Steering Study of Linear Differential Microphone Arrays

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***Abstract*—Differential microphone arrays (DMAs) can achieve high directivity and frequency-invariant spatial response with small apertures; they also have a great potential to be used in a wide spectrum of applications for high-fidelity sound acquisition. Although many efforts have been made to address the design of linear DMAs (LDMAs), most developed methods so far only work for the situation where the source of interest is incident from the endfire direction. This paper studies the steering problem of differential beamformers with linear microphone arrays. We present new insights into beam steering of LDMAs and propose a series of steerable differential beamformers. The major contributions of this paper are as follows. 1) A series of ideal functions are defined to describe the ideal, target beampatterns of LDMAs. 2) We prove that first-order differential beamformers with linear microphone arrays are not steerable and their mainlobes can only be at the endfire directions. 3) We deduce the fundamental conditions for designing steerable differential beamformers with LDMAs. 4) We develop a method to design steerable beamformers with LDMAs using null constraints. Simulations and experiments validate the properties of the developed method.**

***Index Terms*—Microphone arrays, uniform linear arrays, differential beamforming, frequency-invariant beamformer, beampattern, beam steering.**

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I. Introduction

MICROPHONE arrays combined with proper beamforming algorithms have been widely used in speech communication and human-machine interface systems to extract the speech signals of interest from unwanted noise and interference. Many beamforming methods have been developed over the last few decades, including the delay-and-sum [1], [2], filter-and-sum and broadband [3]–[6], modal [7]–[11], superdirective [12]–[21], differential [22]–[32], and adaptive [33]–[51] beamformers, etc. Among those, the so-called differential beamformer has attracted much research interest since it is able to form frequency-invariant beampatterns and has the potential to attain high directional gains with small size arrays [13], [25], [26], [52]–[55].

The principle of differential beamforming can be traced back to the 1940s when directional microphones were invented [56], [57], but it was not introduced into the regime of microphone arrays until the 1990s [22], [23], [52]. The earliest differential beamforming method is implicitly embedded in the so-called differential microphone arrays (DMAs), which are designed to measure the spatial derivatives of the sound pressure field. Basically, a first-order DMA consists of two closely spaced omnidirectional microphones and its output is the subtraction of one sensor's output from that of the other. The two omnidirectional microphones measure the sound pressure field, each from its own viewpoint. Since they are closely spaced, the difference between their outputs approximates very well the spatial derivative of the sound pressure field along the axis that connects the two sensors. So, this simple subtraction operation gives a differential beamformer that is responsive to the first-order differential of the acoustic pressure field. Similarly, one can construct second- and higher-order DMAs. Generally, an Nth-order DMA, which uses a total of $N + 1$ omnidirectional microphones, is formed by subtractively combining the outputs of two DMAs of order $N - 1$ [23], [25], [52]. This seemingly simple way of constructing microphone arrays lays out the foundation for differential beamforming and, interestingly, the resulting beampattern is frequency independent, which is an algebraic polynomial of the cosine function of the azimuth angle. The order of this polynomial is equal to the order of the differential beamformer, which in turn measures the same order of the differential pressure field. From a beampattern perspective, a higher-order differential beamformer generally has narrower beamwidth and a higher directivity factor (DF), which measures how directive a beampattern is. As a result, one would want to go with as high order as possible in practical applications for sufficient spatial noise and interference suppression. However, this way of doing differential beamforming amplifies significantly spatial white noise, particularly at low frequencies, making the beamformer sensitive to array imperfections such as the sensors' self noise, sensors placement errors, and mismatch among different sensors. The higher the order, the more serious is white noise amplification.

Inspired by the frequency-independent polynomial form of the DMA beampatterns, approximation-based approaches to differential beamforming were developed in the short-time Fourier transform (STFT) domain, which design the beamforming filter in such a way that the resulting beampattern resembles the ideal, target DMA beampattern. If the mainlobe is pointed to the endfire direction, which is assumed to be true in most differential beamforming methods with linear arrays, a DMA beampattern is uniquely determined by its nulls. Based on this fact, a null-constrained approach was developed in [25], [26], where the differential beamformer is designed in each STFT subband by solving a linear system of equations constructed from constraints on the nulls' directions of the target directivity pattern. A prominent advantage of this approach is that one can maximize the so-called white noise gain (WNG) by increasing the number of microphones with a given order of the DMA beampattern, leading to an optimal way in dealing with the problem of white noise amplification. Alternatively, one can also design differential beamformers using a series expansion technique, which approximates the beamformer's beampattern using, e.g., the Jacobi-Anger expansion and then identifies the beamforming coefficients using the relationship between the beamformer's beampattern and the target directivity pattern [10], [11]. While it provides a better way to control the designed beampattern so that it closely matches the target beampattern as compared to the null-constrained method, the series expansion approach needs to know the target beampattern as well as its analytic form as the *a priori* information, which may not be accessible in real-world applications.

Although, a great deal of efforts have been devoted to differential beamforming with linear DMAs (LDMAs), most methods developed so far assume that the mainlobe is at the endfire direction, i.e., $0°$. The major reasons for this include: 1) differential beamformers with LDMAs generally achieve the maximum DF at the endfire directions so it is natural to assume that the look direction is at $0°$, and 2) differential beamformers with LDMAs have limited steering flexibility. In applications such as hearing aids and bluetooth headsets, the endfire assumption holds true and steering is not really needed. But in many other applications, such as smart televisions as the one illustrated in Fig. 1, steering is desired as the source position can vary. In the context of LDMAs, when one attempts to steer the mainlobe of a differential beamformer to a different look direction other than $0°$, three cases can happen. 1) Steerable: in this case, the resulting beampattern is simply a rotation of the beampattern at $0°$. 2) Partially steerable: in this case, the beampattern has a gain of 1 at the look direction and gains of less than or equal to 1 at other directions, but the beampattern steered to the look direction is different from the one at $0°$. 3) Non-steerable: the beampattern has a gain of 1 at the look direction, but also gains greater than 1 at some other directions. Differential beamformers with LDMAs have limited steering flexibility, which means that the steerable case cannot happen and the best case that is achievable is the second one; though, in many other scenarios the beamformer may not be partially steerable at all. Consequently, it is important to study under what conditions a differential beamformer becomes partially steerable and how to steer it to a specified look direction if it can be steered, which is the main focus of this work.

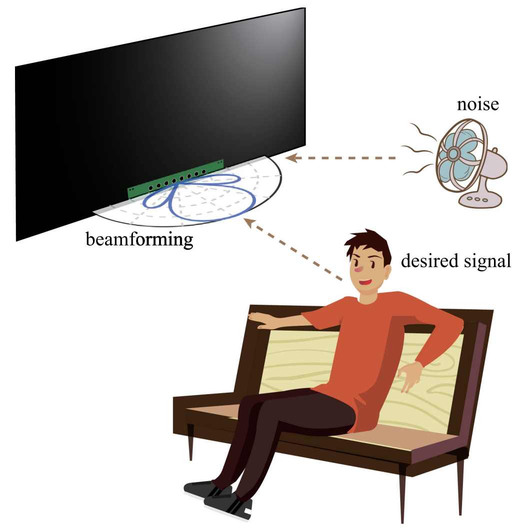

Fig. 1. Illustration of an LDMA integrated into a smart television. This LDMA is mounted at the bottom side to form a directivity pattern in the broadside.

There have been some efforts in the literature to address the problem of steerable DMAs, which can be classified into two categories. The first one addresses how to steer beamformers with LDMAs [22], [58]–[60], but they focus only on first- or second-order DMAs. Besides, not much analysis has been provided whether the steering is successful or not. The second group attempts to use two or three dimensional microphone arrays, e.g., circular arrays [10], [29], square arrays [31], [61], [62], concentric circular arrays [11], and spherical arrays [7], [8], [63]–[67]. However, in many applications, such as the one illustrated in Fig. 1, linear arrays are preferable and widely used.

Apparently, there is a great need to study the problem of beam steering with LDMAs, which is the focus of this paper. By analyzing the polynomial form of an Nth-order LDMA, we prove that the mainlobe of a first-order LDMA can only be at the endfire direction. We then deduce the fundamental conditions for designing steerable second- and third-order LDMAs. These results are subsequently generalized to the general case of Nth-order LDMAs. Based on the analysis of the fundamental conditions, we propose a method to design Nth-order steerable LDMAs (SLDMAs) using null constraints.

The remainder of this paper is organized as follows. In Section II, we present the signal model, problem formulation, and performance metrics. We then briefly describe the null-constrained DMA design method in Section III. In Section IV,

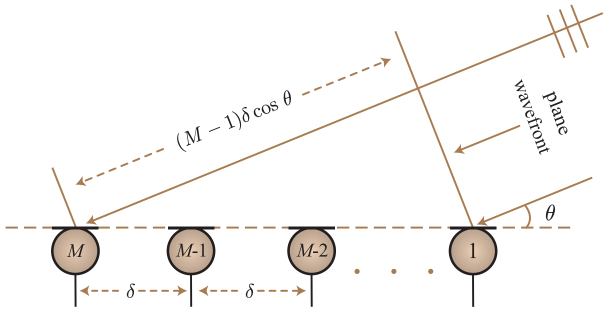

Fig. 2. Illustration of a uniform LDMA consisting of M omnidirectional microphones, where δ is the interelement spacing and θ denotes the incidence azimuth angle.

we introduce the ideal function of DMAs and deduce the conditions for designing steerable LDMAs. In Section V, we propose a method to design steerable LDMAs with null constraints. Some simulations and design examples are presented in Section VI to validate the proposed method for the design of SLDMAs. Finally, conclusions are given in Section VII.

II. Signal Model, Problem Formulation, and Performance Metrics

Consider a plane wave, in the farfield, that propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on a uniform linear array (ULA) consisting of M omnidirectional microphones with an interelement spacing δ, as illustrated in Fig. 2. If the incidence angle is parameterized by θ, then the corresponding phase vector (of length M) is given by [68]

$$\mathbf{d}(\omega, \cos\theta) \triangleq \left[1 \; e^{-\jmath\varpi\cos\theta} \; \cdots \; e^{-\jmath(M-1)\varpi\cos\theta} \right]^T, \quad (1)$$

where $\jmath$ is the imaginary unit, with $\jmath^2 = -1$, $\varpi = \omega\delta/c$, with $\omega = 2\pi f$ being the angular frequency and $f > 0$ the temporal frequency, and the superscript ^{T} is the transpose operator. The acoustic wavelength is $\lambda = c/f$. In this paper, we consider small values of the interelement spacing, i.e., δ is much smaller than the smallest wavelength in the frequency band of interest, such that the acoustic pressure differentials can be approximated by finite differences of the microphones outputs.

Considering the general case where the signal of interest (i.e., the desired signal) comes from the direction θ_s, we can express the frequency-domain observation signal vector of length M as [2]

$$\begin{aligned}\mathbf{y}(\omega) &\triangleq \left[Y_1(\omega) \; Y_2(\omega) \; \cdots \; Y_M(\omega) \right]^T \\ &= \mathbf{d}(\omega, \cos\theta_\mathrm{s}) X(\omega) + \mathbf{v}(\omega), \end{aligned} \quad (2)$$

where $X(\omega)$ is the zero-mean source signal of interest and $\mathbf{v}(\omega)$ is the zero-mean additive noise signal vector defined similarly to $\mathbf{y}(\omega)$.

Beamforming is a process to design a spatial filter, $\mathbf{h}(\omega)$, that we apply to the observation vector in order to obtain a good estimate of $X(\omega)$. The output of the beamformer is

$$Z(\omega) = \mathbf{h}^H(\omega)\mathbf{y}(\omega), \quad (3)$$

where the superscript ^{H} is the conjugate-transpose operator. The distortionless constraint in the desired (assumed to be the look) direction is generally needed, i.e.,

$$\mathbf{h}^H(\omega)\mathbf{d}(\omega, \cos\theta_\mathrm{s}) = 1. \quad (4)$$

So, the problem of differential beamforming is to find an optimal beamforming filter subject to the distortionless constraint in (4). The optimality is generally evaluated using three performance measures: beampattern, DF, and WNG.

The beampattern describes the sensitivity of a beamformer to a plane wave impinging on the array from the direction θ. It is defined as [13], [25]

$$\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta] \triangleq \mathbf{h}^H(\omega)\mathbf{d}(\omega, \cos\theta). \quad (5)$$

One can check that with a ULA as illustrated in Fig. 2, the beampattern is symmetric with respect to the endfire directions, i.e.,

$$\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta] = \mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), -\theta]. \quad (6)$$

In the context of steerable DMAs, we have the following three scenarios.

1) Steerable. In this case, for any given $\theta_\mathrm{s} \in [0^\circ, 180^\circ]$, we expect to have $|\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta_\mathrm{s}]|^2 = 1$, $|\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta]|^2 \leq 1$, and $\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta]$ is a rotation of $\mathcal{B}_0[\mathbf{h}(\omega), \theta]$ by θ_s. This case cannot be achieved with LDMAs.
2) Partially steerable. In this case, for any given $\theta_\mathrm{s} \in [0^\circ, 180^\circ]$, we expect to have $|\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta_\mathrm{s}]|^2 = 1$, $|\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta]|^2 \leq 1$, but $\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta]$ varies with θ_s and may not be a rotation of $\mathcal{B}_0[\mathbf{h}(\omega), \theta]$ by θ_s. This is the best we can do with LDMAs. This scenario is studied in this paper.
3) Non-steerable. In this case, for a given $\theta_\mathrm{s} \in [0^\circ, 180^\circ]$, we have $|\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta_\mathrm{s}]|^2 = 1$, but $|\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta]|^2 \geq 1$ for some angles. Therefore, the beampattern is not a valid one and the corresponding beamformer may amplify noise and interference. We will discuss later how to avoid this from happening.

Note that in the rest of this paper, we will drop the subscript θ_s from $\mathcal{B}_{\theta_\mathrm{s}}[\mathbf{h}(\omega), \theta]$ to simplify the notation, which should not lead to any confusion.

The WNG, which evaluates the sensitivity of a microphone array to some of its imperfections, is defined as [25]

$$\mathcal{W}[\mathbf{h}(\omega)] \triangleq \frac{\left|\mathbf{h}^H(\omega)\mathbf{d}(\omega, \cos\theta_\mathrm{s})\right|^2}{\mathbf{h}^H(\omega)\mathbf{h}(\omega)}. \quad (7)$$

The DF, which quantifies the spatial gain of a beamformer in a spherical isotropic noise field, is defined as [13], [25], [69]

$$\mathcal{D}[\mathbf{h}(\omega)] \triangleq \frac{|\mathcal{B}[\mathbf{h}(\omega), \theta_\mathrm{s}]|^2}{\frac{1}{2}\int_0^\pi |\mathcal{B}[\mathbf{h}(\omega), \theta]|^2 \sin\theta d\theta}, \quad (8)$$

which can be rewritten as

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{\left|\mathbf{h}^H(\omega)\mathbf{d}(\omega, \cos\theta_\mathrm{s})\right|^2}{\mathbf{h}^H(\omega)\boldsymbol{\Gamma}_\mathrm{d}(\omega)\mathbf{h}(\omega)}, \quad (9)$$

where the (i,j)th element (for $i,j = 1,2,\ldots,M$) of $\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)$ is

$$[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)]_{ij} = \frac{\sin[\omega(j-i)\delta/c]}{\omega(j-i)\delta/c}, \tag{10}$$

with $[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)]_{ii} = 1$.

III. Conventional Design of LDMAs

The ideal spatial response to the Nth-order derivative of the sound pressure field is of the following form [23], [25]:

$$\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n}\cos^n\theta = \mathbf{a}_N^T\mathbf{p}(\theta), \tag{11}$$

where $a_{N,n},\ n = 0,1,\ldots,N$ are real coefficients and

$$\mathbf{a}_N = \begin{bmatrix} a_{N,0} \ a_{N,1} \ \cdots \ a_{N,N}\end{bmatrix}^T, \tag{12}$$

$$\mathbf{p}(\theta) = \begin{bmatrix} 1 \ \cos\theta \ \cdots \ \cos^N\theta\end{bmatrix}^T. \tag{13}$$

So, the problem of designing an Nth-order LDMA becomes one of designing the beamforming filter so that the corresponding beampattern is as close as possible to $\mathcal{B}_N(\theta)$. Therefore, we can call $\mathcal{B}_N(\theta)$ the Nth-order ideal (or target) beampattern. Taking (11) as the target beampattern, one way to design the differential beamformer is to optimize the filter, $\mathbf{h}(\omega)$, such that its beampattern is as close as possible to the target one. A representative method is to design the differential beamformer using the null information. Generally, an Nth-order DMA beampattern has N nulls. So, a straightforward way to find the filter is by building the relationship between the nulls of the beamformer's beampattern and those of the target beampattern [25], which can be viewed as an extension of the traditional multistage subtraction method [28].

Let us assume that the Nth-order DMA target beampattern has N distinct nulls[1] at $\theta_1,\theta_2,\ldots,\theta_N$. Combining these constraints with the distortionless one given in (4), we can form the following linear system of equations [25], [26]:

$$\mathbf{D}(\omega,\mathbf{x}_N)\,\mathbf{h}(\omega) = \mathbf{i}_1, \tag{14}$$

where

$$\mathbf{x}_N = \begin{bmatrix} x_{\mathrm{s}} \ x_1 \ \cdots \ x_N\end{bmatrix}^T, \tag{15}$$

with $x_{\mathrm{s}} = \cos\theta_{\mathrm{s}}$ and $x_n = \cos\theta_n,\ n = 1,2,\ldots,N$,

$$\mathbf{D}(\omega,\mathbf{x}_N) = \begin{bmatrix} \mathbf{d}^H(\omega,x_{\mathrm{s}}) \\ \mathbf{d}^H(\omega,x_1) \\ \vdots \\ \mathbf{d}^H(\omega,x_N)\end{bmatrix}, \tag{16}$$

and $\mathbf{i}_1 = [1\ 0 \cdots 0]^T$.

To design an Nth-order LDMA, at least $N+1$ microphones are needed. If the number of microphones M is equal to $N+1$, then the solution of (14) is

$$\mathbf{h}_{\mathrm{E}}(\omega) = \mathbf{D}^{-1}(\omega,\mathbf{x}_N)\,\mathbf{i}_1. \tag{17}$$

[1]If there are multiple nulls occurring in the same direction, we invite the reader to see [26] Thus, the method discussed in this paper can be easily extended to such cases.

If we have more than $N+1$ microphones, i.e., $M > N+1$, then the minimum-norm solution of (14) can be derived, i.e.,

$$\mathbf{h}_{\mathrm{MN}}(\omega) = \mathbf{D}^H(\omega,\mathbf{x}_N)\left[\mathbf{D}(\omega,\mathbf{x}_N)\,\mathbf{D}^H(\omega,\mathbf{x}_N)\right]^{-1}\mathbf{i}_1. \tag{18}$$

It has been shown that this solution yields an Nth-order DMA while improving the WNG, which increases with the number of microphones [25], [26]. In conventional methods with LDMAs, it is generally assumed that the look direction is at the endfire, i.e., $\theta_{\mathrm{s}} = 0^\circ$, which greatly restricts their practical applications. Therefore, it is important to study under what conditions a differential beamformer can be steered to a desired look direction other than the endfire, which will be studied in the next section.

IV. Mainlobe Steering Conditions of LDMAs

Taking $x = \cos\theta$, the ideal beampattern in (11) can be rewritten as an algebraic polynomial of order N with respect to x as

$$\mathcal{P}_N(x) = \sum_{n=0}^{N} a_{N,n}x^n. \tag{19}$$

Since an Nth-order polynomial has N zeros, (19) can be rewritten as

$$\mathcal{P}_N(x) = \frac{1}{\xi_N}\prod_{n=1}^{N}(x - x_n), \tag{20}$$

where $\xi_N = \prod_{n=0}^{N}(x_{\mathrm{s}} - x_n)$ is a normalization factor to satisfy the distortionless constraint in the desired look direction. By comparing (20) and (19), it is not difficult to check that

$$\frac{1}{\xi_N} = a_{N,N}. \tag{21}$$

So, (20) can be rewritten as

$$\mathcal{P}_N(x) = a_{N,N}\prod_{n=1}^{N}(x - x_n). \tag{22}$$

Since the function (22) is directly derived from the ideal beampattern, in our context, we name $\mathcal{P}_N(x)$ as the Nth-order ideal function.

Now, let us study the relationship between the ideal function and the ideal beampattern. It can be checked that the phase vector with respect to x is periodic, i.e.,

$$\mathbf{d}(\omega,x) = \mathbf{d}\left(\omega, x + \frac{2k\pi}{\varpi}\right), \tag{23}$$

where k is an integer number and the period is $\dfrac{c}{f\delta}$. Then, from (5) and (23), the beampattern is also periodic with respect to x, i.e.,

$$\mathcal{B}[\mathbf{h}(\omega), x] = \mathcal{B}\left[\mathbf{h}(\omega), x + \frac{c}{f\delta}\right]. \tag{24}$$

For small-size LDMAs, we have $c \gg f\delta$. So, only the part of the beampattern in the interval $-1 \le x \le 1$ can be seen and is responsible for sound acquisition. Therefore, for ideal functions, we define the "visible zone" of which the boundary consists of

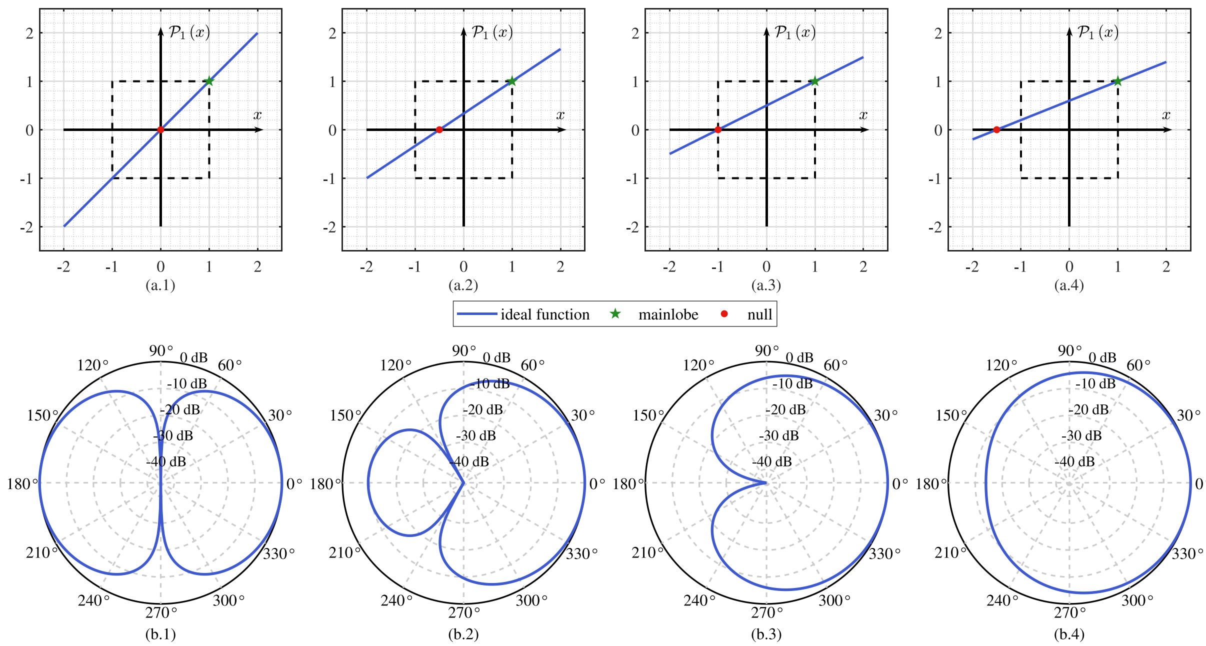

Fig. 3. First-order ideal function, $\mathcal{P}_1(x)$, and the corresponding first-order ideal beampattern, $\mathcal{B}_1(\theta)$. The dashed line is the boundary of the visible zone, and the part of the ideal function outside the visible zone is invisible in the ideal beampattern. The values of the null are respectively: (a.1) (b.1) dipole, $x_1 = \cos(90^\circ)$; (a.2) (b.2) hypercardioid, $x_1 = \cos(120^\circ)$; (a.3) (b.3) cardioid, $x_1 = \cos(180^\circ)$; and (a.4) (b.4) subcardioid, $x_1 = -1.5$.

$x = \pm 1$ and $\mathcal{P}_N(x) = \pm 1$. The part of the ideal function inside the visible zone corresponds to the ideal beampattern $\mathcal{B}_N(\theta)$ in $0 \leq \theta \leq \pi$.

We first consider the simplest case with $N = 1$. Then, the first-order ideal function is

$$\mathcal{P}_1(x) = a_{1,1}x + a_{1,0}, \tag{25}$$

with $a_{1,1} \neq 0$, which is a linear function of x. For the conventional first-order LDMA, the mainlobe direction is set to the endfire direction, i.e., $\theta_{\mathrm{s}} = 0^\circ$, so we have $\mathcal{P}_1(1) = 1$. Then, $\mathcal{P}_1(x)$ is uniquely determined by the single null at x_1. To see more clearly, we present the following four scenarios.

1) First-order dipole: $x_1 = 0$, and $a_{1,1} = 1$, $a_{1,0} = 0$.
2) First-order hypercardioid: $x_1 = -1/2$, and $a_{1,1} = 2/3$, $a_{1,0} = 1/3$.
3) First-order cardioid: $x_1 = -1$, and $a_{1,1} = 1/2$, $a_{1,0} = 1/2$.
4) First-order subcardioid: $x_1 = -3/2$, and $a_{1,1} = 2/5$, $a_{1,0} = 3/5$.

For LDMAs, the role of the invisible part cannot be neglected. Fig. 3 plots the first-order ideal functions and their corresponding ideal beampatterns. In particular, Fig. 3(a.4) and (b.4) illustrate that the invisible null can also be used to design LDMAs.

The derivative of $\mathcal{P}_1(x)$ with respect to θ is

$$\frac{d\mathcal{P}_1(x)}{d\theta} = -a_{1,1}\sqrt{1 - x^2}. \tag{26}$$

This derivative is equal to 0 if and only if $x = \pm 1$ since $a_{1,1} \neq 0$. Therefore, the maximum and minimum of $\mathcal{P}_1(x)$ in the range of $-1 \leq x \leq 1$ can only appear at $x = \pm 1$. Consequently, the look direction of the first-order LDMA can only be at 0° or 180° and cannot be steered to other directions regardless of what method is used.

It is easy to check that

$$\frac{d\mathcal{P}_N(x)}{d\theta} = -\sqrt{1 - x^2} \times \frac{d\mathcal{P}_N(x)}{dx}. \tag{27}$$

Therefore, in the range of $-1 < x < 1$, analyzing $\frac{d\mathcal{P}_N(x)}{d\theta}$ is equivalent to analyzing $\frac{d\mathcal{P}_N(x)}{dx}$. So, in the rest part of this paper, we study the beam steering problem by directly analyzing the derivative of $\mathcal{P}_N(x)$ with respect to x.

The second-order ideal function is

$$\mathcal{P}_2(x) = a_{2,2}x^2 + a_{2,1}x + a_{2,0} \tag{28}$$

and its derivative with respect to x is

$$\frac{d\mathcal{P}_2(x)}{dx} = 2a_{2,2}x + a_{2,1}. \tag{29}$$

Since $\mathcal{P}_2(x_{\mathrm{s}})$ must be a maximum, the derivative of $\mathcal{P}_2(x)$ should be equal to 0 at x_{s}. It follows immediately that

$$x_{\mathrm{s}} = \frac{-a_{2,1}}{2a_{2,2}}. \tag{30}$$

If $-1 < x_{\mathrm{s}} < 1$, $\mathcal{P}_2(x)$ corresponds to the ideal beampattern of a second-order DMA. To achieve the mainlobe steering, a core issue in the design is how to reasonably set the null constraints. In other words, to steer a second-order DMA, the steering direction, x_{s}, and the values of the nulls, x_1 and x_2, must satisfy certain conditions.

From (22), the second-order ideal function can be written as

$$\mathcal{P}_2(x) = a_{2,2}x^2 - a_{2,2}(x_1 + x_2)x + a_{2,2}x_1x_2. \tag{31}$$

Comparing (28) with (31), one can have

$$x_1 + x_2 = -\frac{a_{2,1}}{a_{2,2}}. \tag{32}$$

Substituting (32) into (30), the relationship among x_s, x_1, and x_2 can be expressed as

$$x_1 + x_2 = 2x_\mathrm{s}, \tag{33}$$

which is the fundamental condition for designing steerable second-order LDMAs.

Similarly, from (19), the third-order ideal function is

$$\mathcal{P}_3(x) = a_{3,3}x^3 + a_{3,2}x^2 + a_{3,1}x + a_{3,0}. \tag{34}$$

The derivative of $\mathcal{P}_3(x)$ with respect to x is

$$\frac{d\mathcal{P}_3(x)}{dx} = 3a_{3,3}x^2 + 2a_{3,2}x + a_{3,1}. \tag{35}$$

Let the derivative of $\mathcal{P}_3(x)$ at x_s be equal to 0, we get

$$3x_\mathrm{s}^2 + 2\frac{a_{3,2}}{a_{3,3}}x_\mathrm{s} + \frac{a_{3,1}}{a_{3,3}} = 0. \tag{36}$$

By comparing (22) with (34), it can be deduced that

$$\frac{a_{3,2}}{a_{3,3}} = -x_1 - x_2 - x_3, \tag{37}$$

$$\frac{a_{3,1}}{a_{3,3}} = x_1x_2 + x_1x_3 + x_2x_3. \tag{38}$$

Substituting (37) and (38) into (36), one can obtain the fundamental condition to design steerable third-order LDMAs:

$$3x_\mathrm{s}^2 - 2\sum_{n=1}^{3} x_n x_\mathrm{s} + x_1x_2 + x_2x_3 + x_1x_3 = 0. \tag{39}$$

For the Nth-order ideal function, its derivative with respect to x is

$$\frac{d\mathcal{P}_N(x)}{dx} = \sum_{n=1}^{N} n a_{N,n} x^{n-1}. \tag{40}$$

By expanding the Nth-order ideal function (20), we have

$$\mathcal{P}_N(x) = a_{N,N}\sum_{n=0}^{N}(-1)^{N-n}\zeta_{N,n}x^n, \tag{41}$$

where

$$\zeta_{N,N} = 1, \tag{42}$$

$$\zeta_{N,N-1} = x_1 + x_2 + \cdots + x_N, \tag{43}$$

$$\zeta_{N,N-2} = x_1x_2 + x_1x_3 + \cdots + x_{N-1}x_N, \tag{44}$$

$$\vdots$$

$$\zeta_{N,1} = x_1x_2\cdots x_{N-1} + \cdots + x_2x_3\cdots x_N, \tag{45}$$

$$\zeta_{N,0} = x_1x_2\cdots x_{N-1}x_N. \tag{46}$$

From (19) and (41), we can obtain

$$\zeta_{N,n}(-1)^{N-n} = \frac{a_{N,n}}{a_{N,N}},\ n = 0, 1, \ldots, N. \tag{47}$$

To find the maximum of the beampattern, the derivative in (40) is set to zero. Then, substituting (47) into (40), we get the fundamental condition for constructing an Nth-order steerable ideal function as

$$\sum_{n=1}^{N} n(-1)^{N-n}\zeta_{N,n}x_\mathrm{s}^{n-1} = 0. \tag{48}$$

V. Design of Steerable LDMAs

Having studied the conditions for SLDMAs, we are now ready to discuss how to design an SLDMA. According to (17) and (18), we need $N+1$ parameters to determine the beamforming filter for an N-order SLDMA, i.e., the steering direction and the directions for the N nulls. For the problem at hand, the steering direction can be assumed to be given. Then, the design problem becomes one of determining the N nulls. Without loss of generality, let us assume that the N nulls are arranged in an ascending order. One straightforward way to determine the directions of the N nulls is to set the locations of the first $N-1$ nulls according to the requirements of the practical application and then determine the last null according to the condition in (48).

1) In the case that $x_\mathrm{s} = 0$, i.e., $\theta_\mathrm{s} = 90^\circ$, (clearly, $x_n \neq 0,\ n = 1, 2, \ldots, N$), the fundamental condition in (48) can be rewritten as

$$\zeta_{N,1} = 0. \tag{49}$$

From the definition of $\zeta_{N,1}$, the last null to control steering is deduced as

$$x_N = \frac{-1}{\sum_{n=1}^{N-1} 1/x_n}. \tag{50}$$

2) In the case that $x_\mathrm{s} \neq 0$, we define the following vector:

$$\mathbf{q}_N(x) = \begin{bmatrix} 1 & x & \cdots & x^N \end{bmatrix}^T, \tag{51}$$

where $x \in \{x_\mathrm{s}, x_1, x_2, \ldots, x_N\}$. Setting the derivative of the ideal function at x_s to 0 gives

$$\mathbf{q}_N^T(x_\mathrm{s})\,\boldsymbol{\Sigma}_N\mathbf{a}_N = 0, \tag{52}$$

where $\boldsymbol{\Sigma}_N = \mathrm{diag}(0, 1, \ldots, N)$ is a diagonal matrix.

If the ideal function has distinct nulls, the coefficients vector, $\mathbf{a}_N$, defined in (12) can be derived from the following linear system of equations according to the study in [25]:

$$\mathbf{Q}(x)\mathbf{a}_N = \mathbf{i}_1, \tag{53}$$

where

$$\mathbf{Q}(x) = \begin{bmatrix} \mathbf{q}_N^T(x_\mathrm{s}) \\ \mathbf{q}_N^T(x_\mathrm{s})\,\boldsymbol{\Sigma}_N \\ \mathbf{q}_N^T(x_1) \\ \vdots \\ \mathbf{q}_N^T(x_{N-1}) \end{bmatrix}. \tag{54}$$

For high order LDMAs, multiple nulls may occur in the same direction. If the ideal function $\mathcal{P}_N(x)$ has P $(1 \leq P \leq N)$ nulls at the same direction, i.e., $x_n = x_{n+1} =$

TABLE I
PARAMETERS OF THE SECOND-ORDER SLDMAS

	$x_{\rm s}$	x_1	x_2
SSLDMA-I	$\cos(90^\circ)$	$\cos(160^\circ)$	0.9397
SSLDMA-II	$\cos(75^\circ)$	$\cos(145^\circ)$	1.3368

$\cdots = x_{n+P-1}$ (in other words, x_n is a null with multiplicity of P), we need to construct $\mathbf{Q}(x)$ according to the method proposed in [26] as

$$\mathbf{Q}(x) = \begin{bmatrix} \mathbf{q}_N^T(x_{\rm s}) \\ \mathbf{q}_N^T(x_{\rm s})\mathbf{\Sigma}_N \\ \mathbf{q}_N^T(x_1) \\ \vdots \\ \mathbf{q}_N^T(x_n) \\ \mathbf{q}_N^T(x_n)\mathbf{\Sigma}_N \\ \vdots \\ \mathbf{q}_N^T(x_n)\mathbf{\Sigma}_N^{P-1} \\ \mathbf{q}_N^T(x_{n+P}) \\ \vdots \\ \mathbf{q}_N^T(x_{N-1}) \end{bmatrix}. \quad (55)$$

The solution for $\mathbf{a}_N$ is

$$\mathbf{a}_N = \mathbf{Q}^{-1}(x)\mathbf{i}_1. \quad (56)$$

Assume that the last two elements of $\mathbf{a}_N$ are $a_{N,N-1}$ and $a_{N,N}$, then the last null, x_N, can be determined from the definition of $\zeta_{N-1,N}$ in (47) and (43) as

$$x_N = -\frac{a_{N,N-1}}{a_{N,N}} - \sum_{n=1}^{N-1} x_n. \quad (57)$$

As a result, the vector $\mathbf{x}_N$ is subsequently obtained according to (15). Finally, by substituting the determined null vector, $\mathbf{x}_N$, into the linear system in (14) and solving the optimal filter by (17) or (18), we obtain the beamforming filter of the Nth-order steerable LDMAs.

VI. SIMULATIONS

In this section, we study the performance of the proposed method for designing steerable LDMAs.

A. SLDMA Design With Distinct Nulls

We first study the performance of second-order SLDMA beamformers with three microphones, where the interelement spacing, δ, is set to 1 cm. We consider two cases: SSLDMA-I and SSLDMA-II, whose mainlobes are at, respectively, 90° and 75°. We assume that one null x_1 is pre-specified and the other null x_2 is obtained from (33). The parameters for the two beamformers are shown in Table I. Fig. 4 plots the ideal functions, designed beampatterns at 1 kHz, and broadband beampatterns versus frequency. As seen, steering is achieved successfully.

The ideal functions of the second-order SLDMAs are parabolas and the nulls are symmetrically distributed on both sides of $x_{\rm s}$, which is clear from Fig. 4. It is also seen from Fig. 4

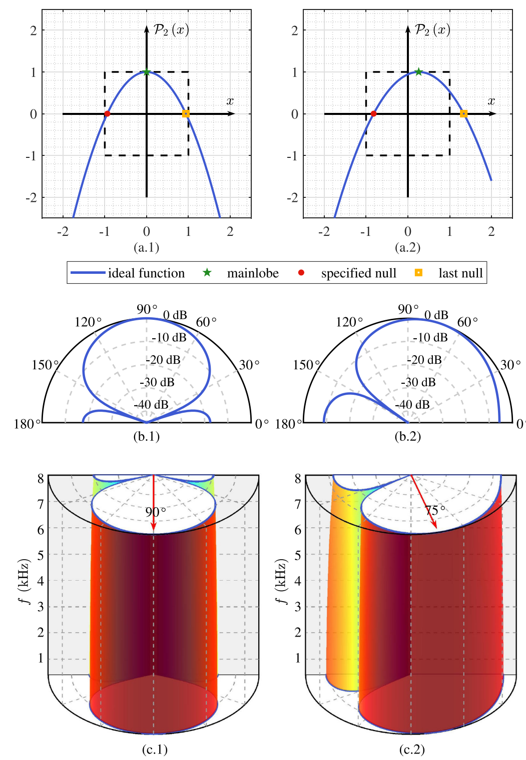

Fig. 4. The ideal functions and beampatterns of the second-order SLDMAs: (a.1) (b.1) (c.1) SSLDMA-I and (a.2) (b.2) (c.2) SSLDMA-II. (a.1) and (a.2) are the ideal functions; (b.1) and (b.2) are the beampatterns at $f = 1$ kHz; and (c.1) and (c.2) are the broadband beampatterns versus frequency, f. Conditions of simulation: $M = 3$ and $\delta = 1$ cm.

TABLE II
PARAMETERS OF THE THIRD-ORDER SLDMAS

	$x_{\rm s}$	x_1	x_2	x_3
TSLDMA-I	$\cos(70^\circ)$	$\cos(125^\circ)$	$\cos(175^\circ)$	0.8857
TSLDMA-II	$\cos(45^\circ)$	$\cos(100^\circ)$	$\cos(150^\circ)$	1.2717

that the beampatterns are frequency invariant, which is an important property of differential beamformers for high-fidelity sound acquisition. Fig. 5 plots the DFs and WNGs of the two beamformers. Not surprisingly, the DF and WNG vary with the steering angle, $\theta_{\rm s}$. Comparing the results in Fig. 5 with those in [25], one can see that a second-order SLDMA has its maximum DF at the endfire directions, which explains why most works on LDMA assume the steering direction at 0°.

Now, we study the design of third-order SLDMAs with four microphones. We still consider two cases: TSLDMA-I and TSLDMA-II, with the mainlobes being at, respectively, 70° and 45°. Similarly, we assume that x_1 and x_2 are pre-specified but the last null x_3 is determined according to (39). These parameters are listed in Table II. Fig. 6 plots the ideal functions and beampatterns. Again, it is seen that the steering is successful. Fig. 7 plots the DFs and WNGs of the third-order SLDMAs. One can see that the DF is higher but the WNG is lower than the second-order SLDMA shown in the previous simulation.

In this simulation, we study the performance of fourth-order SLDMAs with five microphones. We consider four different

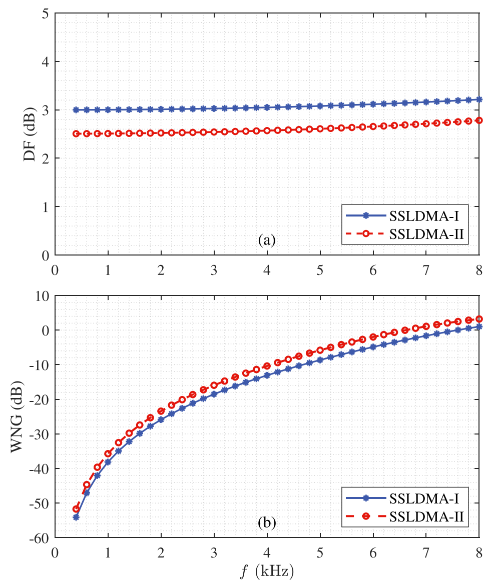

Fig. 5. DFs and WNGs of the second-order SLDMAs as a function of frequency, f: (a) DF and (b) WNG. Conditions of simulation: $M = 3$ and $\delta = 1$ cm.

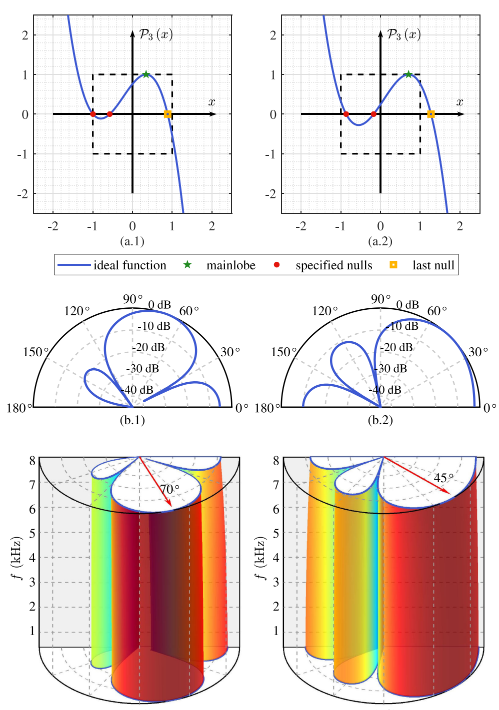

Fig. 6. The ideal functions and beampatterns of the third-order SLDMAs: (a.1) (b.1) (c.1) TSLDMA-I and (a.2) (b.2) (c.2) TSLDMA-II. (a.1) and (a.2) are the ideal functions; (b.1) and (b.2) are the beampatterns at $f = 1$ kHz; and (c.1) and (c.2) are the broadband beampatterns versus frequency, f. Conditions of simulation: $M = 4$ and $\delta = 1$ cm.

cases: FSLDMA-I, FSLDMA-II, FSLDMA-III, and FSLDMA-IV, where the coefficients vector, $\mathbf{a}_N$, and null, x_4, are computed according to (56) and (57), respectively. All the parameters are shown in Table III. Fig. 8 plots the beampatterns and Fig. 9 plots the broadband beampatterns versus frequency. It is clearly seen

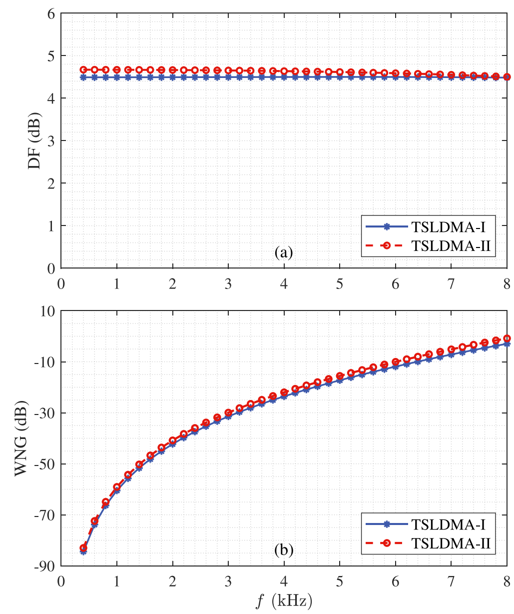

Fig. 7. DFs and WNGs of the third-order SLDMAs as a function of frequency, f: (a) DF and (b) WNG. Conditions of simulation: $M = 4$ and $\delta = 1$ cm.

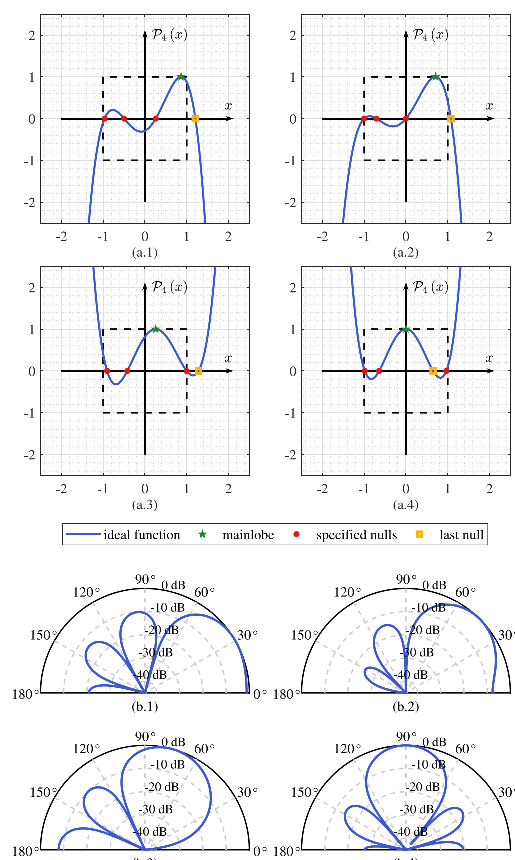

Fig. 8. The ideal functions and beampatterns of the fourth-order SLDMAs: (a.1) (b.1) FSLDMA-I; (a.2) (b.2) FSLDMA-II; (a.3) (b.3) FSLDMA-III; and (a.4) (b.4) FSLDMA-IV. (a.1) (a.2) (a.3) (a.4) are the ideal functions and (b.1) (b.2) (b.3) (b.4) are the beampatterns at $f = 1$ kHz. Conditions of simulation: $M = 5$ and $\delta = 1$ cm.

TABLE III
PARAMETERS OF THE FOURTH-ORDER SLDMAS

	x_s	x_1	x_2	x_3	x_4
FSLDMA-I	$\cos(30^\circ)$	$\cos(75^\circ)$	$\cos(120^\circ)$	$\cos(165^\circ)$	1.2079
FSLDMA-II	$\cos(45^\circ)$	$\cos(90^\circ)$	$\cos(135^\circ)$	$\cos(180^\circ)$	1.0765
FSLDMA-III	$\cos(75^\circ)$	$\cos(0^\circ)$	$\cos(115^\circ)$	$\cos(155^\circ)$	1.2828
FSLDMA-IV	$\cos(90^\circ)$	$\cos(15^\circ)$	$\cos(130^\circ)$	$\cos(170^\circ)$	0.6511

TABLE IV
PARAMETERS FOR TSLDMA-I AND FSLDMA-II (BOTH HAVE A NULL OF MULTIPLICITY)

	x_s	x_1	x_2	x_3	x_4
TSLDMA-I	$\cos(60^\circ)$	$\cos(135^\circ)$	$\cos(135^\circ)$	1.1036	—
FSLDMA-II	$\cos(60^\circ)$	$\cos(135^\circ)$	$\cos(135^\circ)$	$\cos(135^\circ)$	0.9024

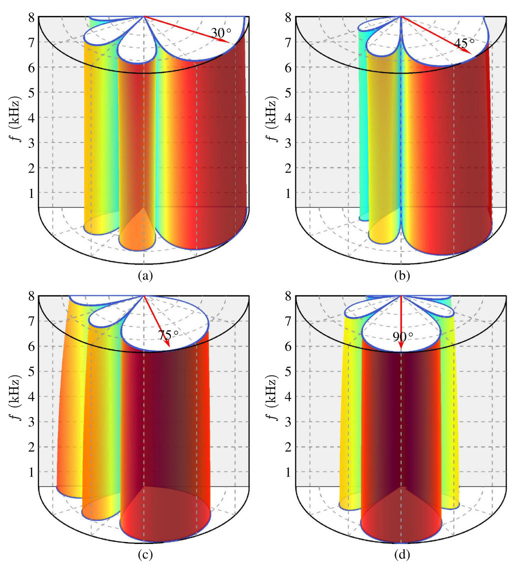

Fig. 9. Broadband beampatterns versus frequency of the fourth-order SLDMAs: (a) FSLDMA-I, (b) FSLDMA-II, (c) FSLDMA-III, and (d) FSLDMA-IV. Conditions of simulation: $M = 5$ and $\delta = 1$ cm.

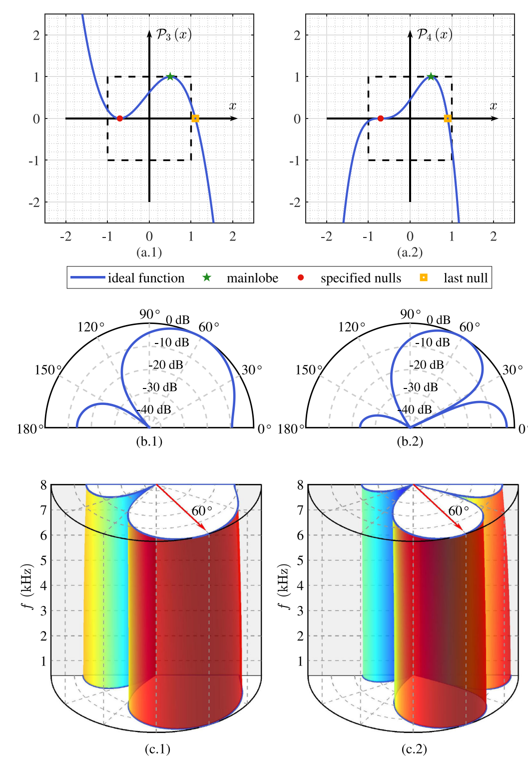

Fig. 10. The ideal functions and beampatterns of TSLDMA-I and FSLDMA-II (both have a null of multiplicity of 2): (a.1) (b.1) (c.1) TSLDMA-I and (a.2) (b.2) (c.2) FSLDMA-II. (a.1) and (a.2) are the ideal functions; (b.1) and (b.2) are the beampatterns at $f = 1$ kHz; and (c.1) and (c.2) are the broadband beampatterns versus frequency, f. Conditions of simulation: TSLDMA-I $M = 4$, FSLDMA-II $M = 5$, and $\delta = 1$ cm.

that the designed beamformers achieve mainlobe steering and the beampatterns are frequency invariant.

B. Design SLDMA Consisting of Nulls With Multiplicity

In this subsection, we investigate two cases: TSLDMA-I and FSLDMA-II. Their look directions are at 60°, and they both have a null at $\cos(135^\circ)$ with multiplicity of 2 and 3, respectively. In this situation, the coefficients vector, $\mathbf{a}_N$, is solved according to (55) and (56). The parameters of both TSLDMA-I and FSLDMA-II are shown in Table IV. Fig. 10 plots the ideal functions and beampatterns. It is seen that both TSLDMA-I and FSLDMA-II have successfully achieved mainlobe steering.

C. Robust SLDMA Design

In this part of simulations, we show that the WNG of SLDMA can also be improved by increasing the number of microphones. We consider to design a second-order SLDMA with $x_s = \cos(90^\circ)$, $x_1 = \cos(0^\circ)$, and $x_2 = \cos(180^\circ)$, using 3, 7, 11, and 15 microphones. Figs. 11 and 12 plot, respectively, the broadband beampatterns and DF and WNG. Clearly, steering is successful, which validates the developed method. As seen, the WNG improves as the number of microphones increase. This is the advantage of the minimum-norm method, which improves the robustness of the designed SLDMA. But one can notice from Fig. 11 that this method may introduce extra nulls into the beampattern, particularly at high frequencies. In other words, at high frequencies, the order of the designed DMA with the minimum-norm method may exceed the specified order. How to design SLDMA with the minimum-norm method to achieve

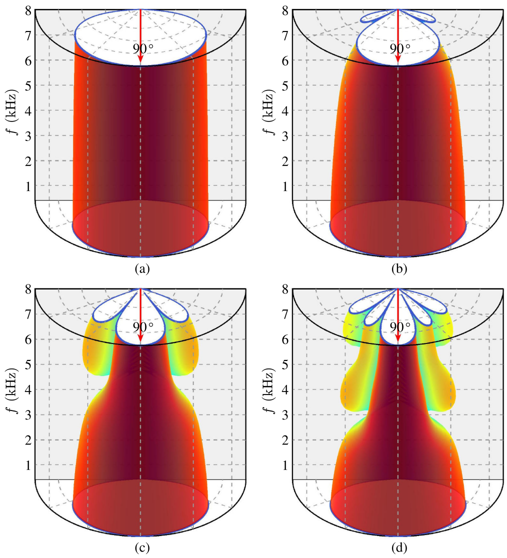

Fig. 11. Broadband beampatterns versus frequency of a second-order SLDMA designed with different numbers of microphones: (a) $M = 3$, (b) $M = 7$, (c) $M = 11$, and (d) $M = 15$.

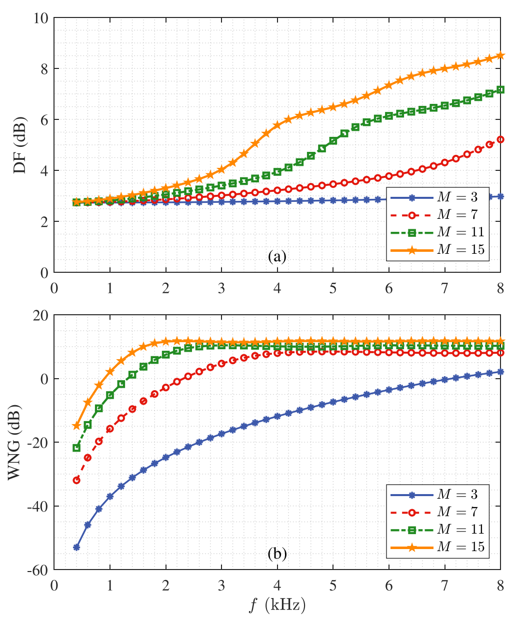

Fig. 12. DF and WNG of a second-order SLDMA as a function of frequency, f: (a) DF and (b) WNG. Conditions of simulation: $\delta = 1$ cm.

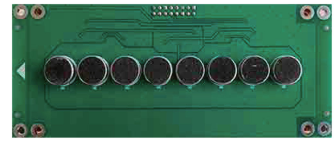

Fig. 13. A photo of the designed linear microphone array consisting of eight electret microphones.

Fig. 14. A photo of measurements with the designed linear microphone array.

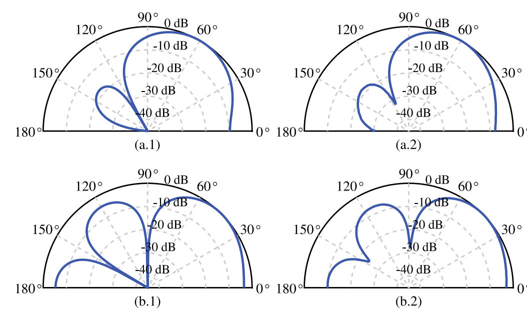

Fig. 15. The designed and measured beampatterns of TSLDMA-I and TSLDMA-II (at 3.5 kHz): (a.1) (a.2) TSLDMA-I; (b.1) (b.2) TSLDMA-II. (a.1) (b.1) are the designed beampatterns; and (a.2) (b.2) are the measured beampatterns.

frequency-invariant beampattern is an interesting topic, which is however beyond the main thrust of this paper.

D. Experiments

In this subsection, we carry out some experiments to further evaluate the performance of the proposed SLDMAs. To perform the experiments, we designed a microphone array, which consists of eight electret microphones (omnidirectional) with the spacing between two neighboring microphones being 1.1 cm. A photo of this designed array is shown in Fig. 13. A number of different beamformers are implemented into the designed array system. The real beampatterns are measured in an anechoic chamber of size $11.8\,\mathrm{m} \times 4.2\,\mathrm{m} \times 3.8\,\mathrm{m}$. The microphone array is placed on a rotating platform and a loudspeaker is placed 2 meters away from the array center. Both the array and the loudspeaker are on a same horizontal plane, which is 60 cm above the sound absorption floor, as shown in Fig. 14. The loudspeaker plays back a narrowband signal of a specified frequency. The rotating platform is configured to rotate the microphone array every 5 seconds with a step size of 1°. The gain in every rotated direction is computed based on the beamformer's output, thereby giving the measured beampattern. In the following, we present some measured results.

The implemented differential beamformers are

1) TSLDMA-I: a third-order SLDMA with $x_\mathrm{s} = \cos(60^\circ)$, $x_1 = \cos(120^\circ)$, $x_2 = \cos(180^\circ)$ and $x_3 = 1.1000$; and
2) TSLDMA-II: a third-order SLDMA with $x_\mathrm{s} = \cos(45^\circ)$, $x_1 = \cos(90^\circ)$, $x_2 = \cos(150^\circ)$ and $x_3 = 1.1949$.

The designed beampatterns of the two SLDMAs are plotted, respectively, in Fig. 15(a.1) and Fig. 15(b.1). The subplots of (a.2) and (b.2) in Fig. 15 show, respectively, the measured beampatterns of TSLDMA-I and TSLDMA-II at 3.5 kHz. It can be clearly seen that the steering is successfully achieved and the measured beampatterns agree very much with the designed ones. There are some difference between the designed beampattern and the measured one, which is caused by array imperfections as well as measurement errors. However, this difference is negligible.

VII. Conclusion

This paper studied the steering problem of LDMAs. By analyzing the ideal function of the differential beampatterns, we first proved that first-order LDMAs are non-steerable regardless of what beamforming method is used and the mainlobe of a first-order LDMA can only be at the endfire directions. Then, we deduced the fundamental conditions for the design of Nth-order ($N > 1$) steerable differential beamformers, i.e., the null position should satisfy a particular equation. Based on those fundamental conditions, a method was proposed to design steerable differential beamformers with LDMAs. With any specified look direction, we first set its $N - 1$ nulls according to the practical needs, and the last null is then determined according to the fundamental condition. The beamforming filter is finally computed by solving a linear system of equations constructed by the null constraints. Simulation and experimental results validated that the proposed method can successfully achieve beam steering with LDMAs.

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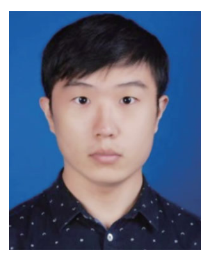

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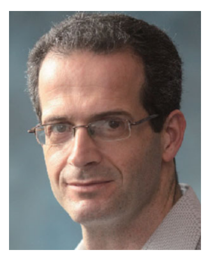

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