WINDOW BEAMFORMER FOR SPARSE CONCENTRIC CIRCULAR ARRAY

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ABSTRACT

This work proposes a practical fixed beamformer for a sparse concentric circular array (CCA) of microphones. Firstly, a method is proposed which enables the reduction of the number of microphones in a CCA based on the optimal placement of the microphones to maximize the directivity-factor (DF). Thereafter, a beamformer is derived which strives to obtain a beamwidth that is not smaller than a desired target beamwidth (in both the elevation and azimuth) without neglecting the DF and white-noise-gain (WNG) across the speech spectrum. The proposed beamformer weights the microphones based on their corresponding ring number or radius, and further uses a Gaussian-window function to emphasize the microphones which are aligned with the direction-of-arrival of the signal. The results showcase the superiority of the proposed Gaussian-window (GW) beamformer over known solutions.

***Index Terms*—** sparse CCA, control elevation and azimuth beamwidths, DF and WNG.

1. INTRODUCTION

A practical beamformer should have a 3D-beampattern that is not-too-narrow in both the elevation and azimuth planes to provide some tolerance against any misestimation of the direction-of-arrival (DOA) of the signal-of-interest. At the same time, the beamwidths should not be too large so that interferences from other directions are inhibited. To avoid signal-distortion, an ideal beamformer should provide the desired beampattern characteristics uniformly across the entire frequency spectrum, which is the goal of frequency-invariant or constant-beamwidth beamforming [1–14]. This is of particular significance for speech signals, since they contain a multitude of information embedded across their entire spectrum, which could be useful for different target applications ranging from speech and speaker recognition to health and emotion analysis [15, 16].

Unfortunately, for arrays that are not 3-D, controlling the beampattern in the elevation plane is a challenging task. As such, known solutions for frequency-invariant or constant-beamwidth beamforming for the concentric circular array (CCA), which is a 2-D array, strive to control the azimuth beampattern or beamwidth only [5–12, 17, 18]. Such solutions are generally confined to the signal-of-interest being in the horizontal plane and have constraints regarding the number of sensors in the rings of the CCA and the presence of the central sensor. Henceforth, in our recent work, we proposed a methodology for controlling both the elevation and azimuth beamwidths of any arbitrary CCA and any arbitrary DOA [19]. The proposed Kaiser-window (KW) beamformer, which is a fixed beamformer, also ensured that high levels of the directivity-factor (DF) and white-noise-gain (WNG) are maintained for the majority of the speech spectrum [19].

This work follows our recent work, with an emphasis on sparse CCA. Firstly, we propose a methodology to design a sparse array by optimally placing the microphones in the rings of the CCA to maximize the DF. The proposed sparse array results in low-frequency beamformer characteristics which are near-identical to that of the standard uniform CCA. Secondly, we introduce some modifications to the process used to derive the KW beamformer. These modifications ensure that at high frequencies better beamwidth-control and high levels of DF and WNG can be achieved, particularly for the sparse CCA.

The rest of this work is organized as follows: Sec. 2 presents the beamforming framework and the sparse CCA design. Sec. 3 describes the proposed beamforer. Sec. 4 presents the results, and Sec. 5 summarizes this work.

2. SIGNAL MODEL AND ARRAY DESIGN

Consider a CCA of P concentric rings. The p^{th} ring of the CCA has a radius, r_p, and consists of $M_p = 2K_p$ microphones. The microphones are uniformly distributed on the ring, with an angular separation of $\phi_p = 2\pi/M_p$. The positions of the sensors are given by $m_p \in [-K_p + 1,\ K_p]$. Consider also a microphone at the center of the CCA, which may be considered as the 0^{th} ring of radius, $r_0 = 0$, and $M_0 = 1,\ m_0 = 0$. A discrete-time broadband speech signal-of-interest, $x(t)$, where t denotes the discrete-time index, impinges on the CCA at an elevation, θ_{d}, and an azimuth, ϕ_{d}, as a plane-wave in the far-field, traveling at the velocity of

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sound, $c \approx 340$ ms^{-1}, through the air-medium. The signal-of-interest detected by the CCA may be represented as:

$$\begin{aligned}\mathbf{x}(f) &= [\mathbf{x}_0^T(f),\ \mathbf{x}_1^T(f),\ ...,\ \mathbf{x}_P^T(f)]^T \\ &= [...,\ \mathbf{d}_p^T(f,\theta_\text{d},\phi_\text{d}),\ ...]^T X(f) \\ &= \mathbf{d}(f,\theta_\text{d},\phi_\text{d})X(f),\ x(t) \leftrightarrow X(f), \quad (1)\\ \mathbf{d}_p(f,\theta,\phi) &= [....,\ d_{p,m_p}(f,\theta,\phi),\ ...]^T, \\ d_{p,m_p}(f,\theta,\phi) &= e^{j2\pi f \frac{r_p}{c}\sin\theta\cos(\phi-\phi_{p,m_p})}.\end{aligned}$$

Let $M = \sum_p M_p$ denote the total number of sensors. An M-dimensional beamformer, $\mathbf{h}(f)$, is applied on the received noisy-data to emphasize the signal corresponding to the steering-vector, $\mathbf{d}(f,\theta_\text{d},\phi_\text{d})$, and reject interferences/noise from other directions/sources. The 3-D beampattern, DF, and WNG of the beamformer represent this capability [1, 2, 19]:

$$\mathcal{B}(f,\theta,\phi) = \mathbf{h}^H(f)\mathbf{d}(f,\theta,\phi),\ \theta,\phi \in [-\pi,\pi]\ ; \quad (2)$$

$$\begin{aligned}\mathcal{D}(f) &= \frac{|\mathbf{h}^H(f)\mathbf{d}(f,\theta_\text{d},\phi_\text{d})|^2}{\mathbf{h}^H(f)\Gamma(f)\mathbf{h}(f)}, \\ \Gamma(f)|_{i,j} &= \text{sinc}\big(2\pi f l_{i,j}/c\big),\ 1 \le\ i,j\ \le M\ ;\end{aligned} \quad (3)$$

$$\mathcal{W}(f) = \frac{|\mathbf{h}^H(f)\mathbf{d}(f,\theta_\text{d},\phi_\text{d})|^2}{\mathbf{h}^H(f)\mathbf{h}(f)}. \quad (4)$$

In equation (3), $\Gamma(f)|_{i,j}$ represents the $(i,j)^\text{th}$ position of the matrix, $\Gamma(f)$, and $l_{i,j}$ represents the Euclidean distance between the two microphones corresponding to that position [11, 12, 19]. The 3-dB elevation-beamwidth, $b_\theta(f)$, is the spread of θ within which $|\mathcal{B}(f,\theta,\phi_\text{d})\ /\ \mathcal{B}(f,\theta_\text{d},\phi_\text{d})| \le 0.5$. Similarly, the 3-dB azimuth-beamwidth, $b_\phi(f)$, is the spread of ϕ within which $|\mathcal{B}(f,\theta_\text{d},\phi)\ /\ \mathcal{B}(f,\theta_\text{d},\phi_\text{d})| \le 0.5$. Our objective is to derive a beamformer which achieves a minimum target-elevation-beamwidth, θ_BW, and a minimum target-azimuth-beamwidth, ϕ_BW, across the speech spectrum, while maintaining a robust DF and WNG.

Consider the speech sampling rate as $F_s = 16$ kHz. Hence, the maximum frequency is $F_\text{max} = F_s/2 = 8$ kHz, and the minimum wavelength is $\lambda_\text{min} = c/F_\text{max} = 2.125$ cm. According to the Nyquist criterion, to avoid spatial aliasing, the sensors must be separated by a distance, $\delta \le \lambda_\text{min}/2$. The number of microphones in the rings is given by [5, 6]

$$\begin{aligned}M_p &= \left\lfloor \pi/\sin^{-1}\big(\frac{\lambda_\text{min}}{4r_p}\big)\right\rceil,\ p = 1,2,...,P, \\ M_p &:= M_p + 1,\ M_p\ \text{ is odd}.\end{aligned} \quad (5)$$

Thus, for a standard CCA with $r_p = \{0,\ 5,\ 10,\ 15,\ 20\}$ cm, $M_p = \{1,\ 16,\ 30,\ 44,\ 60\}$, with a total of $M = 151$ microphones. It is natural to strive to obtain the desired performance with fewer microphones. With this objective, we devise a simple methodology that takes advantage of the relationship of the DF with the inter-sensor distances (hence the microphones' positions). The broadband DF for any given frequency-range, $[F_1, F_2]$, is given by

$$\mathcal{D}_\text{bb} = \frac{\int_{F_1}^{F_2} |\mathbf{h}^H(f)\mathbf{d}(f,\theta_\text{d},\phi_\text{d})|^2 df}{\int_{F_1}^{F_2} \mathbf{h}^H(f)\Gamma(f)\mathbf{h}(f)df}. \quad (6)$$

In this work, we choose $\{F_1 = 0,\ F_2 = 2\ \text{kHz}\}$, $\{F_1 = 2\ \text{kHz},\ F_2 = F_s/2\}$, and $\{F_1 = 0,\ F_2 = F_s/2\}$ to obtain the low-frequency, high-frequency, and full-spectrum DFs, respectively, denoted as $\mathcal{D}_\text{lf}$, $\mathcal{D}_\text{hf}$, and $\mathcal{D}$, respectively. The bifurcation frequency is considered as 2 kHz since the pitch and the principal vocal tract resonances of speech are generally contained within 2 kHz [15, 16].

Consider that only the central microphone is present in the CCA, and all other microphones in the rings are removed. We distribute the remaining $M-1$ microphones in pairs over $(M-1)/2$ iterations. For the n^th $(1 \le n \le \frac{M-1}{2})$ iteration:

(a) Hypothetically place two additional microphones in the p^th $(p = 1,2,...,P)$ ring. The number of microphones in the CCA is $M_n = 1+2n$. The number of microphones in the ring becomes $M_p := M_p + 2$, which are uniformly separated. We consider the broadside DOA ($\theta_\text{d} = 0^o$) and apply the delay-and-sum (DS) beamformer:

$$\mathbf{h}(f) = \frac{1}{M_n}\mathbf{d}(f,\theta_\text{d},\phi_\text{d}) = \frac{1}{M_n}[1,...,1,...1]^T. \quad (7)$$

This is the simplest and most unbiased case of beamforming: all the microphones are given equal emphasis ($1/M_n$), and the received signals are simply added.

(b) Determine the full-spectrum DF, $\mathcal{D}_p,\ p = 1,2,...,P$, for the placement of the microphone-pair in each of the rings. The optimal ring for the pair of microphones is $p_\text{opt} = \underset{p:\ 1\ \le\ p\ \le P}{\text{argmax}}\ \mathcal{D}_p$. Update the CCA design by placing the sensors on the optimal (p_opt) ring.

We subject the CCA of 151 microphones to the aforementioned process and track the three broadband DFs as a microphone-pair is placed in its optimal ring after each iteration. Fig. 1 shows the broadband DFs as a function of the number of microphones optimally placed in the CCA. As is evident, the full-spectrum DF saturates long before all the microphones are placed. The low-frequency DF saturates even earlier, and in fact, shows marginal degradation as the process continues. Only the high-frequency DF continues to increase, though, after about 51 microphones, the curve flattens out relatively. Thus, we may consider only the optimally distributed first 51 microphones (or even fewer) as the sparse CCA design, which we denote as CCA-I. The CCA-I has rings with $r_p = \{0,\ 10,\ 15,\ 20\}$ cm, with $M_p = \{1,\ 8,\ 10,\ 32\}$. None of the 51 sensors were placed in the 5 cm ring in the process.

3. SYMMETRIC-WINDOW BEAMFORMER

In general, at any given frequency, the 3-D beampattern of any circular array becomes wider (in both the elevation and

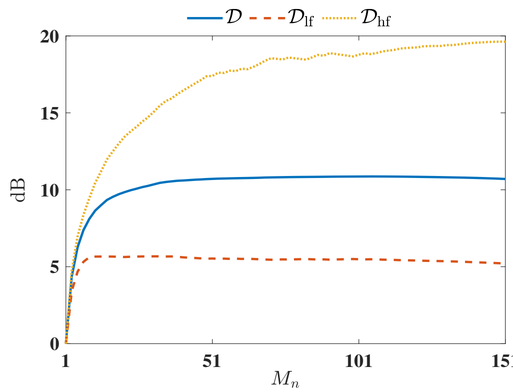

Fig. 1: Evolution of the low-frequency, high-frequency, and full-spectrum DFs as microphone-pairs are optimally placed in the CCA.

the azimuth) as its ring-radius decreases (hence the number of microphones) and vice versa. Also, the higher the frequency, the lesser are the beamwidths. Hence, by weighting the different rings of any given CCA differently at different frequencies, one may strive to achieve uniform beamwidths across the entire spectrum. In addition, one may differently weight the sensors in any given ring of the CCA to control the beamwidths. We implement these two ideas in the form of the following symmetric-window beamformer:

$$\begin{aligned}
&\mathbf{h}(f) = [\mathbf{h}_0^T(f),\ \mathbf{h}_1^T(f),\ \dots,\ \mathbf{h}_P^T(f)]^T,\\
&\mathbf{h}_p(f) = [....,\ h_{p,m_p}(f),\ ...]^T,\\
&h_{p,m_p}(f) = w_p(f)\,\mathcal{N}(v_{p,m_p}\mid 0,\sigma_p^2)\,d_{p,m_p}(f,\theta_{\text{d}},\phi_{\text{d}}),\\
&h_{p,m_p}(f) := \frac{h_{p,m_p}(f)}{\sum_p\sum_{m_p}|h_{p,m_p}(f)|}\,;\\
&w_p(f)\in[0,\ 1],\ \sum_p w_p(f)=1,\ \text{and}
\end{aligned} \tag{8}$$

$$\begin{aligned}
&v_{p,m_p} =\\
&\begin{cases}||\mathbf{u}_{\theta_{\text{d}},\phi_{\text{d}}}-\mathbf{u}_{\frac{\pi}{2},\phi_{p,m_p}}||_2, & \phi_{p,m_p}\in[\phi_{\text{d}}-\frac{\pi}{2},\phi_{\text{d}}+\frac{\pi}{2}]\\ ||\mathbf{u}_{\theta_{\text{d}}+\pi,\phi_{\text{d}}+\pi}-\mathbf{u}_{\frac{\pi}{2},\phi_{p,m_p}}||_2, & \text{otherwise}\end{cases}\,;
\end{aligned} \tag{9}$$

$$\begin{aligned}
&v_{p,m_p} := (v_{p,m_p}-v_{\min})/v_{\max},\\
&v_{\min} = ||\mathbf{u}_{\theta_{\text{d}},\phi_{\text{d}}}-\mathbf{u}_{\frac{\pi}{2},\phi_{\text{d}}}||_2,\\
&v_{\max} = ||\mathbf{u}_{\theta_{\text{d}},\phi_{\text{d}}}-\mathbf{u}_{\frac{\pi}{2},\phi_{\text{d}}+\frac{\pi}{2}}||_2.
\end{aligned} \tag{10}$$

In (8), $w_p(f)$ provides weight to the microphone based on its ring-number (hence radius). $\mathcal{N}(v_{p,m_p}\mid 0,\sigma_p^2)$ is a Gaussian-window function of 0-mean and σ_p^2 variance, which further weights the microphone based on its alignment with the DOA. The distance (v_{p,m_p}) between the unit vector of the DOA and that of the microphone-position is used as input to the Gaussian-window function. Diametrically opposite microphones in a ring are identically weighted. With respect to our recently proposed KW beamformer, we have incorporated two modifications [19]. Firstly, we have modified the symmetric-window function from Kaiser to Gaussian to investigate if there is any advantage of one over the other. Secondly, in equation (10), unlike in the KW, we do not scale the distances between [0,1] irrespective of the elevation angle. When $\theta_{\text{d}}\sim 90^0$, the distances are better scaled, whereas when $\theta_{\text{d}}\sim 0^0$, the distances are more concentrated towards zero. We now utilize the following modified gradient-descent algorithm to determine the optimal parameters:

Algorithm: Given $\theta_{\text{BW}},\ \phi_{\text{BW}},\ \mathcal{D}_{\min}(f),\ \theta_{\text{d}},\ \phi_{\text{d}}$. Define:

$$\begin{aligned}
&\sigma_p'^2(f) = \log_{100}\sigma_p^2(f)\in(-\infty,1],\ p=1,2,...,P,\\
&\Delta b_\theta'(f) = \frac{\theta_{\text{BW}}-b_\theta(f)}{\pi/2},\ \Delta b_\phi'(f) = \frac{\phi_{\text{BW}}-b_\phi(f)}{\pi/2},\\
&\mathcal{D}'(f) = -\log_{10}\mathcal{D}(f)\,;
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\mathcal{L}(f) =\\
&\begin{cases}\Delta b_\theta'(f),\ \Delta b_\theta'(f)>0\ \&\ \Delta b_\phi'(f)\le 0\ \&\ \mathcal{D}(f)\ge\mathcal{D}_{\min}(f)\\ \Delta b_\phi'(f),\ \Delta b_\theta'(f)\le 0\ \&\ \Delta b_\phi'(f)>0\ \&\ \mathcal{D}(f)\ge\mathcal{D}_{\min}(f)\\ \Delta b_\theta'(f)+\Delta b_\phi'(f),\ \Delta b_\theta'(f),\ \Delta b_\phi'(f)>0\ \&\ \mathcal{D}(f)\ge\mathcal{D}_{\min}(f)\\ \mathcal{D}'(f),\ \mathcal{D}(f)<\mathcal{D}_{\min}(f)\end{cases}\,;
\end{aligned} \tag{12}$$

$$\begin{aligned}
&\mathbf{s}(f)\\
&= [w_0(f),\ ....,\ w_P(f),\ \sigma_1'^2(f),\ ....,\ \sigma_P'^2(f)]^T\\
&= [s_1(f),\ ....,\ s_{P+1}(f),\ s_{P+2}(f),\ ....,\ s_{2P+1}(f)]^T,\\
&\Delta s(f)\in(0,1),\ \mu(f)\in(0,1).
\end{aligned} \tag{13}$$

Initialize $\mathbf{s}^{(0)}(f)$. We use $s_l^{(0)}(f)=1/(P+1),\ l\in[1,\ P+1]$, and $s_l^{(0)}(f)=0.5,\ l\in[P+2,\ 2P+1]$. Also, we use $\mu(f)=0.1$, and $\Delta s(f)=0.2$, which are made adaptive. For the n^{th} $(n\ge 0)$ iteration:

(a) Determine $\mathbf{h}(f)$ using $\mathbf{s}^{(n)}(f)$ and equations (8),(9),(10). Hence, determine $\Delta b_\theta(f)$ and $\Delta b_\phi(f)$, and select the optimization function, as per (11),(12). Determine the gradient of the optimization function for each parameter:

$$\frac{\partial\mathcal{L}(f)}{s_l(f)} = \frac{\mathcal{L}_l^+(f)-\mathcal{L}_l^-(f)}{\Delta s(f)},\ l\in[1,\ 2P+1],$$

where $\mathcal{L}_l^\pm(f)$ is obtained for

$$\mathbf{s}^\pm(f) = [s_1^{(n)}(f),...,s_l^{(n)}(f)\pm\Delta s(f)/2,...,s_{2P+1}^{(n)}(f)]^T.$$

(b) Update: $s_l^{(n+1)}(f) = s_l^{(n)}(f)-\mu(f)\frac{\partial\mathcal{L}(f)}{s_l(f)}$.

(c) Terminate the algorithm after a pre-defined maximum number of iterations, or when $\mathcal{L}(f)$ saturates.

The final parameters result in the Gaussian-window (GW) beamformer. With respect to the KW, the GW uses a modified optimization function, which ensures that the DF $\ge\mathcal{D}_{\min}(f)$ is maintained, wherever possible in the spectrum [19].

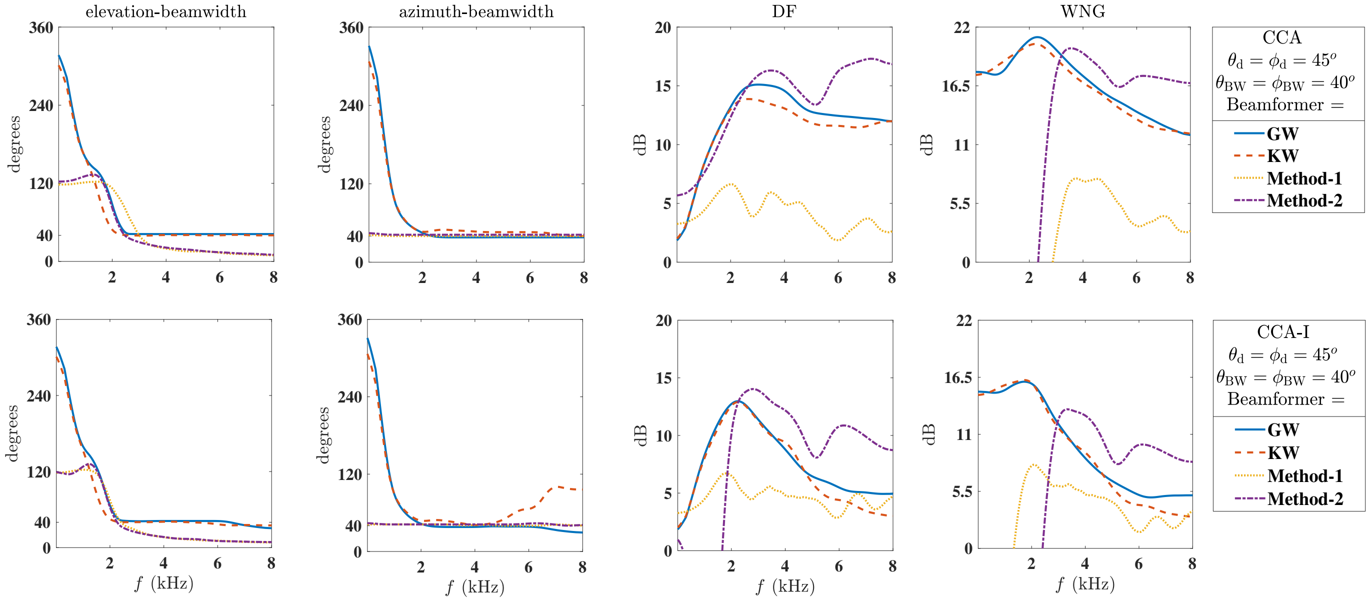

Fig. 2: Performance of the GW, KW, Method-1, and Method-2 beamformers applied on the CCA (top row) and the CCA-I (bottom row).

4. RESULTS

We now compare the performances of the proposed GW beamformer with respect to the KW and two other state-of-the-art beamformers: Method-1 [7], and Method-2 [11, 12]. We consider the general oblique DOA ($\theta_{\text{d}} = \phi_{\text{d}} = 45^o$), $\theta_{\text{BW}} = \phi_{\text{BW}} = 40^o$, and $\mathcal{D}_{\text{min}}(f) = 5$ dB. The MATLAB codes are available online [20].

Fig. 2 shows the performance of the beamformers for the CCA and CCA-I designs. Consider first the CCA. The GW and KW provide almost identical performances, which indicates that there is no significant advantage of the Gaussian-window over the Kaiser-window. The marginal advantage of the GW at higher frequencies may be credited to the modified distance computation in equation (10). The other two methods, Method-1 and Method-2, are able to exactly attain the target-azimuth-beamwidth. However, they provide no control over the elevation-beamwidth. Additionally, their solutions are arguably impractical at low frequencies since the WNG < 0 dB. In fact, the beampatterns (not shown) of Method-1 and Method-2 are misdirected towards the horizontal plane, particularly at low frequencies. Such fallacies of conventional methods are aptly discussed in [19].

Consider now the CCA-I. We first compare the performance of the GW and KW for the CCA-I vs. the CCA. Up to ~ 2 kHz, their performances are nearly identical for both the designs. This is a vindication of our proposed methology of designing the sparse CCA. For higher frequencies, the beampatterns are naturally narrower, and hence the two window beamformers strive to increase the beamwidths. In this endeavor, many microphones are de-emphasized. For the CCA-I, since it has much fewer sensors than the CCA, de-emphasizing the microphones results in a stark decrease in the DF and WNG. The $\mathcal{D}_{\text{min}}(f)$ parameter in the case of the GW, however, ensures that at higher frequencies there is a trade-off between optimizing the DF vs. beamwidth-control. Optimizing the DF automatically enhances the WNG, since the entire process is based on microphone-weighting. Additionally, one can observe that compared to the KW, the GW provides better control on the beamwidth. This is, again, due to the modified distance computation. The limitations of Method-1 and Method-2, mentioned earlier, persist for the CCA-I.

5. SUMMARY AND CONCLUSIONS

Exploiting the relationship of the DF and the inter-sensor distances, a method is devised to obtain a sparse CCA. The proposed method sequentially places microphone-pairs in their optimal rings such that the full-spectrum DF is maximized. The much smaller set of microphones that saturates the full-spectrum DF constitutes the sparse CCA, denoted as CCA-I. Thereafter, following our recent work, we propose a methodology to derive a beamformer which can control the elevation and azimuth beamwidths, while enhancing the DF wherever possible in the spectrum. The proposed GW beamformer is able to outperform our recently proposed KW beamformer, particularly in the case of the CCA-I. Both the beamformers provide superior performances over conventional frequency-invariant and constant-beamwidth beamformers. Further refinement of the CCA-I design may be explored in the future.

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