ROBUST STEERABLE DIFFERENTIAL BEAMFORMERS WITH NULL CONSTRAINTS FOR CONCENTRIC CIRCULAR MICROPHONE ARRAYS

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ABSTRACT

Differential beamformers with concentric circular microphone arrays (CCMAs) are desirable for use in various applications since they can form frequency-invariant spatial responses, have better beam steering flexibility than linear arrays, and suffer less with beampattern irregularity and white noise amplification than circular microphone arrays (CMAs). The methods developed previously for differential beamforming with CCMAs are based on the series expansion. Such methods need to know the analytic form of the target beampattern, which may not be accessible in practice. Furthermore, expansion error may lead to erroneous solution, which can cause noise amplification instead of reduction. In this paper, we extend our recently developed beamforming method for CMAs to the design of differential beamformers with CCMAs, which takes advantage of the symmetric null constraints from the beampattern. Simulations are performed to justify the properties of the proposed approach.

***Index Terms*—** Microphone arrays, differential beamforming, concentric circular microphone arrays, frequency-invariant beampattern.

1. INTRODUCTION

In voice communication and human-machine speech interfaces, signals of interest picked up by microphone sensors are inevitably contaminated by such adverse effects as noise, reverberation, and interference, which impair the quality and/or intelligibility of the signal of interest [1, 2]. To deal with those adverse effects and recover the signal of interest from its corrupted observations, microphone arrays and associated beamforming techniques have been widely studied and used [3–7]. Among the numerous beamforming methods developed in the literature, one of the most widely used ones is the differential beamforming [8–13], which can achieve high spatial gain and frequency-invariant spatial response [14–16]. An important issue with the design of differential beamformers is the beampattern steering flexibility, which motivates the use of circular microphone arrays (CMAs) [17–21]. However, CMAs-based differential beamformers often suffer from the problem of irregularity, which leads to irregular beampatterns and deep nulls in array gains [3, 22]. To circumvent this issue, concentric CMAs (CCMAs) are used [10, 23–26].

Previously, the CCMA differential beamformers were designed based on the method of series expansion, where the beamformer's beampattern is approximated with a Jacobi-Anger expansion [10]. By substituting the Jacobi-Anger expansion and making the designed beampattern equal to the target beampattern, a system of linear equations are constructed and the beamforming filter is subsequently obtained by solving this system. The steering direction is included in the target beampattern, so the designed beampattern can be steered to any directions in the plane where the sensors are located. Since the beamformer's beampattern corresponding to different rings can be approximated with different orders depending on the number of sensors [3], the placement of the sensors in each ring can be very flexible. However, the series expansion based method needs to know the analytical form of the target beampattern, which is often not accessible in practice. Furthermore, expansion error is generally unavoidable, particularly at high frequencies. This may lead to erroneous solution with irregular beampatterns and deep nulls in the array gain. To deal with these issues, we extend a method developed previously for differential beamforming with CMAs [27] to the design of differential beamformers with CCMAs by exploiting symmetric null constraints, which only requires to know the directions of the nulls and the steering angle.

2. SIGNAL MODEL AND PROBLEM FORMULATION

We consider a CCMA composed of P rings, as shown in Fig. 1, where the pth ($p = 1, 2, \dots, P$) ring, with a radius of r_p, consists of M_p omnidirectional microphones, the angular position of the mth array element on the pth ring is

$$\psi_{p,m} = \psi_{p,1} + \frac{2\pi(m-1)}{M_p} \quad (1)$$

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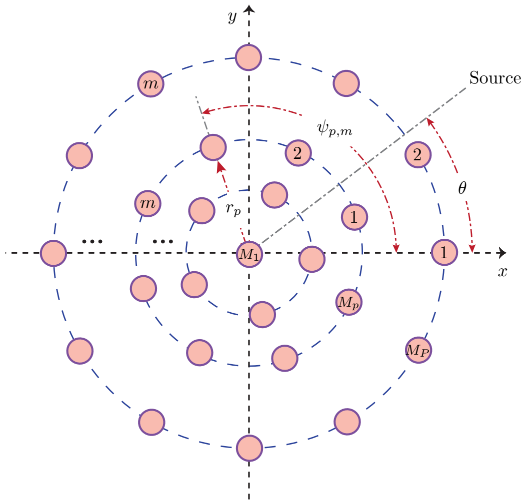

Fig. 1. Illustration of a CCMA consisting of P rings, placed on a polar plane, where the pth ($p = 1, 2, \ldots, P$) ring, with a radius of r_p, consists of M_p omnidirectional microphones.

with $\psi_{p,1} \geq 0$ being the angular position of the first microphone on the pth ring. Assume that the center of the CCMA coincides with the origin of Cartesian system and the coordinates of the mth microphone on the pth ring can be represented as $\mathbf{r}_{p,m} = r_p \left[\cos \psi_{p,m} \ \sin \psi_{p,m}\right]^T$, where the superscript ^{T} is the transpose operator. The steering vector of length $\underline{M}$, where $\underline{M} = \sum_{p=1}^{P} M_p$ is the total number of microphones, is

$$\underline{\mathbf{d}}(\omega, \theta) = \left[\begin{array}{cccc} \mathbf{d}_1^T(\omega, \theta) & \mathbf{d}_2^T(\omega, \theta) & \cdots & \mathbf{d}_P^T(\omega, \theta) \end{array}\right]^T, \quad (2)$$

where

$$\mathbf{d}_p(\omega, \theta) = \left[e^{\jmath \varpi_p \cos(\theta - \psi_{p,1})} \ \ldots \ e^{\jmath \varpi_p \cos(\theta - \psi_{p,M_p})} \right]^T \quad (3)$$

is the pth ring's steering vector with $\varpi_p = \omega r_p / c$, $\omega = 2\pi f$ being the angular frequency, f being the temporal frequency, and c is the speed of sound in air.

We consider a source signal of interest (plane wave), in the farfield, that propagates in an anechoic acoustic environment to the CCMA. The observation signals can then be expressed as

$$\begin{aligned} \underline{\mathbf{y}}(\omega) &= \left[\begin{array}{cccc} \mathbf{y}_1^T(\omega) & \mathbf{y}_2^T(\omega) & \cdots & \mathbf{y}_P^T(\omega) \end{array}\right]^T \\ &= \underline{\mathbf{d}}(\omega, \theta_{\mathrm{s}}) X(\omega) + \underline{\mathbf{v}}(\omega), \end{aligned} \quad (4)$$

where

$$\mathbf{y}_p(\omega) = \left[\begin{array}{cccc} Y_{p,1}(\omega) & Y_{p,2}(\omega) & \cdots & Y_{p,M_p}(\omega) \end{array}\right]^T, \quad (5)$$

$\underline{\mathbf{d}}(\omega, \theta_{\mathrm{s}})$ is the signal propagation vector, which is same as the steering vector corresponding to the direction θ_{s}, $X(\omega)$ is the desired signal, and $\underline{\mathbf{v}}(\omega)$ is defined in a similar way to $\underline{\mathbf{y}}(\omega)$.

Microphone array beamforming is a process of applying a complex weight at the output of each microphone and then summing all the weighted outputs together to get an estimate of the source signal, i.e.,

$$\begin{aligned} Z(\omega) &= \underline{\mathbf{h}}^H(\omega) \underline{\mathbf{y}}(\omega) \\ &= \underline{\mathbf{h}}^H(\omega) \underline{\mathbf{d}}(\omega, \theta_{\mathrm{s}}) X(\omega) + \underline{\mathbf{h}}^H(\omega) \underline{\mathbf{v}}(\omega), \end{aligned} \quad (6)$$

where the superscript ^{H} is the conjugate-transpose operator, and

$$\underline{\mathbf{h}}(\omega) = \left[\begin{array}{cccc} \mathbf{h}_1^T(\omega) & \mathbf{h}_2^T(\omega) & \cdots & \mathbf{h}_P^T(\omega) \end{array}\right]^T \quad (7)$$

is the spatial filter of length $\underline{M}$ with

$$\mathbf{h}_p(\omega) = \left[\begin{array}{cccc} H_{p,1}(\omega) & H_{p,2}(\omega) & \cdots & H_{p,M_p}(\omega) \end{array}\right]^T. \quad (8)$$

In microphone array beamforming, the distortionless constraint in the look direction is generally needed, i.e.,

$$\underline{\mathbf{h}}^H(\omega) \underline{\mathbf{d}}(\omega, \theta_{\mathrm{s}}) = 1. \quad (9)$$

Now, the beamforming problem becomes one of finding a filter $\underline{\mathbf{h}}(\omega)$ that can achieve an optimal performance with the distortionless constraint in (9). Generally, the optimality is evaluated through three metrics, i.e., the beampattern, the white noise gain (WNG), and the directivity factor (DF). The beampattern which describes the sensitivity of the beamformer to a plane wave impinging on the CCMA from the direction θ, is given by [9]

$$\mathcal{B}\left[\underline{\mathbf{h}}(\omega), \theta\right] = \underline{\mathbf{h}}^H(\omega) \underline{\mathbf{d}}(\omega, \theta). \quad (10)$$

The WNG, which evaluates the robustness of the beamformer to the array imperfections, is defined as [9]

$$\mathcal{W}\left[\underline{\mathbf{h}}(\omega)\right] = \frac{\left|\underline{\mathbf{h}}^H(\omega) \underline{\mathbf{d}}(\omega, \theta_{\mathrm{s}})\right|^2}{\underline{\mathbf{h}}^H(\omega) \underline{\mathbf{h}}(\omega)}. \quad (11)$$

The DF, which, as its name indicates, quantifies how directive is the beampattern, is expressed as

$$\mathcal{D}\left[\underline{\mathbf{h}}(\omega)\right] = \frac{\left|\underline{\mathbf{h}}^H(\omega) \underline{\mathbf{d}}(\omega, \theta_{\mathrm{s}})\right|^2}{\underline{\mathbf{h}}^H(\omega) \mathbf{\Gamma}_{\mathrm{d}}(\omega) \underline{\mathbf{h}}(\omega)}, \quad (12)$$

where $\mathbf{\Gamma}_{\mathrm{d}}(\omega)$ is the pseudo-coherence matrix of the diffuse noise, whose (i, j)th element is $\left[\mathbf{\Gamma}_{\mathrm{d}}(\omega)\right]_{ij} = \mathrm{sinc}(\omega \delta_{ij} / c)$, with $\delta_{ij} = \|\mathbf{r}_i - \mathbf{r}_j\|_2$ being the distance between microphones i and j, $\| \cdot \|$ being the Euclidean distance, and $\mathbf{r}_i, \mathbf{r}_j \in \left\{\mathbf{r}_{1,1}, \ldots, \mathbf{r}_{1,M_1}, \ldots, \mathbf{r}_{p,M_p}, \ldots, \mathbf{r}_{P,M_P}\right\}$.

3. ROBUST DMA BEAMFORMERS BASED ON SYMMETRIC NULL CONSTRAINTS

Differential beamformers attempt to measure the differential acoustic pressure field [28]. They generally use finite differences between the microphone sensors' outputs to approximate the true acoustic pressure differentials. This requires the sensor spacing to be much smaller than the smallest acoustic wavelength in the frequency band of interest. The earliest effort for differential beamforming with microphone arrays is the multistage DMA, in which an Nth-order DMA is formed by subtractively combining the outputs of two DMAs of order $N-1$ [8]. Following this principle, one can show that the frequency-independent directivity pattern of an Nth-order circular DMA corresponding to any desired direction, $\theta_{\rm s}$, can be written as [3, 18]

$$\mathcal{B}_N\left(\theta-\theta_{\rm s}\right)=\sum_{n=0}^{N} a_{N,n}\cos^n\left(\theta-\theta_{\rm s}\right), \tag{13}$$

where $a_{N,n}$, $n=0,1,\ldots,N$ are real coefficients. It is generally required that $\sum_{n=0}^{N} a_{N,n}=1$.

Given the aforementioned form of directivity pattern, the problem of differential beamforming now becomes one of approximation, i.e., finding a beamforming filter, $\underline{\mathbf{h}}\left(\omega\right)$, such that the designed beampattern, $\mathcal{B}\left[\underline{\mathbf{h}}\left(\omega\right),\theta\right]$, is as close as possible to the target frequency-invariant beampattern $\mathcal{B}_N\left(\theta-\theta_{\rm s}\right)$.

Different methods can be used to achieve this approximation. One way is through the Jacobi-Anger expansion [10]. With this expansion, the exponentials in (3) can be written as

$$e^{\jmath\varpi_p\cos\left(\theta-\psi_{p,m}\right)}\approx\sum_{n=-N_p}^{N_p}\jmath^n J_n\left(\varpi_p\right)e^{\jmath n\left(\theta-\psi_{p,m}\right)}, \tag{14}$$

where N_p ($N_p\leq N$) is the expansion order corresponding to the pth ring, $J_n(\varpi_p)$ is the nth-order Bessel function of the first kind with $J_{-n}\left(\varpi_p\right)=(-1)^n J_n\left(\varpi_p\right)$. In this case, the pth ring requires to have at least $2N_p+1$ microphones, and to design an Nth-order symmetric beampattern, we need to ensure $\max\{N_p,\ p=1,2,\ldots,N_P\}=N$. By substituting the Jacobi-Anger expansion and making the designed beampattern equal to the target beampattern, a system of linear equations can be constructed and the beamforming filter can subsequently be obtained through solving the linear system. For detailed description of such method, please refer to [10] and the references therein.

Although it works well with CCMAs, this series expansion method requires the analytic expression of the target beampattern, which may not be accessible in practice. Another way to design the differential beamformer so that its beampattern resembles the target frequency-invariant beampattern is through the so-called null-constrained method. In [27], a null-constrained method is proposed to design differential beamformers with CMAs, which does not require the knowledge of the target beampattern. However, as widely discussed in literature [3, 10], differential beamformers with circular arrays may suffer from irregularity problems at some frequencies. So, we propose to design differential beamformer with CCMAs by using symmetric null-constraints that can overcome the aforementioned problem.

It can be observed from (13) that the frequency-independent directivity pattern is symmetric with respect to $\theta_{\rm s}\ \leftrightarrow\ (\theta_{\rm s}+\pi)$. Therefore, when designing a differential beamformer with a CCMA, the symmetry condition should be considered, i.e.,

$$\mathcal{B}\left[\underline{\mathbf{h}}\left(\omega\right),\theta_{\rm s}+\theta\right]=\mathcal{B}\left[\underline{\mathbf{h}}\left(\omega\right),\theta_{\rm s}-\theta\right],\ \ \theta\in[0,\ \pi]. \tag{15}$$

Assume that the Nth-order differential beamformer with the main beam pointing to $\theta_{\rm s}$ has N distinct nulls: $\theta_{\rm s}<\theta_1<\cdots<\theta_N\leq\theta_{\rm s}+\pi$. The symmetric constraints with a CCMA can be written as

$$\underline{\mathbf{d}}^H\left(\omega,\theta_{\rm s}\pm\Delta\theta_n\right)\underline{\mathbf{h}}\left(\omega\right)=0, \tag{16}$$

where $\Delta\theta_n=\theta_n-\theta_{\rm s}$, $n=1,2,\ldots,N$. In the special case of $\theta_n=\theta_{\rm s}+\pi$, i.e., $\Delta\theta_n=\pi$, the two constraints at (16) should be combined.

By combining these symmetric null constraints and the distortionless constraint together, we get the following linear system of equations

$$\underline{\mathbf{D}}\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)=\mathbf{i}_{2N+1}, \tag{17}$$

where

$$\underline{\mathbf{D}}\left(\omega\right)=\begin{bmatrix}\underline{\mathbf{d}}^H\left(\omega,\theta_{\rm s}\right)\\ \underline{\mathbf{d}}^H\left(\omega,\theta_{\rm s}+\Delta\theta_1\right)\\ \underline{\mathbf{d}}^H\left(\omega,\theta_{\rm s}-\Delta\theta_1\right)\\ \vdots\\ \underline{\mathbf{d}}^H\left(\omega,\theta_{\rm s}+\Delta\theta_N\right)\\ \underline{\mathbf{d}}^H\left(\omega,\theta_{\rm s}-\Delta\theta_N\right)\end{bmatrix} \tag{18}$$

is an $(2N+1)\times\underline{M}$ matrix, and $\mathbf{i}_{2N+1}$ is a vector of length $(2N+1)$, whose first element is 1 and all other components are 0.

Given the constraints in (17), one can derive CCMA differential beamforming filter by optimizing certain performance measure. One way is through maximizing the WNG, which can be expressed as the following optimization problem:

$$\min\ \underline{\mathbf{h}}^H\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)\ \ \text{s.t.}\ \ \underline{\mathbf{D}}\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)=\mathbf{i}_{2N+1}. \tag{19}$$

The solution is the maximum WNG (MWNG) CCMA differential beamformer:

$$\underline{\mathbf{h}}_{\rm MWNG}\left(\omega\right)=\underline{\mathbf{D}}^H\left(\omega\right)\left[\underline{\mathbf{D}}\left(\omega\right)\underline{\mathbf{D}}^H\left(\omega\right)\right]^{-1}\mathbf{i}_{2N+1}. \tag{20}$$

Alternately, the CCMA differential beamforming filter can be derived from the maximization of the DF, which is equivalent to

$$\min\ \underline{\mathbf{h}}^H\left(\omega\right)\mathbf{\Gamma}_{\rm d}\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)\ \ \text{s.t.}\ \ \underline{\mathbf{D}}\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)=\mathbf{i}_{2N+1}. \tag{21}$$

The solution is the maximum DF (MDF) CCMA differential beamformer:

$$\underline{\mathbf{h}}_{\rm MDF}\left(\omega\right)=\mathbf{\Gamma}_{\rm d}^{-1}\left(\omega\right)\underline{\mathbf{D}}^H\left(\omega\right)\left[\underline{\mathbf{D}}\left(\omega\right)\mathbf{\Gamma}_{\rm d}^{-1}\left(\omega\right)\underline{\mathbf{D}}^H\left(\omega\right)\right]^{-1}\mathbf{i}_{2N+1}. \tag{22}$$

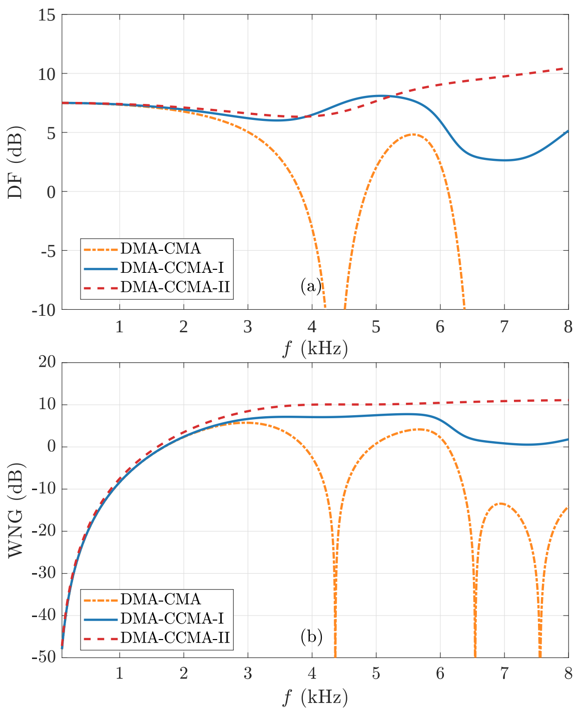

Fig. 2. Performance of the differential beamformers designed based on symmetric null-constrained method with CMAs and CCMAs: (a) DF and (b) WNG. Conditions: $\theta_\mathrm{s} = 30°$, $\theta_1 = 120°$, and $\theta_2 = 150°$.

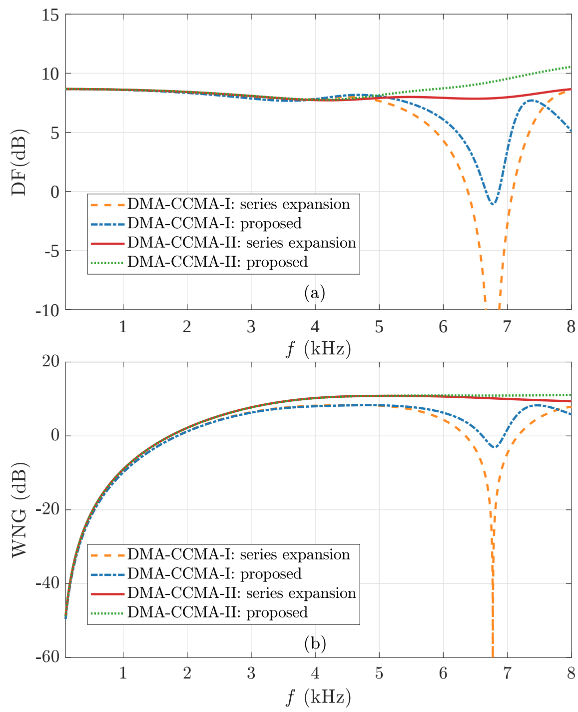

Fig. 3. Performance of the differential beamformers designed based on symmetric null-constrained method and series expansion method with CCMAs: (a) DF and (b) WNG. Conditions: $\theta_\mathrm{s} = 30°$, $\theta_1 = 102°$, and $\theta_2 = 174°$.

4. EXPERIMENTAL RESULTS

In this section, we study the performance of the proposed CCMA differential beamformers.

We first compare the proposed method with the null-constrained CMA differential beamformers [27]. We consider the design of a second-order differential beamformer with $\theta_\mathrm{s} = 30°$, where the two nulls are arbitrarily chosen as $120°$ and $150°$. We consider the following three array configurations: DMA-CMA: a CMA with $M = 6$, $r = 3.0$ cm; DMA-CCMA-I: a CMA plus a microphone at the center with $M_2 = 6$, $r_2 = 3.0$ cm; DMA-CCMA-II: a CCMA with two rings plus a microphone at the center, $r_2 = 2.0$ cm, $M_2 = 6$, $r_3 = 3.0$ cm, $M_3 = 6$, microphones at the two rings are not aligned with $\psi_{2,1} = 30°$, $\psi_{3,1} = 0°$. The differential beamformers with CCMAs are computed according to (20). Figure 2 plots the DFs and WNGs as a function of frequency for the aforementioned three cases. It is seen that the DMA-CMA suffers from serious degradation in DFs and WNGs at some frequencies. In contrast, the DF and WNG performances of the DMA-CCMAs are significantly improved, which indicates that using CCMAs with multiple rings can mitigate the irregularity problem. Not surprisingly, DMA-CCMA-II has better performance than DMA-CMA and DMA-CCMA-I since it uses more rings.

We then compare the proposed method with the series expansion method with CCMAs [10], where we use the DMA-CCMA-I and DMA-CCMA-II arrays configurations. Since the series expansion method requires the analytic form of the target beampattern, the target beampattern is chosen as the second order hypercardioid, which is

$$\mathcal{B}_2(\theta - \theta_\mathrm{s}) = 0.2 + 0.4\cos(\theta - \theta_\mathrm{s}) + 0.4\cos[2(\theta - \theta_\mathrm{s})], \tag{23}$$

when $\theta_\mathrm{s} = 30°$, the two nulls are at $102°$ and $174°$ respectively. Figure 3 plots the value of DFs and WNGs as a function of frequency. It is clearly seen that the proposed symmetric null-constrained method outperforms the series expansion method at high frequencies where the expansion errors are not negligible.

5. CONCLUSIONS

This paper presented a null-constrained method for the design of steerable differential beamformers with CCMAs. With the proposed approach, two different DMA beamformers were developed: the maximum WNG CCMA differential beamformer and the maximum DF CCMA differential beamformer. These beamformers suffer less (or even no) irregularity problem and yield better performance than their CMA counterparts. Furthermore, the proposed method does not require the analytic expression of the target beampattern as compared to the series expansion based CCMA differential beamformers. Simulation results demonstrated the superiority of the proposed method as compared to the series expansion approach as well as the CMA differential beamformers.

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