ON THE DESIGN OF SQUARE DIFFERENTIAL MICROPHONE ARRAYS WITH A MULTISTAGE STRUCTURE

Xudong Zhao[1,2], *Gongping Huang*[3], *Jacob Benesty*[2], *Jingdong Chen*[1], *and Israel Cohen*[3]

[1]: CIAIC and Shaanxi Provincial Key Laboratory of Artificial Intelligence, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China
[2]: INRS-EMT, University of Quebec, Montreal, QC H5A 1K6, Canada
[3]: Technion–Israel Institute of Technology, Technion City, Haifa 3200003, Israel

ABSTRACT

This paper studies the problem of designing square differential microphone arrays (SDMAs). It presents a multistage approach, which first divides an SDMA composed of M^2 microphones into $(M-1)^2$ subarrays with each subarray being a 2×2 square array formed by four adjacent microphones. Then, differential beamforming is performed with each subarray in the first-stage. The first-stage differential beamformers' outputs are subsequently used as the inputs of the second stage to form $(M-2)^2$ subarrays and a second-stage differential beamforming is then performed. Continuing this process till the $(M-1)$th stage, we obtain the final output of the SDMA. The SDMA designed in such a multistage structure has two important properties. First, the global weighting matrix is equal to the two dimensional convolution of weighting matrices from the first stage to the last one. Second, the global beampattern is equal to the product of beampatterns from all stages. Consequently, we can combine different kinds of beamformers in different stages and have better control of the performance metrics.

Index Terms—Microphone arrays, square arrays, differential beamforming, multistage structure, frequency invariance.

1. INTRODUCTION

Design of microphone arrays and associated beamformers to enhance a signal of interest from noisy observations have attracted a significant amount of research attention [1–9]. Among the many beamforming methods developed in the literature, the so called differential beamforming has attracted much research and engineering interest over the past few decades as it can form frequency-invariant beampatterns and achieve high directivity gains with small size arrays [10–14]. One of the most widely-used methods to design differential beamformers, or equivalently differential microphone arrays (DMAs), is based on the multistage structure [15–18], in which a first-order DMA is obtained by subtracting the outputs of two closely spaced omnidirectional sensors and then a second-order DMA is formed by subtracting the outputs of two first-order DMAs. With the same principle, an Nth-order DMA is obtained by subtracting the outputs of two DMAs of order $N-1$. With the same kind of beampattern shapes, the higher the DMA order, the larger is the directivity factor (DF). In other words, in this multistage approach, more stages will generally lead to higher DFs, which are desirable for practical applications. This multistage method has been extended to different forms. For example, a method was recently developed to design DMAs based on null constraints on the beampattern in the short-time Fourier transform (STFT) domain [11], which is equivalent to the multistage method if the number of microphone sensors is equal to the DMA order plus one [19]. The advantage of this method is that it brings flexibility not only for the design of differential beamformers but also in dealing with DMA robustness by exploiting more sensors.

In [2, 20–22], a differential Kronecker product beamformer was proposed, which decomposes a uniform linear array into two virtual sub-arrays and the corresponding sub-beamformers are then designed individually. In [23, 24], the multistage approach is extended by defining a spatial difference operator, where differential beamformers are constructed in two main stages: Differential and beamforming. While it serves as the basis for DMA design and differential beamforming, the multistage method has been investigated mainly for linear DMAs so far. How to design DMAs with array geometries other than linear is still a challenging problem and further efforts in this perspective is indispensable.

This paper extends the multistage theory to the design of square differential microphone arrays (SDMAs). We divide the design of SDMAs composed of M^2 microphones into $M-1$ stages and in every stage we deal with the design of SDMAs with only four microphones. With this decomposition, the global beamforming weighting matrix is equal to the two dimensional (2-D) convolution of weighting matrices from the $M-1$ stages, and the global beampattern is equal to the product of beampatterns from all stages. The proposed multistage approach makes the design of SDMAs easy and flexible. Since in each stage a different beamforming method can be used, we can combine different kinds of beamformers through this multistage structure, thereby gaining flexible control of the beampattern shape, directivity factor, and white noise gain of the overall SDMA.

2. SIGNAL MODEL AND PERFORMANCE MEASURES

Consider a square array, which consists of M^2 (with $M \geqslant 2$) omnidirectional microphones. In the Cartesian coordinate system with axes x, y, and z, the square array which lies on the first draqant of the xy plane, is composed of M identical uniform linear arrays (ULAs), with M elements each, parallel to the x axis, and the interelement in any of these ULAs is equal to δ; this is also equivalent to having M identical ULAs parallel to the y axis.

Let us consider the farfield and anechoic acoustic model and denote elevation angle by θ ($0 \leqslant \theta \leqslant \pi$) and azimuth angle by φ ($0 \leqslant \varphi \leqslant 2\pi$). The steering matrix $\mathbf{D}_{\theta,\varphi}(\omega)$ of size $M \times M$ can then be expressed as

$$[\mathbf{D}_{\theta,\varphi}(\omega)]_{mn} = e^{\jmath \varpi \sin\theta [(m-1)\cos\varphi + (n-1)\sin\varphi]}, \quad m,n = 1,2,\ldots,M, \tag{1}$$

where $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\varpi = \omega\delta/c$, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, c is the

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speed of sound in air, i.e., $c = 340$ m/s, and the steering vector of length M^2 is defined as

$$\mathbf{d}_{\theta,\varphi}(\omega) = \text{vec}\left[\mathbf{D}_{\theta,\varphi}(\omega)\right], \tag{2}$$

with vec$[\cdot]$ being an operator that stacks the columns of a matrix into a vector. In the rest, in order to simplify the notation, we drop the dependence on the angular frequency, ω.

Now, assume that the signal of interest (also called the desired signal), propagates from the direction $\{\theta_\text{d}, \varphi_\text{d}\}$. Throughout this paper, we consider fixed beamforming with small values of the interelement spacing, δ, of the array, like in differential [11, 15] or superdirective [25] beamforming, where the main lobe points to $\{\theta_\text{d}, \varphi_\text{d}\}$ from which the desired signal propagates. For that, we apply a beamforming filter $\mathbf{h}$ of length M^2 to the outputs of the square array, whose coefficients depend on what kind of response we want. Equivalently, the filter can be written as $\mathbf{h} = \text{vec}(\mathbf{H})$, where $\mathbf{H}$ is a weighting matrix of size $M \times M$, whose elements are $H_{mn},\ m, n = 1, 2, \ldots, M$.

To evaluate the fixed beamformers, we use three common measures. They are

- the beampattern:

$$\begin{aligned}\mathcal{B}_{\theta,\varphi}(\mathbf{h}) &= \mathbf{d}_{\theta,\varphi}^H \mathbf{h} \\ &= \sum_{m=1}^{M}\sum_{n=1}^{M} H_{mn} e^{-\jmath\varpi \sin\theta[(m-1)\cos\varphi + (n-1)\sin\varphi]},\end{aligned} \tag{3}$$

  where the superscript ^{H} is the conjugate-transpose operator,

- the white noise gain (WNG):

$$\mathcal{W}(\mathbf{h}) = \frac{\left|\mathbf{h}^H \mathbf{d}_{\theta_\text{d},\varphi_\text{d}}\right|^2}{\mathbf{h}^H\mathbf{h}}, \tag{4}$$

- and the directivity factor (DF):

$$\mathcal{D}(\mathbf{h}) = \frac{\left|\mathbf{h}^H \mathbf{d}_{\theta_\text{d},\varphi_\text{d}}\right|^2}{\mathbf{h}^H\mathbf{\Gamma}\mathbf{h}}, \tag{5}$$

  where the elements of the $M^2 \times M^2$ matrix $\mathbf{\Gamma}$ are given by

$$\begin{aligned}(\mathbf{\Gamma})_{ij} &= \text{sinc}\left[\varpi\sqrt{(m_i - m_j)^2 + (n_i - n_j)^2}\right], \\ & i, j = 1, 2, \ldots, M^2,\end{aligned} \tag{6}$$

  with $\text{sinc}(x) = \sin x/x$, $n_i = \lceil i/M \rceil$, $m_i = i - M(n_i - 1)$, $n_j = \lceil j/M \rceil$, $m_j = j - M(n_j - 1)$, and $\lceil\cdot\rceil$ is the ceiling function.

Furthermore, the distortionless constraint in the direction of the source of interest is always desired, i.e.,

$$\mathbf{h}^H \mathbf{d}_{\theta_\text{d},\varphi_\text{d}} = 1. \tag{7}$$

3. THE MULTISTAGE STRUCTURE

The basic unit in the multistage structure is a 2×2 square array formed by 4 adjacent microphones. This structure divides the square array with M^2 microphones into $(M-1)^2$ subarrays. In a multistage approach, a first-stage differential beamformer is applied in each subarray, then a second-stage differential beamformer is applied to the first-stage outputs, as shown in Fig. 1. We denote the frequency-domain signal received by the microphone at the mth row and nth column as X_{mn}; then, the first-stage outputs are

$$\begin{aligned}Y_{mn}^{(1)} &= \sum_{i=1}^{2}\sum_{j=1}^{2} X_{(m+i-1)(n+i-1)}\left[H_{ij}^{(1)}\right]^*, \\ & m, n = 1, 2, \ldots, M-1,\end{aligned} \tag{8}$$

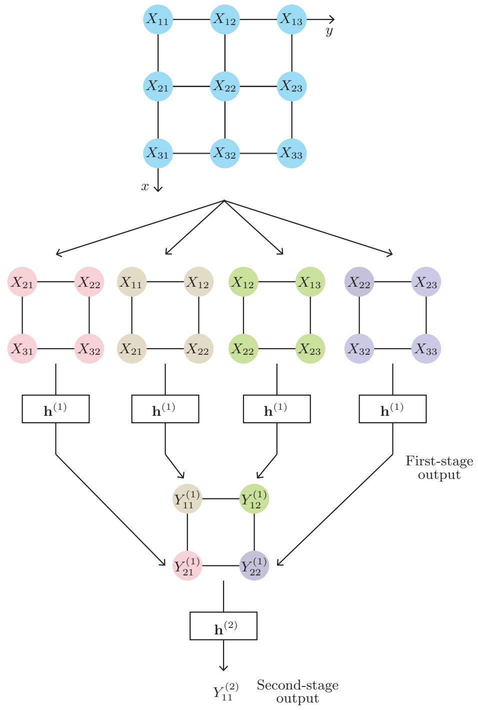

Fig. 1. Illustration of the multistage structure for SDMAs.

where the superscript $^{()}$ is added to emphasize the stage with the number inside the parentheses indicating the stage, the complex weight $H_{ij}^{(1)}$ is the (i, j)th element of the first-stage weighting matrix $\mathbf{H}^{(1)}$ of size 2×2, the first-stage beamforming filter is obtained by $\mathbf{h}^{(1)} = \text{vec}\left[\mathbf{H}^{(1)}\right]$, and the superscript * is the complex-conjugate operator. Then, the first-stage outputs are treated as the inputs of the second stage and, similarly, the second-stage outputs are obtained by

$$\begin{aligned}Y_{mn}^{(2)} &= \sum_{i=1}^{2}\sum_{j=1}^{2} Y_{(m+i-1)(n+i-1)}^{(1)}\left[H_{ij}^{(2)}\right]^*, \\ & m, n = 1, 2, \ldots, M-2,\end{aligned} \tag{9}$$

where the definition of the second-stage beamforming filter is similar to the one of the first stage. Continuing this process, the final output is obtained as

$$Y_{11}^{(M-1)} = \sum_{i=1}^{2}\sum_{j=1}^{2} Y_{ij}^{(M-2)}\left[H_{ij}^{(M-1)}\right]^*. \tag{10}$$

SDMAs implemented with the multistage structure have two prominent properties, which are listed bellow.

Property 3.1. The global weighting matrix is equal to the 2-D convolution of weighting matrices from the first stage to the last one, i.e.,

$$\mathbf{H} = \mathbf{H}^{(1)} \star \mathbf{H}^{(2)} \star \cdots \star \mathbf{H}^{(M-1)}, \tag{11}$$

where $\star$ denotes the 2-D convolution [26], $\mathbf{H}$ is the global weighting matrix of size $M \times M$, and $\mathbf{H}^{(i)}$ of size 2×2 is the weighting matrix in the ith ($i = 1, 2, \ldots, M-1$) stage.

Proof. By substituting (8) into (9), we get

$$
\begin{aligned}
Y^{(2)}_{mn} = & \sum_{i'=1}^{2}\sum_{j'=1}^{2} \left[H^{(2)}_{i'j'}\right]^* \times \\
& \sum_{i=1}^{2}\sum_{j=1}^{2} X_{(m+i+i'-2)(n+j+j'-2)} \left[H^{(1)}_{ij}\right]^* . \qquad (12)
\end{aligned}
$$

If we set $p = i + i' - 1$ and $q = j + j' - 1$, (12) can be written as

$$
\begin{aligned}
Y^{(2)}_{mn} = & \sum_{p=1}^{3}\sum_{q=1}^{3} X_{(m+p-1)(n+q-1)} \times \\
& \sum_{i=1}^{2}\sum_{j=1}^{2} \left[H^{(1)}_{ij} H^{(2)}_{(p-i+1)(q-j+1)}\right]^* \qquad (13) \\
= & \sum_{p=1}^{3}\sum_{q=1}^{3} X_{(m+p-1)(n+q-1)} \left[\mathbf{H}^{(1)} \star \mathbf{H}^{(2)}\right]^*_{pq} .
\end{aligned}
$$

Continuing on, we obtain the final output as

$$
\begin{aligned}
Y^{(M-1)}_{11} &= \sum_{p=1}^{M}\sum_{q=1}^{M} X_{pq} \left[\mathbf{H}^{(1)} \star \mathbf{H}^{(2)} \star \cdots \star \mathbf{H}^{(M-1)}\right]^*_{pq} \\
&= \sum_{p=1}^{M}\sum_{q=1}^{M} X_{pq} H^*_{pq}, \qquad (14)
\end{aligned}
$$

which completes the proof. □

Property 3.2. The global beampattern of SDMAs is the product of beampatterns from all stages, i.e.,

$$
\mathcal{B}_{\theta,\varphi}(\mathbf{h}) = \prod_{i=1}^{M-1} \mathcal{B}_{\theta,\varphi}\left[\mathbf{h}^{(i)}\right]. \qquad (15)
$$

Proof. Taking the 2-D $\mathcal{Z}$-transform of the convolution of weighting matrices in the first two stages, we have

$$
\begin{aligned}
& \mathcal{Z}\left[\mathbf{H}^{(1)} \star \mathbf{H}^{(2)}\right] \\
&= \sum_{p=1}^{3}\sum_{q=1}^{3} \left[\sum_{i=1}^{2}\sum_{j=1}^{2} H^{(1)}_{ij} H^{(2)}_{(p-i+1)(q-j+1)}\right] z_1^{-(p-1)} z_2^{-(q-1)} \\
&= \sum_{i=1}^{2}\sum_{j=1}^{2} H^{(1)}_{ij} z_1^{-(i-1)} z_2^{-(j-1)} \sum_{i'=1}^{2}\sum_{j'=1}^{2} H^{(2)}_{i'j'} z_1^{-(i'-1)} z_2^{-(j'-1)} \\
&= \mathcal{Z}\left[\mathbf{H}^{(1)}\right] \times \mathcal{Z}\left[\mathbf{H}^{(2)}\right], \qquad (16)
\end{aligned}
$$

where $\mathcal{Z}[\cdot]$ stands for the 2-D $\mathcal{Z}$-transform. Continuing this process, we get

$$
\mathcal{Z}\left[\mathbf{H}^{(1)} \star \mathbf{H}^{(2)} \star \cdots \star \mathbf{H}^{(M-1)}\right] = \prod_{i=1}^{M-1} \mathcal{Z}\left[\mathbf{H}^{(i)}\right]. \qquad (17)
$$

Since the beampattern can be written as [27]

$$
\mathcal{B}_{\theta,\varphi}(\mathbf{h}) = \mathcal{Z}(\mathbf{H})\big|_{z_1 = e^{\jmath\varpi\sin\theta\cos\varphi},\, z_2 = e^{\jmath\varpi\sin\theta\sin\varphi}}, \qquad (18)
$$

it follows immediately that the global beampattern is the product of beampatterns from all stages. This completes the proof. □

Given the multistage structure, the design of SDMAs with M^2 microphones can be divided into the design of $M-1$ SDMAs with four microphones, which provides a lot of flexibility.

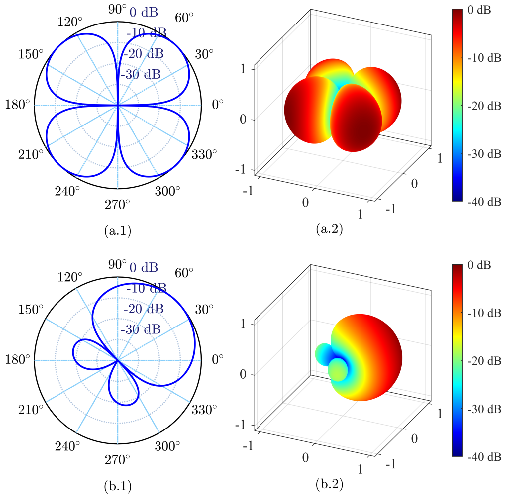

Fig. 2. Beampatterns of the proposed beamformers in the xy plane: (a.1) $\mathbf{h}_{\mathrm{Q}}$ and (b.1) $\mathbf{h}_{\mathrm{C}}$; and in 3-D: (a.2) $\mathbf{h}_{\mathrm{Q}}$ and (b.2) $\mathbf{h}_{\mathrm{C}}$. Conditions of simulation: $M = 2$, $\delta = 1$ cm, and $f = 1$ kHz.

4. DESIGN EXAMPLES

For the design examples, in this paper we follow the null-constrained method in the design of circular DMAs [28], where the beampattern can be steered to different azimuth angles that correspond to the direction of sensor positions. In this example, we focus on the design of beampatterns that are symmetric about the axis $\varphi = \varphi_{\mathrm{d}} \leftrightarrow \varphi_{\mathrm{d}} + \pi$. We set the desired direction at $\{\theta_{\mathrm{d}}, \varphi_{\mathrm{d}}\} = \{\pi/2, \pi/4\}$, so we have $\mathcal{B}_{\theta,\varphi}(\mathbf{h}) = \mathcal{B}_{\theta,\pi/2-\varphi}(\mathbf{h})$ and $H^{(i)}_{12} = H^{(i)}_{21}$ with $i = 1, 2, \ldots, M-1$. Therefore, the symmetric SDMA with four microphones can be determined by solving the following linear system of equations:

$$
\begin{bmatrix}
\mathbf{d}^H_{\theta_{\mathrm{d}},\varphi_{\mathrm{d}}} \\
\mathbf{d}^H_{\theta_1,\varphi_1} \\
\mathbf{d}^H_{\theta_2,\varphi_2} \\
\mathbf{c}^T
\end{bmatrix}
\mathbf{h} =
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (19)
$$

where

$$
\mathbf{c} = \begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}^T
$$

is the symmetry constraint, $0 \leqslant \theta_1, \theta_2 \leqslant \pi/2$, and $\pi/4 \leqslant \varphi_1, \varphi_2 \leqslant 5\pi/4$.

There are many other design methods that can be used, such as the one based on series approximations [13, 29], which may achieve a better steering flexibility, but the design process is similar.

4.1. SDMAs with One Stage

First, we consider the design of SDMAs with only one stage, i.e., SDMAs with four microphones. The first example is the quadrupole, which has two distinct nulls at $\{\theta, \varphi\} = \{\pi/2, \pi/2\}$ and $\{\pi/2, \pi\}$. The solution to (19) is $\mathbf{h}_{\mathrm{Q}}$ and the corresponding weighting matrix is

$$
\mathbf{H}_{\mathrm{Q}} = \frac{1}{\left(e^{-\jmath\sqrt{2}\varpi/2} - 1\right)^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \qquad (20)
$$

Substituting $\mathbf{h}_{\mathrm{Q}} = \mathrm{vec}(\mathbf{H}_{\mathrm{Q}})$ into (4), we get the WNG as

$$
\mathcal{W}(\mathbf{h}_{\mathrm{Q}}) = \left[1 - \cos\left(\sqrt{2}\varpi/2\right)\right]^2. \qquad (21)
$$

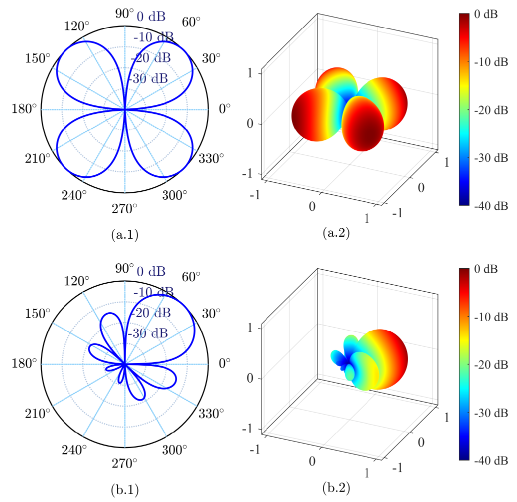

Fig. 3. Beampatterns of the proposed beamformers in the xy plane: (a.1) $\mathbf{h}_{\mathrm{Q/Q}}$ and (b.1) $\mathbf{h}_{\mathrm{C/Q}}$; and in 3-D: (a.2) $\mathbf{h}_{\mathrm{Q/Q}}$ and (b.2) $\mathbf{h}_{\mathrm{C/Q}}$. Conditions of simulation: $M = 3$, $\delta = 1$ cm, and $f = 1$ kHz.

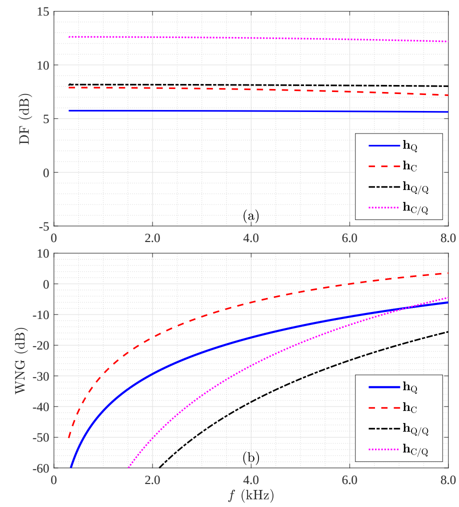

Fig. 4. Performance of proposed beamformers: $\mathbf{h}_{\mathrm{Q}}$ and $\mathbf{h}_{\mathrm{C}}$ with $M = 2$; $\mathbf{h}_{\mathrm{Q/Q}}$ and $\mathbf{h}_{\mathrm{C/Q}}$ with $M = 3$. (a) DF and (b) WNG. Conditions of simulation: $\delta = 1$ cm.

For the second example, we consider the design of the second-order cardioid, which has two distinct nulls at $\{\theta, \varphi\} = \{\pi/2, 3\pi/4\}$ and $\{\pi/2, 5\pi/4\}$. In this scenario, the solution to (19) is $\mathbf{h}_{\mathrm{C}}$ and the corresponding weighting matrix is

$$\mathbf{H}_{\mathrm{C}} = \frac{1}{2\cos\left(\sqrt{2}\varpi\right) - 2} \begin{bmatrix} e^{\jmath\sqrt{2}\varpi} & -e^{\jmath\sqrt{2}\varpi/2} \\ -e^{\jmath\sqrt{2}\varpi/2} & 1 \end{bmatrix}. \quad (22)$$

Substituting $\mathbf{h}_{\mathrm{C}} = \mathrm{vec}\left(\mathbf{H}_{\mathrm{C}}\right)$ into (4), we get the WNG as

$$\mathcal{W}\left(\mathbf{h}_{\mathrm{C}}\right) = \left[1 - \cos\left(\sqrt{2}\varpi\right)\right]^2. \quad (23)$$

Figure 2 plots the beampatterns of the proposed beamformers, $\mathbf{h}_{\mathrm{Q}}$ and $\mathbf{h}_{\mathrm{C}}$, at frequency $f = 1$ kHz. It is seen from Figs 2(a.1) and (b.1) that both beampatterns satisfy the constraints. Figure 2(a.2) also shows that the three dimensional (3-D) beampattern of $\mathbf{h}_{\mathrm{Q}}$ has a null at the top, which can be verified as $\mathcal{B}_{0,\varphi}\left(\mathbf{h}_{\mathrm{Q}}\right) = \sum_{m=1}^{2}\sum_{n=1}^{2} H_{\mathrm{Q},mn} = 0$. This will be useful in applications for suppressing the noise from the top.

4.2. SDMAs with Two Stages

Next, we consider the design of SDMAs with two stages, i.e., 3×3 SDMAs. As discussed, SDMAs with two stages can be formed from the combination of two SDMAs with one stage, so the first example is the combination of two quadrupoles. The global weighting matrix can be obtained by

$$\begin{aligned}\mathbf{H}_{\mathrm{Q/Q}} &= \mathbf{H}_{\mathrm{Q}} \star \mathbf{H}_{\mathrm{Q}} \\ &= \frac{1}{\left(e^{-\jmath\sqrt{2}\varpi/2} - 1\right)^4} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \end{aligned} \quad (24)$$

and the global beamforming filter is $\mathbf{h}_{\mathrm{Q/Q}} = \mathrm{vec}\left(\mathbf{H}_{\mathrm{Q/Q}}\right)$. Substituting $\mathbf{h}_{\mathrm{Q/Q}}$ into (4), we can get the WNG:

$$\mathcal{W}\left(\mathbf{h}_{\mathrm{Q/Q}}\right) = \frac{4}{9}\left[1 - \cos\left(\sqrt{2}\varpi/2\right)\right]^4. \quad (25)$$

The second example is the combination of the second-order cardioid and quadrupole. The global weighting matrix can be obtained by

$$\mathbf{H}_{\mathrm{C/Q}} = \mathbf{H}_{\mathrm{C}} \star \mathbf{H}_{\mathrm{Q}} \quad (26)$$

and the global beamforming filter is $\mathbf{h}_{\mathrm{C/Q}} = \mathrm{vec}\left(\mathbf{H}_{\mathrm{C/Q}}\right)$. Substituting $\mathbf{h}_{\mathrm{C/Q}}$ into (4), we get the WNG as

$$\mathcal{W}\left(\mathbf{h}_{\mathrm{C/Q}}\right) = \frac{4\left[1 - \cos\left(\sqrt{2}\varpi/2\right)\right]^2\left[1 - \cos\left(\sqrt{2}\varpi\right)\right]^2}{\left[2 + \cos\left(\sqrt{2}\varpi/2\right)\right]^2}. \quad (27)$$

Figures 3(a.1) and (a.2) plot the beampatterns of $\mathbf{h}_{\mathrm{Q/Q}}$ at $f = 1$ kHz. Compared to the quadrupole, the beamformer $\mathbf{h}_{\mathrm{Q/Q}}$ has a similar pattern but with wider nulls. Figures 3(b.1) and (b.2) plot the beampatterns of $\mathbf{h}_{\mathrm{C/Q}}$. It is clearly seen that the combination of the second-order cardioid and quadrupole leads to four nulls in the beampattern, i.e., $\{\theta, \varphi\} = \{\pi/2, \pi/2\}$, $\{\pi/2, 3\pi/4\}$, $\{\pi/2, \pi\}$, and $\{\pi/2, 5\pi/4\}$, and there is also a null at the top.

Figure 4 plots the DF and WNG of the proposed differential beamformers, $\mathbf{h}_{\mathrm{Q}}$, $\mathbf{h}_{\mathrm{C}}$, $\mathbf{h}_{\mathrm{Q/Q}}$, and $\mathbf{h}_{\mathrm{C/Q}}$. In Fig. 4(a), we see that the DFs remain almost constant in the studied frequency range, which further illustrates an interesting property of SDMAs, i.e., frequency-invariant beampattern and DF. In Fig. 4(b), it can be seen that the WNG of the differential beamformers is small at low frequencies, indicating the problem of white noise amplification; but the WNG increases with the frequency; so, the problem of white noise amplification becomes less serious as the frequency increase. If the frequency is sufficiently high, there is even white noise reduction.

5. CONCLUSIONS

We have studied the design of SDMAs with a multistage structure. In the first stage, every four adjacent microphones are used to form a 2×2 square subarray, and a differential beamformer is designed and applied to such a subarray. Then, the outputs of the first stage are used as the inputs of the second stage. In a similar manner, the outputs of one stage are used as the inputs of the next one, and this operation is continued till we get the final output. The multistage approach divides the design of SDMAs with M^2 microphones into the design of $M - 1$ SDMAs with four microphones, where the global weighting matrix is equal to the 2-D convolution of weighting matrices from the $M - 1$ stages, and the global beampattern is equal to the product of beampatterns from all stages. Clearly, with the proposed approach, we can design different kinds of beamformers in different stages, leading to flexible method to combine different kinds of beamformers. Consequently, we can better control the beamforming performance in terms of the beampattern shape, DF, and WNG.

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