aligned} \quad (10)$$

whose solution yields the jointly-sparse filters

$$\mathbf{h}_K(\omega) = \mathbf{T}_\mathrm{s}(\mathbf{i}_K)\mathbf{h}(\omega), \forall \omega \in \Omega. \quad (11)$$

An efficient solution to (10) was proposed in [13] based on an incoherent approach. It comprises four steps, including analysis, dimensionality reduction, clustering, and synthesis. Yet, it lacks an intelligent way to determine K a priori. In the next section, we introduce a reliable and efficient way to determine K by modifying the clustering step. We focus on that step while briefly overviewing the other steps.

4. INCOHERENT SPARSE BROADBAND DESIGN

Let $\{\omega_j\}_{j=1}^J \in \Omega$ denote a set of frequency bins. Then, the first analysis step solves an ℓ_1 optimization problem under constraints similar to $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4\}$ for each frequency ω_j. The result is an $M \times J$ analysis matrix

$$\mathbf{H}_\mathrm{A} = [\mathbf{h}_\mathrm{A}(\omega_1), \mathbf{h}_\mathrm{A}(\omega_2), ..., \mathbf{h}_\mathrm{A}(\omega_J)], \quad (12)$$

where $\mathbf{h}_\mathrm{A}(\omega_j)$ is the solution of the optimization problem for ω_j (see details in [13]). Due to the sparsity nature of $\mathbf{H}_\mathrm{A}$, we can apply principle component analysis (PCA) [15], to reduce its dimensionality and form an $M \times Q$ ($Q \ll J$) compact representative matrix

$$\mathbf{H}_\mathrm{R} = \overline{\mathbf{H}}_\mathrm{A} \cdot \overline{\mathbf{U}}, \quad (13)$$

where $\overline{\mathbf{H}}_\mathrm{A}$ is a centralized version of $\mathbf{H}_\mathrm{A}$, i.e., the sampled mean of each column of $\mathbf{H}_\mathrm{A}$ has been shifted to zero. The subscript R stands for reduced, and the matrix $\overline{\mathbf{U}} \in \mathbb{R}^{J \times Q}$ contains Q eigenvectors corresponding to the Q largest eigenvalues of the sampled correlation matrix of $\mathbf{H}_\mathrm{A}$ defined by:

$$\mathbf{R}_\mathrm{A} = \mathbf{H}_\mathrm{A}^H\mathbf{H}_\mathrm{A} \quad (14)$$

Specifically, we pick the first Q largest eigenvalues which satisfy

$$\Sigma_Q = \frac{\Sigma_{j=1}^Q \lambda_j}{\Sigma_{j=1}^J \lambda_j} \leq \Sigma, \quad (15)$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_J$ are the eigenvalues of the analysis correlation matrix $\mathbf{R}_\mathrm{A}$ and Σ is a parameter that determines the portion of the overall variability in $\mathbf{H}_\mathrm{A}$ to be preserved in $\mathbf{H}_\mathrm{R}$, where its value is typically set between $0.6 \leq \Sigma \leq 0.9$.

The reduced matrix, $\mathbf{H}_\mathrm{R}$, is used as an input to a clustering step, which returns K clusters, from which one representative sensor is selected. To determine K, we present in the following subsections an alternative approach based on [16], which offers a parameter-free clustering tool that extracts the optimal number of clusters automatically. This approach relies on the assumption that the distributions of angles subtended between sensors are distinct in each cluster. Note that there are several heuristic methods that deal with the problem of finding an optimal number of clusters in a dataset [17, 18]. Yet, most of them are domain-dependent and strictly rely on suitable parameters adjustment.

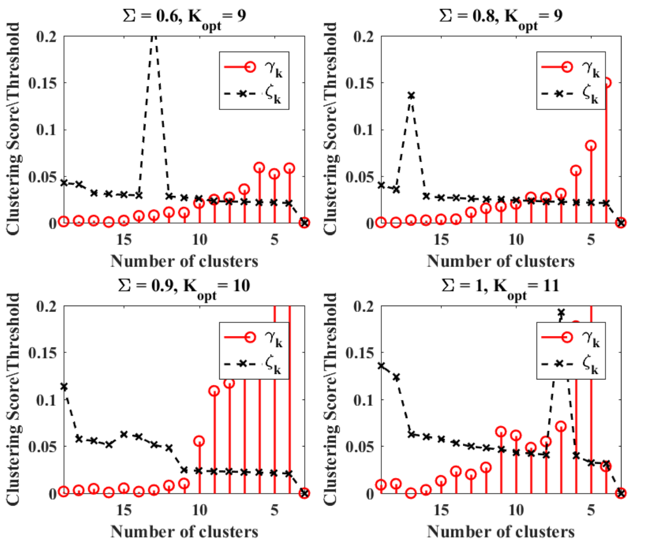

Fig. 1. Clustering score γ_K and threshold ζ_K for different Σ values.

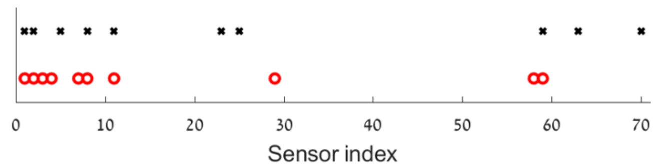

Fig. 2. Array layout obtained by the original incoherent approach (black 'x') and the modified version (red circles).

4.1. Clustering of Angles Subtended Between Sensors

Let $\mathbf{x}_m \in \mathbb{R}^{1\times Q}$ denote the normalized version of the sensors' reduced analysis vectors calculated $\forall m = \{1, 2, ..., M\}$ by:

$$\mathbf{x}_m = \frac{\mathbf{H}_{\mathrm{R}}(m, :)}{||\mathbf{H}_{\mathrm{R}}(m, :)||_2^2}. \tag{16}$$

Then, clustering is performed over the angles unfolded by sensors' pairs (i.e., $\mathbf{x}_m$ and $\mathbf{x}_n$, $m \neq n$) and are calculated according to

$$\theta_{m,n} = \cos^{-1}(\mathbf{x}_m^T \mathbf{x}_n), \tag{17}$$

where $\theta_{m,n} \in [0, \pi]$ is the angle subtended between sensor m and sensor n. These angles are first clustered across a large number of groups according to a given initialization process. Herein, we consider the k-means algorithm to yield the initial clusters.

4.2. Clustering Score

Let $\mathcal{C}_K = \{I_1, I_2, ..., I_K\}$ denote K clusters of datasets, where I_c's are mutually disjoint index sets with $U_{c=1}^K\ I_c = \{1, 2, ..., M\}$. We refer to I_c's as constituent clusters. For each group in $\mathcal{C}_K$ we calculate the angles between all possible data-point pairs to produce a set of approximately Gaussian within-cluster angle distributions, $\{\mathcal{W}_c\}_{c=1}^K$, i.e.,

$$\mathcal{W}_c = \{\theta_{m,n} | m, n \in I_c, m < n\} \sim \mathcal{N}(\mu_{w_c}, \sigma_{w_c}^2), \tag{18}$$

where μ_{w_c} and $\sigma_{w_c}^2$ are, respectively, the sample mean and sample variance of $\mathcal{W}_c$.

Similarly, we calculate the angles subtended by data-points between different clusters to establish a set of between-clusters I_c and I_l angles distributions $\{\mathcal{B}_{cl} | c, l = 1, \ldots, K,\ c \neq l\}$ characterized by a Gaussian distribution as well:

$$\mathcal{B}_{cl} = \{\theta_{m,n} | m \in I_c, n \in I_l\} \sim \mathcal{N}(\mu_{b_{cl}}, \sigma_{b_{cl}}^2) \tag{19}$$

where $\mu_{b_{cl}}$ and $\sigma_{b_{cl}}^2$ are, respectively, the sample mean and sample variance of the distribution elements between clusters I_c and I_l.

We evaluate the distance of within-cluster $\mathcal{W}_c$ and between-clusters $\mathcal{B}_{cl}$ distributions using a well-known distributions distance measure called the Bhattacharyya distance [19]:

$$d_{cl} = \frac{1}{4}\left[\frac{(\mu_{w_c} - \mu_{b_{cl}})^2}{\sigma_{w_c}^2 + \sigma_{b_{cl}}^2} + \log_e\left(\frac{1}{4}\left[\frac{\sigma_{w_c}^2}{\sigma_{b_{cl}}^2} + \frac{\sigma_{b_{cl}}^2}{\sigma_{w_c}^2}\right] + \frac{1}{2}\right)\right] \tag{20}$$

where small values of d_{cl} imply two distributions suspected to share the same subspace, and larger Bhattacharyya distances indicate that the clusters are separate. The clustering score is defined as the value dictated by the minimum possible distance:

$$\eta_c = \min_{l=1,\ldots,K,\ l\neq c} \quad d_{cl} \tag{21}$$

and

$$\gamma_K = \min_{c=1,\ldots,K} \quad \eta_c \tag{22}$$

where η_c stands for the score of cluster I_c, and γ_K is the overall score of $\mathcal{C}_K$. We refer to the two constituent clusters that produce the clustering score as a mergeable pair of $\mathcal{C}_K$.

4.3. Threshold

A theoretical derivation of a threshold value has been presented in [16]. Under a Gaussian assumption on the nature of distributions of angles, it has been shown that the clustering score holds $\gamma_K \leq \zeta_K$, as long as there are at least two groups in the current clustering $\mathcal{C}_K$ that share the same subspace. The threshold ζ_K is strictly related to the number of independent angles in the mergeable pair of $\mathcal{C}_K$, i.e., the number of angles within the cluster that formed by merging the mergeable pair of $\mathcal{C}_K$. Let T_K denote the number of elements in the merged cluster of $\mathcal{C}_K$. Hence, the threshold ζ_K can be obtained by

$$\zeta_K = \frac{1}{\sqrt{T_K - 1}}, \tag{23}$$

4.4. Clustering Algorithm

We start with a large number of initial clusters $K_{\mathrm{init}} \gg K_{\mathrm{opt}}$. The algorithm then runs iteratively from $K = K_{\mathrm{init}}$ to $K = 2$, where in each iteration, the clustering score γ_K and the threshold ζ_K are calculated using (22) and (23), respectively. The corresponding mergeable pairs are merged into one cluster yielding a new set of clusters $\mathcal{C}_{K-1}$ that is passed to the next iteration. In this way, we expect to obtain clustering scores around zero for $\mathcal{C}_{K_{\mathrm{init}}}, \mathcal{C}_{K_{\mathrm{init}-1}}, ..., \mathcal{C}_{K_{\mathrm{opt}+1}}$, and observing a sharp increase when

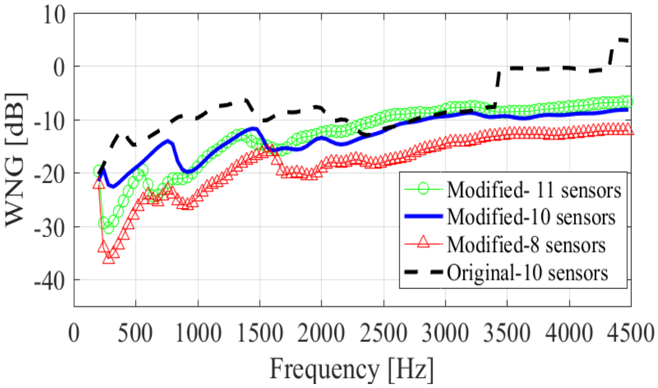

Fig. 3. WNG for the proposed modified incoherent version using 8 sensors (red triangles line), 10 sensors (solid blue line), 11 sensors (green circles line), and for the original approach using 10 sensors (black dashed line).

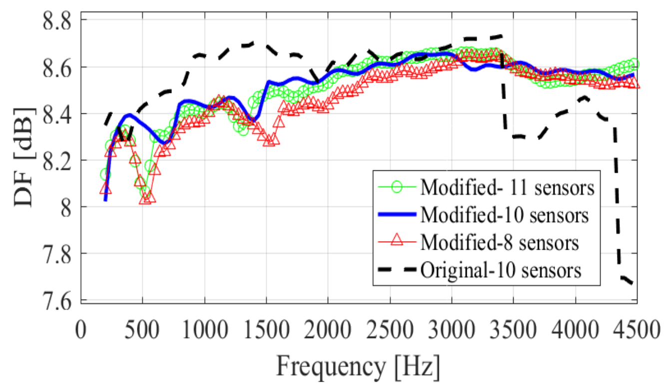

Fig. 4. DF for the proposed modified incoherent version using 8 sensors (red triangles line), 10 sensors (solid blue line), 11 sensors (green circles line), and for the original approach using 10 sensors (black dashed line).

reaching the optimal number of clusters K_{opt}, since no more mergeable pairs are left. From that point to $K = 2$, the scoring behavior is unpredictable as the remaining clusters are mixed subspaces. The optimal number of clusters is determined automatically when the clustering score crosses the threshold for the first time (i.e., when $\gamma_{K_{\text{opt}}} > \zeta_{K_{\text{opt}}}$). Ultimately, in each cluster within $\mathcal{C}_{K_{\text{opt}}}$, we pick one data point which is closest to the cluster centroid in terms of Euclidean distance. This way, the most significant sensors are selected to form the jointly-sparse support vectors, $\mathbf{h}_{K_{\text{opt}}}(\omega_j), \forall \omega_j \in \Omega$.

5. DESIGN EXAMPLE

We consider the design of a superdirective beamformer, i.e, a beamformer that maximizes the DF when steered to the endfire [20]. We apply the original incoherent approach and our modified version to design an FI broadband beampattern for a range of frequencies between $f_{\text{low}} = 200\text{Hz}$ and $f_{\text{high}} = 4480\text{Hz}$. We consider a typical frequency resolution of $\Delta f = 40\text{Hz}$ comprising $J = \frac{f_{\text{high}} - f_{\text{low}}}{\Delta f} = 108$ frequency bins, and a typical waveform's speed of $c = 340\text{m/s}$. We set an initial array of $M = 70$ candidate microphones, with an element spacing of $\delta = 1\text{cm}$. For the modified version we use k-means to produce $K_{\text{init}} = 20$ initial clusters that serve as an input to the proposed clustering scheme. The beamformer's induced WNG and DF are calculated by [21]

$$\mathcal{W}(\mathbf{h}(\omega_j)) = \frac{\left|\mathbf{h}^H(\omega_j)\mathbf{d}\big((k_{\omega_j}(\theta)\big)\right|^2}{\mathbf{h}^H(\omega_j)\mathbf{h}(\omega_j)} \tag{24}$$

and

$$\mathcal{D}(\mathbf{h}(\omega_j)) = \frac{\left|\mathbf{h}^H(\omega_j)\mathbf{d}\big((k_{\omega_j}(\theta)\big)\right|^2}{\mathbf{h}^H(\omega_j)\mathbf{\Gamma}_{\text{dn}}\mathbf{h}(\omega_j)}, \tag{25}$$

where

$$[\mathbf{\Gamma}_{\text{dn}}(\omega_j)]_{uv} = \text{sinc}\left(\frac{\omega_j \delta}{c}(v-u)\right) \tag{26}$$

is an $M \times M$ pseudo-coherence matrix of the diffuse noise field.

We test the performance of our proposed approach across the following dimensionality reduction factors: $\Sigma = 0.6, 0.8, 0.9, 1$, where $\Sigma = 1$ represents an attempt to skip the dimensionality reduction step.

Figure 1 demonstrates the performance of the proposed method for different values of Σ. As discussed, the optimal number of clusters is determined when the clustering score γ_K intersects with the empirical threshold ζ_K for the first time. One can see that for $\Sigma = 0.9$, our method coincides with the analytical solution of the coherent approach in terms of the optimal number of sensors, that is, $K_{\text{opt}} = 10$ [13]. Moreover, in this case, the algorithm yields the sharpest increase at the optimal point, indicating the best clustering performance among all depicted candidates. Figure 2 shows the arrays layouts obtained by the original incoherent approach (black 'x') and the modified proposed approach (red circles). We notice that the modified design yielded a selection of sensors that spread over a smaller aperture. In particular, 7 out of 10 sensors are centered in the first 11 positions.

Figures 3 and 4 show the WNG and DF for both designs. For the modified version, we observe that a favorable tradeoff between WNG and DF is obtained for the optimal number of clusters $K_{\text{opt}} = 10$. Specifically, performance is spread more equally than the original approach, especially for higher frequencies.

6. CONCLUSIONS

We have presented a modified incoherent sparse design of FI beamformers that optimizes the number of sensors in the array. In addition, an iterative clustering methodology fuses sensors' groups that share similar angle distributions. Simulations comparing the proposed and the original incoherent designs show that both implementations achieve a good compromise between DF and WNG with a preference to the modified one, whose performance appears to be more stable across frequencies. This characteristic is reflected in an array layout with a smaller aperture. Hence, we conclude that observing the angles subtended between data points better reveals the inter-connections structure between sensors in large arrays. However, despite the encouraging results, for our approach to enabling beamformers to be operated online, its robustness to the clustering initialization should be further tested. The design of a suitable initialization technique is a subject for future work.

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