Design and Analysis of Quadratic and Multistage Beamformers

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2/2/22

Research Overview

Quadratic

Beamforming

Multistage

Beamforming

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Quadratic Beamforming

KP (Quadratic) Approach

for MCNR[1][2]

Quadratic Approach

for SCNR[3]

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[1] G. Itzhak, J. Benesty, and I. Cohen, ``Nonlinear Kronecker product filtering for multichannel noise reduction," *Speech Communication*, vol. 114, pp. 49--59, 2019

[2] G. Itzhak, J. Benesty, and I. Cohen, ``Quadratic Beamforming for Magnitude Estimation," in *Proc. 29th European Signal Processing Conference (EUSIPCO),* August 2021

[3] G. Itzhak, J. Benesty, and I. Cohen, ``Quadratic approach for single-channel noise reduction," *EURASIP Journal of Audio, Speech and Music Processing*, 7 (2020), pp. 1--14, April 2020

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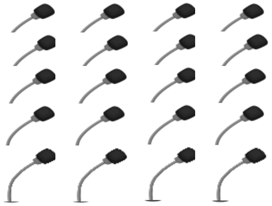
Multistage Beamforming

Multistage Differential KP Beamforming [4]



Multistage Steerable Differential Beamforming [5][6]

[4] G. Itzhak, J. Benesty, and I. Cohen, ``On the Design of Differential Kronecker Product Beamformers," *IEEE/ACM Transactions on Audio, Speech and Language Processing*, vol. 29, pp.1397--1410, 2021

[5] G. Itzhak, J. Benesty, and I. Cohen, ``Multistage Approach for Steerable Differential Beamforming with Rectangular Arrays," submitted to *Speech Communication*, December 2021

[6] G. Itzhak, I. Cohen, and J. Benesty, ``Robust Differential Beamforming with Rectangular Arrays," in *Proc. 29th the European Signal Processing Conference (EUSIPCO)*, August 2021

Outline

Introduction

Multistage Differential KP Beamforming

Quadratic Approach for MCNR

Future Research Leads

Introduction

Beamforming

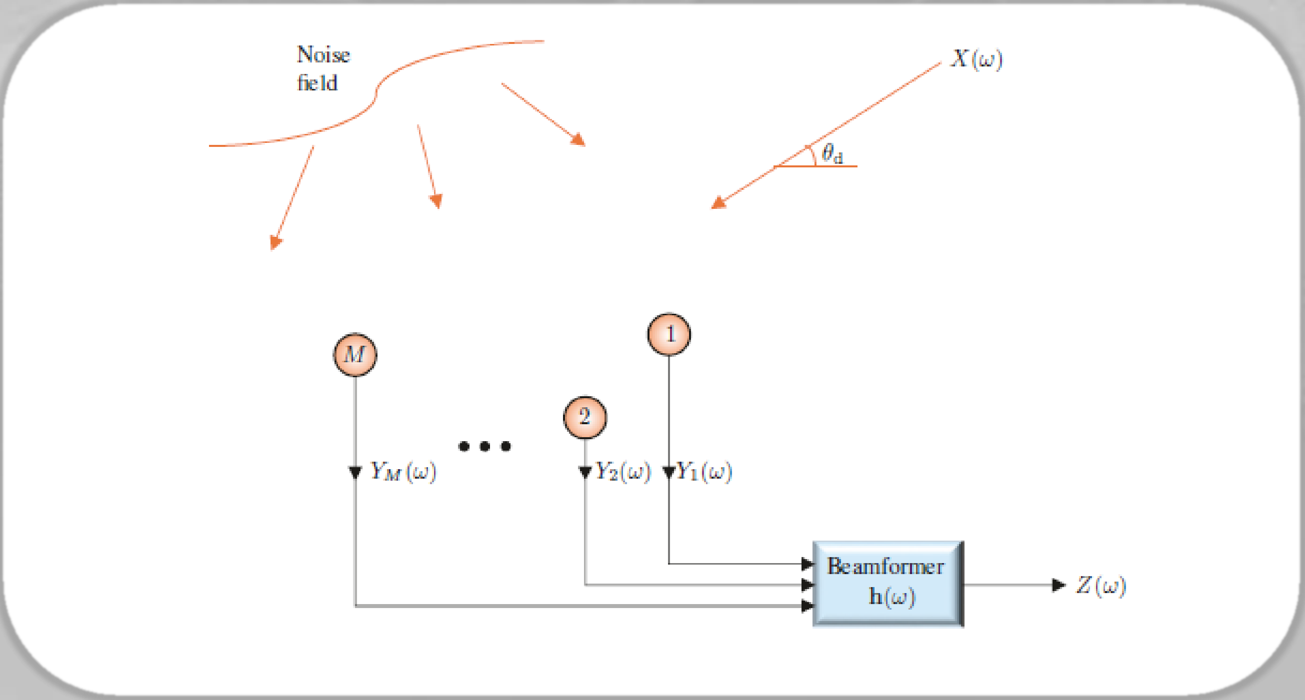
Beamforming or spatial filtering is an active and central research area of array signal processing.

Beamforming and multichannel signal enhancement are at the heart of many fundamental applications such as

- hands-free voice communications
- sonar
- ultrasound
- audio, music and sound applications, etc

Beamforming

A beamformer is a spatial filter that has the ability to form a main beam in the direction of the desired signal and, possibly, place nulls in the directions of interferences.



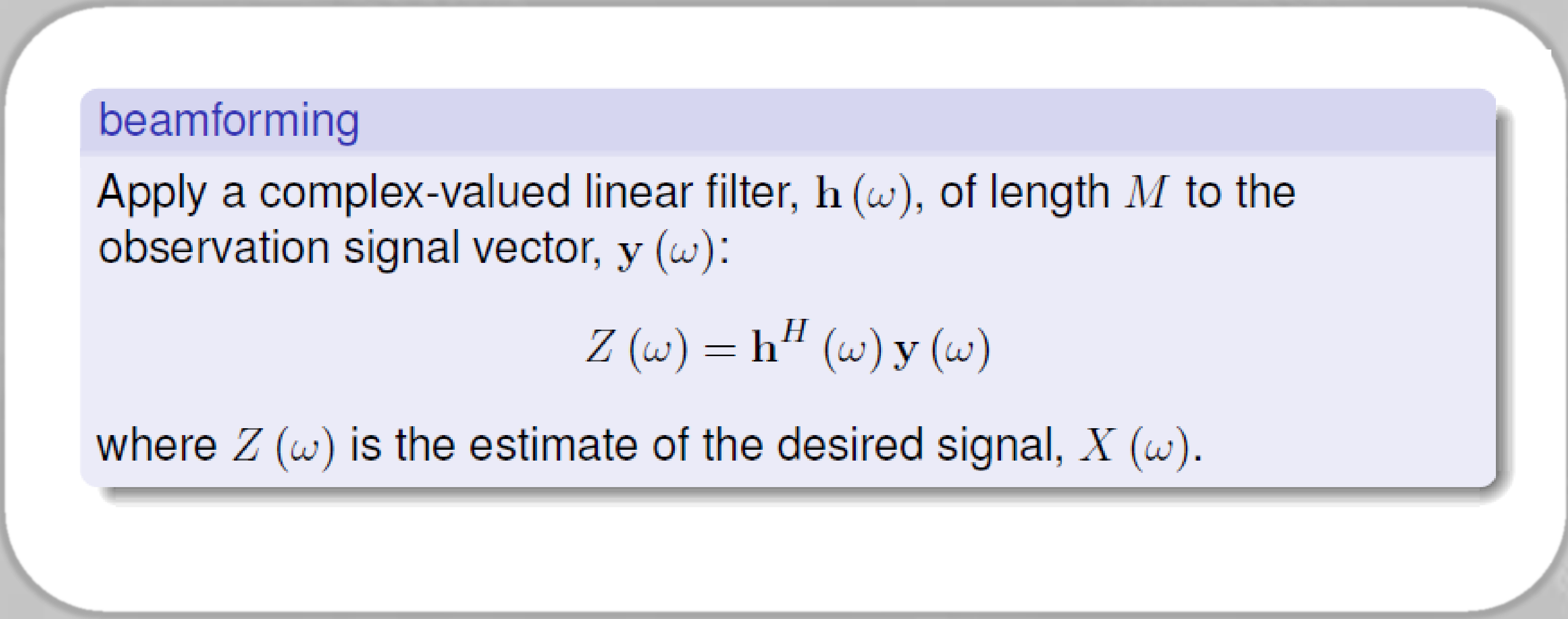
Beamforming

Let us formulate. Considering a microphone array in the frequency domain, the observation signal vector is

$$\begin{aligned}\mathbf{y}(\omega) &= \begin{bmatrix} Y_1(\omega) & Y_2(\omega) & \cdots & Y_M(\omega) \end{bmatrix}^T \\ &= \mathbf{d}_{\theta_{\mathrm{d}},M}(\omega)\, X(\omega) + \mathbf{v}(\omega),\end{aligned}$$

Steering vector   desired signal   noise term

Beamforming



beamforming

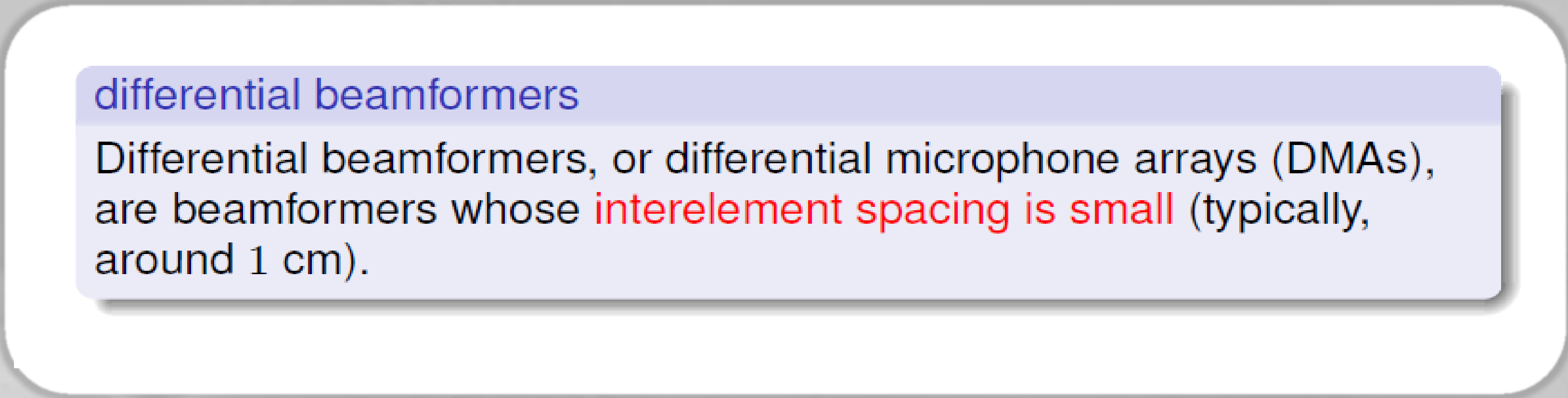
Apply a complex-valued linear filter, $\mathbf{h}(\omega)$, of length M to the observation signal vector, $\mathbf{y}(\omega)$:

$$Z(\omega) = \mathbf{h}^H(\omega)\,\mathbf{y}(\omega)$$

where $Z(\omega)$ is the estimate of the desired signal, $X(\omega)$.

On the Design of Differential Kronecker Product Beamformers

Differential Beamforming



differential beamformers

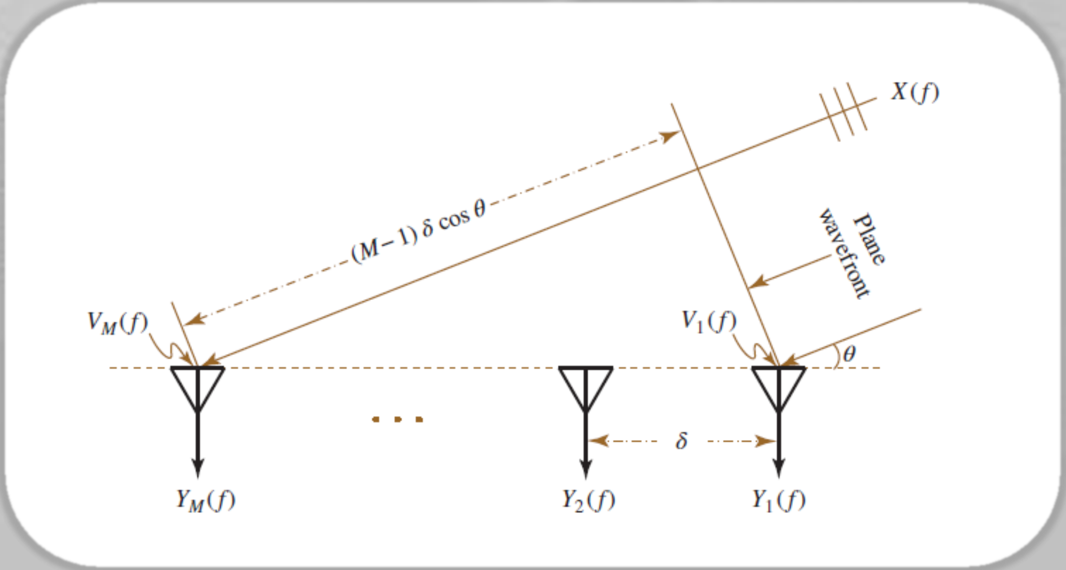
Differential beamformers, or differential microphone arrays (DMAs), are beamformers whose interelement spacing is small (typically, around 1 cm).

DMAs are known to exhibit the following characteristics:

- Small array size.
- Frequency-invariant beampattern.
- Potentially high array directivity.

Signal Model

We consider a uniform linear array (ULA) consisting of M omnidirectional microphones, with an interelement spacing equal to δ



Steering vector:

$$\mathbf{d}_{\theta,M}\left(f\right) = \left[1 \quad e^{-\jmath 2\pi f\delta\cos\theta/c} \quad \cdots \quad e^{-\jmath(M-1)2\pi f\delta\cos\theta/c}\right]^T$$

Multistage Differential KP Beamforming

The differential Kronecker product beamforming approach consists of two successive steps:

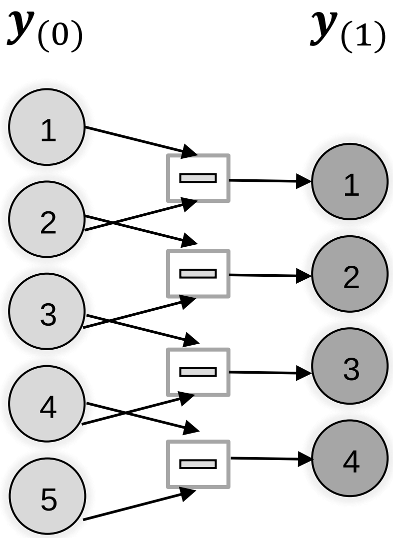
- **Application of a multistage differential operator** onto the noisy observations.
- **Design of a differential KP beamformer** which is applied to the differentials of the noisy observations.

First-order Multistage Differentiation

We define the first-order forward spatial difference operator Δ of $\boldsymbol{y}$ as

$$\Delta Y_i = Y_{i+1} - Y_i = Y_{(1),i}, \; i = 1, 2, \ldots, M-1$$

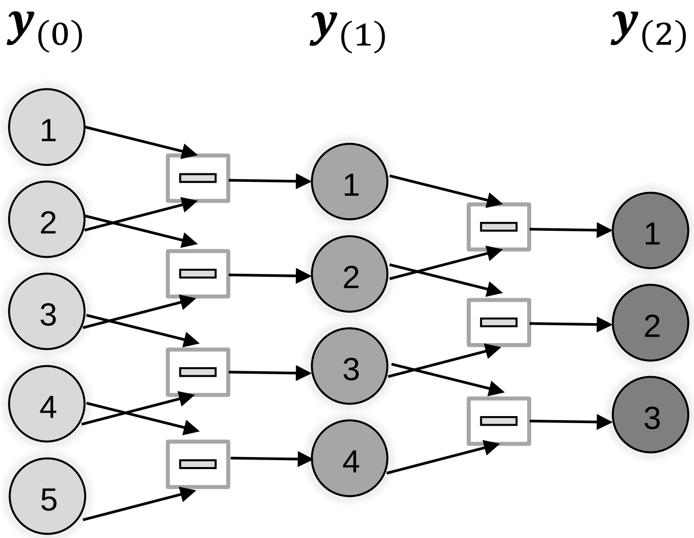
For example, with a ULA of $M = 5$ microphones:



Second-order Multistage Differentiation

Similarly, we define the second-order forward spatial difference operator Δ^2 of $\boldsymbol{y}$ as

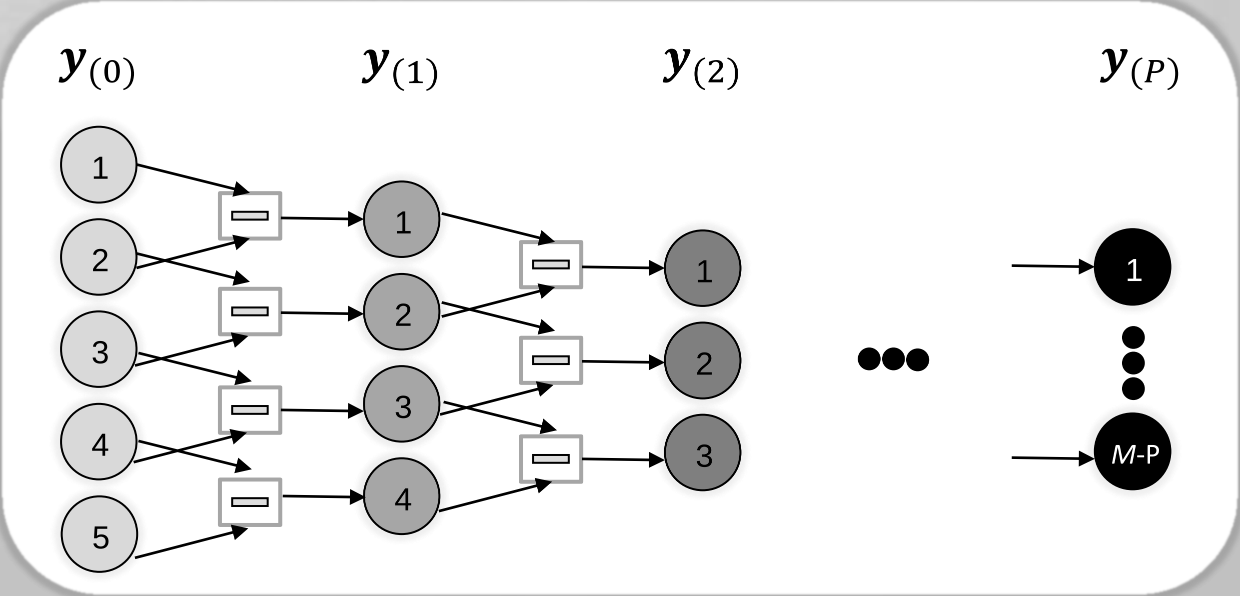
$$\begin{aligned}\Delta^2 Y_i &= \Delta\left(\Delta Y_i\right) = \Delta Y_{i+1} - \Delta Y_i \\ &= Y_{i+2} - 2Y_{i+1} + Y_i = Y_{(2),i}\end{aligned}$$



Pth-order Multistage Differentiation

In general, we define the Pth-order forward spatial difference operator Δ^P of $\boldsymbol{y}$ as

$$\Delta^P Y_{,i} = \Delta^{P-1} Y_{i+1} - \Delta^{P-1} Y_i$$
$$i = 1, 2, \ldots, M - P$$



Noisy Observations Differentials

Considering Δ^P, which can be expressed as an $(M-P)\ x\ M$ matrix $\Delta_{(P)}$:

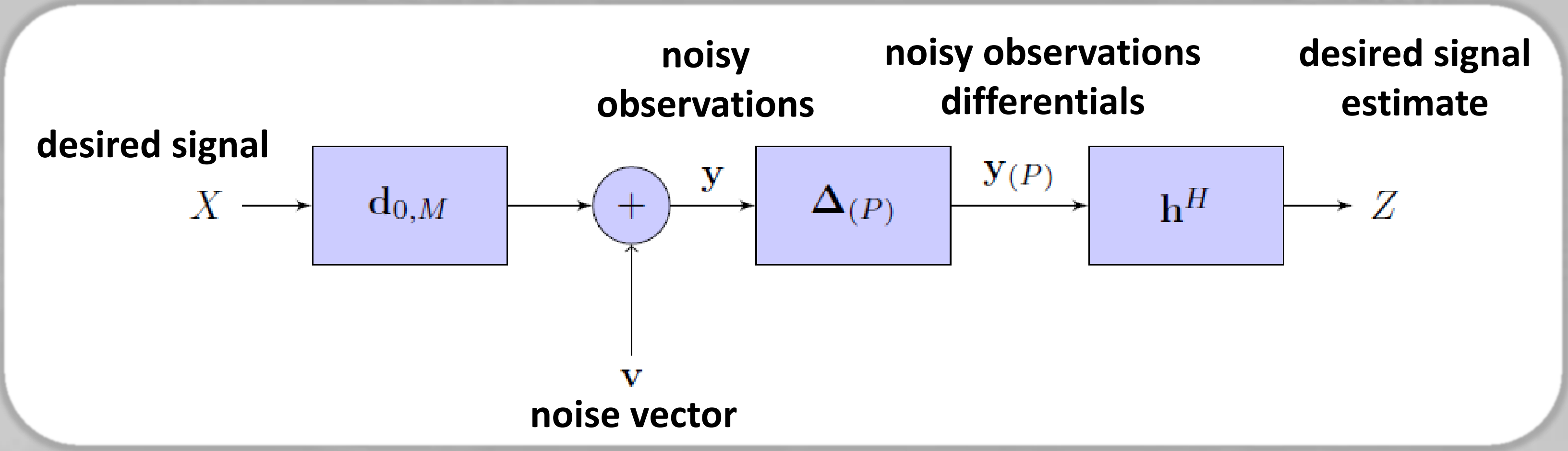
$$\begin{aligned}\mathbf{y}_{(P)} &= \Delta_{(P)}\mathbf{y} \\ &= \underbrace{\tau_0^P}_{\text{directivity term}} \underbrace{\mathbf{d}_{0,M-P}}_{\text{steering vector}} \underbrace{X}_{\text{desired signal}} + \underbrace{\mathbf{v}_{(P)}}_{\text{noise differentials term}}\end{aligned}$$

- endfire direction: $\theta_{\mathrm{d}} = 0$
- directivity term: $\tau_0 = e^{-\jmath\omega\delta/c} - 1$

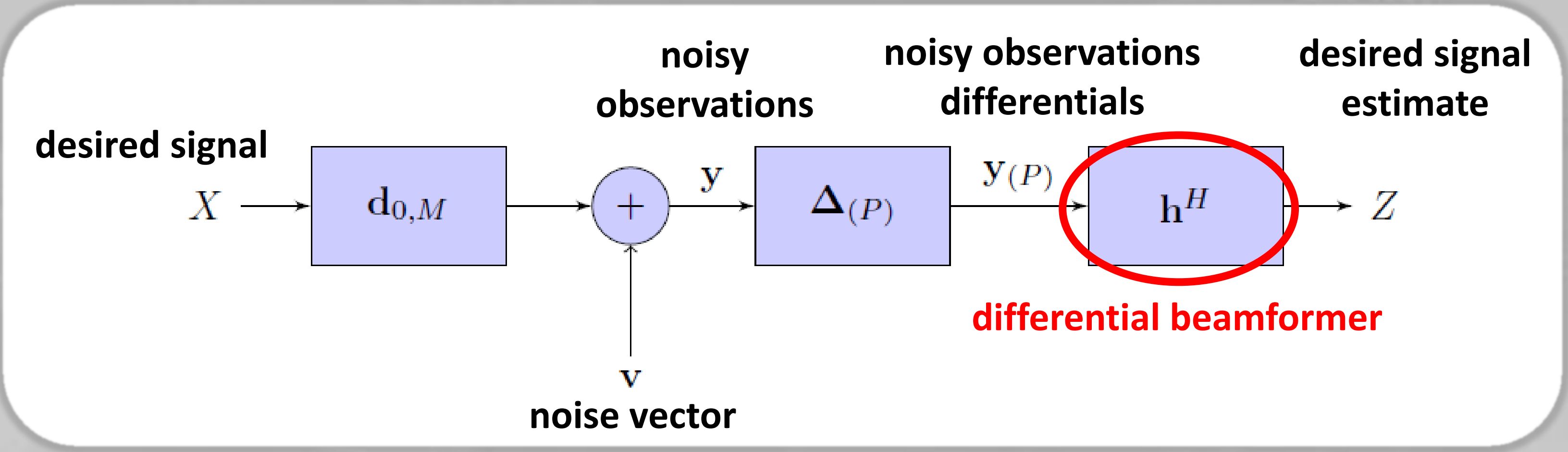
noisy observations differentials

$\mathbf{y}_{(P)}$ is a vector of Pth-order differentials of the noisy observations.

Schematic View



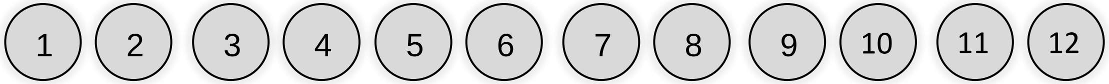
Schematic View



Differential KP Beamforming

Assume $M - P = 12$:

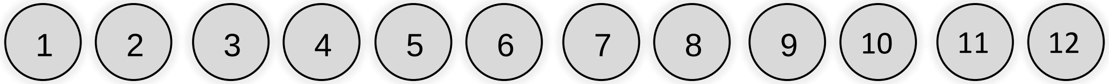
$$\mathbf{d}_{0,M-P}(f) = \left[1 \quad e^{-\jmath 2\pi f\delta/c} \quad \cdots \quad e^{-\jmath 11*2\pi f\delta/c}\right]^T$$



Differential KP Decomposition

Assume $M - P = M_{\mathbf{a}} \times M_{\mathbf{b}} = 12$:

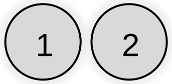
$$\mathbf{d}_{0,M-P}(f) = \begin{bmatrix}1 & e^{-\jmath 2\pi f\delta/c} & \cdots & e^{-\jmath 11*2\pi f\delta/c}\end{bmatrix}^T$$



We can decompose $\mathbf{d}_{0,M-P}(f) = \boldsymbol{a}_0(f) \otimes \mathbf{b}_0(f)$:

$$\mathbf{a}_0(f) = \begin{bmatrix}1 & e^{-\jmath 2\pi f(2\delta)/c} & \cdots & e^{-\jmath 5*2\pi f(2\delta)/c}\end{bmatrix}^T$$

$$\mathbf{b}_0(f) = \begin{bmatrix}1 & e^{-\jmath 2\pi f\delta/c}\end{bmatrix}^T$$



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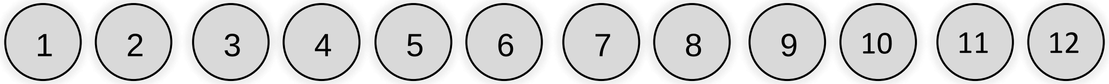


We can decompose $\mathbf{d}_{0,M-P}(f) = \boldsymbol{a}_0(f) \otimes \mathbf{b}_0(f)$:

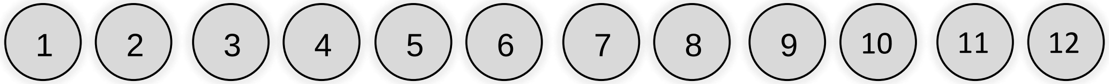
$$\mathbf{a}_0(f) = \left[1 \quad e^{-\jmath 2\pi f(3\delta)/c} \quad \cdots \quad e^{-\jmath 3*2\pi f(3\delta)/c}\right]^T$$

$$\mathbf{b}_0(f) = \left[1 \quad e^{-\jmath 2\pi f\delta/c} \quad e^{-\jmath 2*2\pi f\delta/c}\right]^T$$

Differential KP Decomposition

Assume $M - P = M_{\mathbf{a}} \times M_{\mathbf{b}} = 12$:

$$\mathbf{d}_{0,M-P}(f) = \begin{bmatrix} 1 & e^{-\jmath 2\pi f\delta/c} & \cdots & e^{-\jmath 11*2\pi f\delta/c} \end{bmatrix}^T$$



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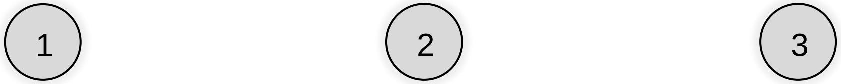
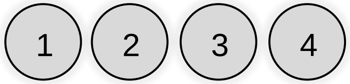
$$\mathbf{a}_0(f) = \begin{bmatrix} 1 & e^{-\jmath 2\pi f(4\delta)/c} & e^{-\jmath 2*2\pi f(4\delta)/c} \end{bmatrix}^T$$



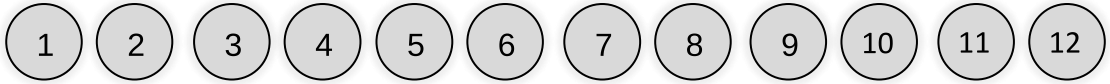
$$\mathbf{b}_0(f) = \begin{bmatrix} 1 & e^{-\jmath 2\pi f\delta/c} & \cdots & e^{-\jmath 3*2\pi f\delta/c} \end{bmatrix}^T$$



Differential KP Decomposition

Assume $M - P = M_{\mathbf{a}} \times M_{\mathbf{b}} = 12$:

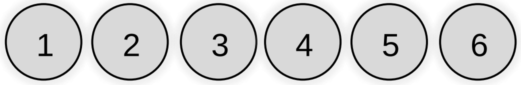
$$\mathbf{d}_{0,M-P}(f) = \begin{bmatrix} 1 & e^{-\jmath 2\pi f\delta/c} & \cdots & e^{-\jmath 11 * 2\pi f\delta/c} \end{bmatrix}^T$$



We can decompose $\mathbf{d}_{0,M-P}(f) = \boldsymbol{a}_0(f) \otimes \mathbf{b}_0(f)$:

$$\mathbf{a}_0(f) = \begin{bmatrix} 1 & e^{-\jmath 2\pi f(6\delta)/c} \end{bmatrix}^T$$

$$\mathbf{b}_0(f) = \begin{bmatrix} 1 & e^{-\jmath 2\pi f\delta/c} & \cdots & e^{-\jmath 5 * 2\pi f\delta/c} \end{bmatrix}^T$$



Noisy Observations Differentials

Considering the KP decomposition:

$$\begin{aligned}\mathbf{y}_{(P)} &= \Delta_{(P)}\mathbf{y} \\ &= \underbrace{\tau_0^P}_{\text{directivity term}} \underbrace{(\mathbf{a}_0 \otimes \mathbf{b}_0)}_{\text{decomposed steering vector}} \underbrace{X}_{\text{desired signal}} + \underbrace{\mathbf{v}_{(P)}}_{\text{noise differentials term}}\end{aligned}$$

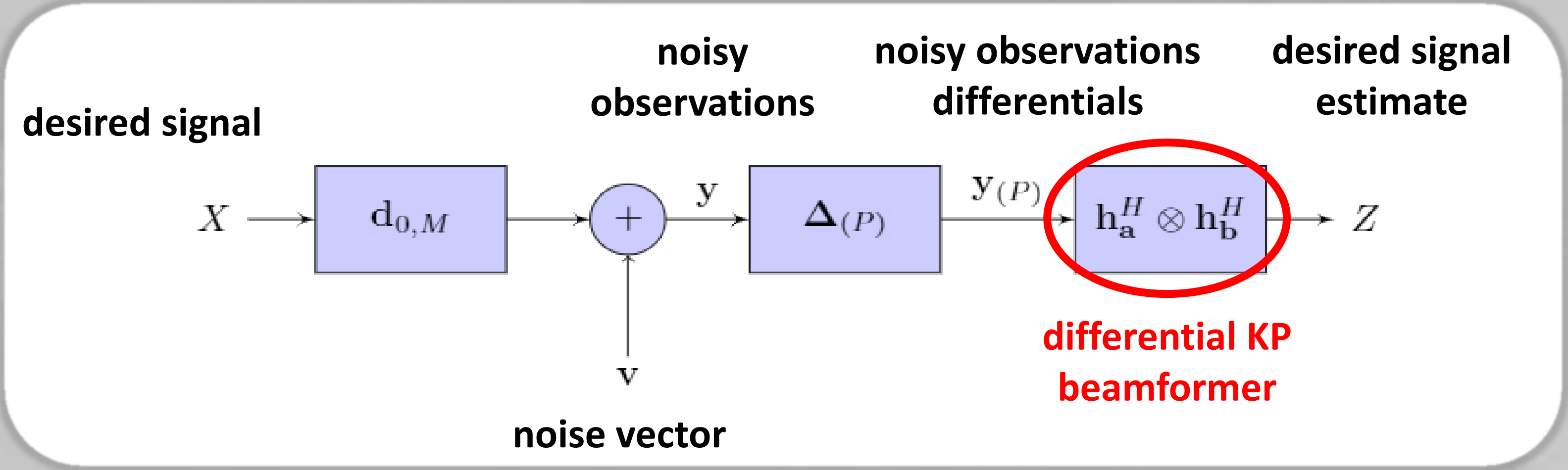
Differential KP Beamforming

Apply $\mathbf{h} = \mathbf{h_a} \otimes \mathbf{h_b}$ to $\mathbf{y}_{(P)}$:

$$\begin{aligned} Z &= \mathbf{h}^H \mathbf{y}_{(P)} \\ &= \tau_0^P \left(\mathbf{h_a} \otimes \mathbf{h_b}\right)^H \left(\mathbf{a}_0 \otimes \mathbf{b}_0\right) X + \left(\mathbf{h_a} \otimes \mathbf{h_b}\right)^H \mathbf{v}_{(P)} \\ &= \tau_0^P \left(\mathbf{h_a}^H \mathbf{a}_0\right) \left(\mathbf{h_b}^H \mathbf{b}_0\right) X + \left(\mathbf{h_a} \otimes \mathbf{h_b}\right)^H \mathbf{v}_{(P)} \end{aligned}$$

$\mathbf{h_a}$ and $\mathbf{h_b}$ may be designed independently.

Schematic View



desired signal

noisy observations

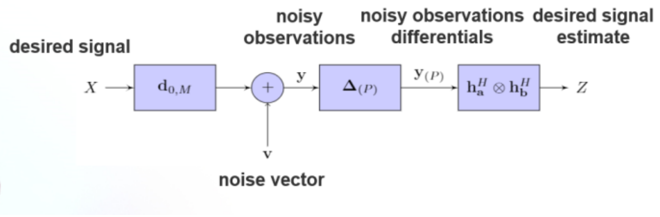
noisy observations differentials

desired signal estimate

noise vector

differential KP beamformer

Differential KP Beamforming



Step 1:

- Set P
- Apply $\Delta_{(P)}$

Step 2:

- Set $M_\mathrm{a}$and M_b
- Design $\mathbf{h_a}$ and $\mathbf{h_b}$
- Apply $\mathbf{h_a} \otimes \mathbf{h_b}$

SNR Gain

The SNR gain indicates the gain obtained by using the beamformer compared to the reference microphone:

$$\mathcal{G}\left(\mathbf{h}_{\mathbf{a}}, \mathbf{h}_{\mathbf{b}}\right) = \frac{\left|\tau_0\right|^{2P} \left|\mathbf{h}_{\mathbf{a}}^H \mathbf{a}_0\right|^2 \left|\mathbf{h}_{\mathbf{b}}^H \mathbf{b}_0\right|^2}{\left(\mathbf{h}_{\mathbf{a}} \otimes \mathbf{h}_{\mathbf{b}}\right)^H \boldsymbol{\Gamma}_{\mathbf{v}_{(P)}} \left(\mathbf{h}_{\mathbf{a}} \otimes \mathbf{h}_{\mathbf{b}}\right)}$$

where

$$\boldsymbol{\Gamma}_{\mathbf{v}_{(P)}} = \Delta_{(P)} \boldsymbol{\Gamma}_{\mathbf{v}} \Delta_{(P)}^T$$

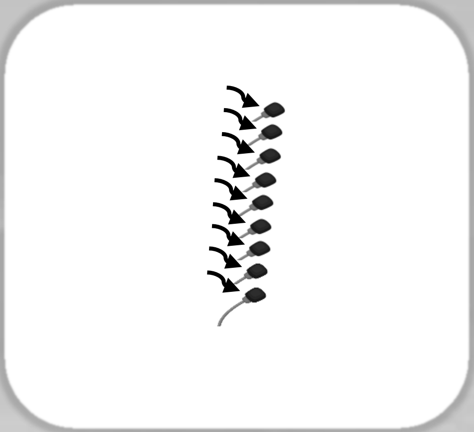
$$\boxed{\boldsymbol{\Gamma}_{\mathbf{v}} = \frac{E\left(\mathbf{v}\mathbf{v}^H\right)}{E\left(|V_1|^2\right)}}$$

noise coherence matrix

SNR Gain

White Noise Gain (WNG):

$$\boldsymbol{\Gamma}_{\mathbf{v}} = \mathbf{I}_M$$



Directivity Factor (DF):

$$\boldsymbol{\Gamma}_{\mathbf{v}} = \boldsymbol{\Gamma}_{\mathrm{d}}$$

$$(\boldsymbol{\Gamma}_{\mathrm{d}})_{ij} = \frac{\sin\left[2\pi f (j-i)\delta/c\right]}{2\pi f (j-i)\delta/c}$$

Beampattern

The beampattern describes the sensitivity of a beamformer to a plane wave impinging on the array from the direction θ:

$$
\begin{aligned}
\mathcal{B}_{\theta}\left(\mathbf{h}\right) &= \tau_{\theta}^{P}\mathbf{h}^{H}\mathbf{d}_{\theta,M-P} \\
&= \underset{\substack{\nearrow \\ \text{directivity term}}}{\tau_{\theta}^{P}} \times \underset{\substack{\uparrow \\ \text{Beampattern of} \\ \text{sub-beamformer } \mathbf{h}_{\mathbf{a}}}}{\mathbf{h}_{\mathbf{a}}^{H}\mathbf{a}_{\theta}} \times \underset{\substack{\nwarrow \\ \text{Beampattern of} \\ \text{sub-beamformer } \mathbf{h}_{\mathbf{b}}}}{\mathbf{h}_{\mathbf{b}}^{H}\mathbf{b}_{\theta}}
\end{aligned}
$$

Optimal Differential KP Beamformers

Now, design $\mathbf{h}_\mathbf{a}$ and $\mathbf{h}_\mathbf{b}$:

- Maximize the WNG.
- Maximize the DF.
- Force spatial constraints (beampattern nulls).
- Other optimization criteria (i.e., maximum front-to-back ratio).

Many possible combinations; we shall focus on three examples.

Maximum WNG (MWNG) Beamformer

Optimize $\mathbf{h}_\mathbf{a}$ and $\mathbf{h}_\mathbf{b}$ iteratively:

- Initialize both sub-beamformers to identity filters.
- Maximize the WNG of $\mathbf{h}$ w.r.t. $\mathbf{h}_\mathbf{a}$ when $\mathbf{h}_\mathbf{b}$ is fixed.
- Maximize the WNG of $\mathbf{h}$ w.r.t. $\mathbf{h}_\mathbf{b}$ when $\mathbf{h}_\mathbf{a}$ is fixed.
- Repeat several time to converge.

Maximum DF (MDF) Beamformer

Optimize $\mathbf{h}_{\mathbf{a}}$ and $\mathbf{h}_{\mathbf{b}}$ iteratively:

- Initialize both sub-beamformers to identity filters.
- Maximize the DF of $\mathbf{h}$ w.r.t. $\mathbf{h}_{\mathbf{a}}$ when $\mathbf{h}_{\mathbf{b}}$ is fixed.
- Maximize the DF of $\mathbf{h}$ w.r.t. $\mathbf{h}_{\mathbf{b}}$ when $\mathbf{h}_{\mathbf{a}}$ is fixed.
- Repeat several time to converge.

Null-steering (NS) Beamformer

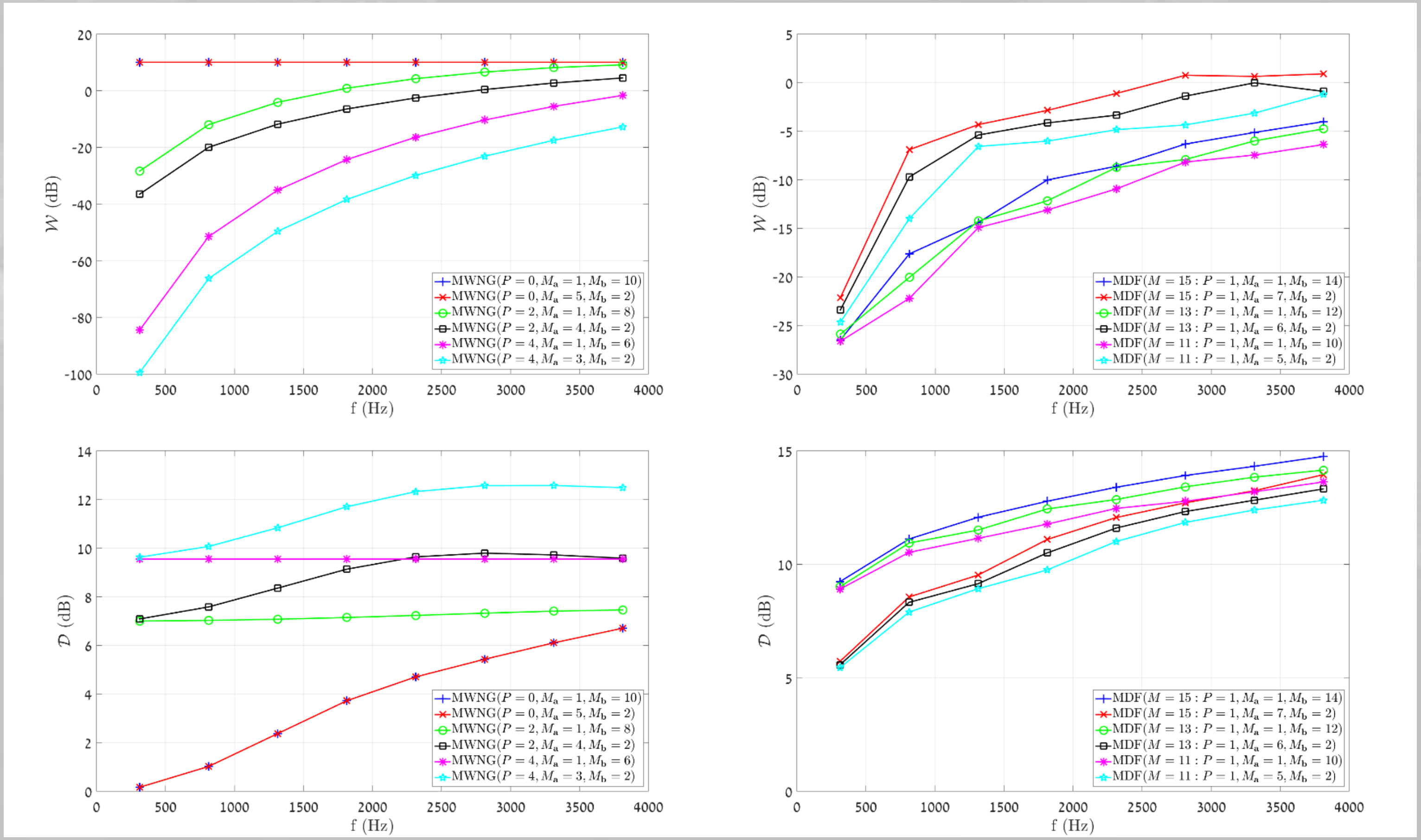
Optimize $\mathbf{h}_{\mathbf{a}}$ and $\mathbf{h}_{\mathbf{b}}$ iteratively:

- Initialize both sub-beamformers to identity filters.
- Maximize the WNG of $\mathbf{h}$ w.r.t. $\mathbf{h}_{\mathbf{a}}$ when $\mathbf{h}_{\mathbf{b}}$ is fixed.
- Maximize the WNG of $\mathbf{h}$ w.r.t. $\mathbf{h}_{\mathbf{b}}$ when $\mathbf{h}_{\mathbf{a}}$ is fixed; set spatial nulls in $\mathbf{h}_{\mathbf{b}}$‘s beampattern.
- Repeat several time to converge.

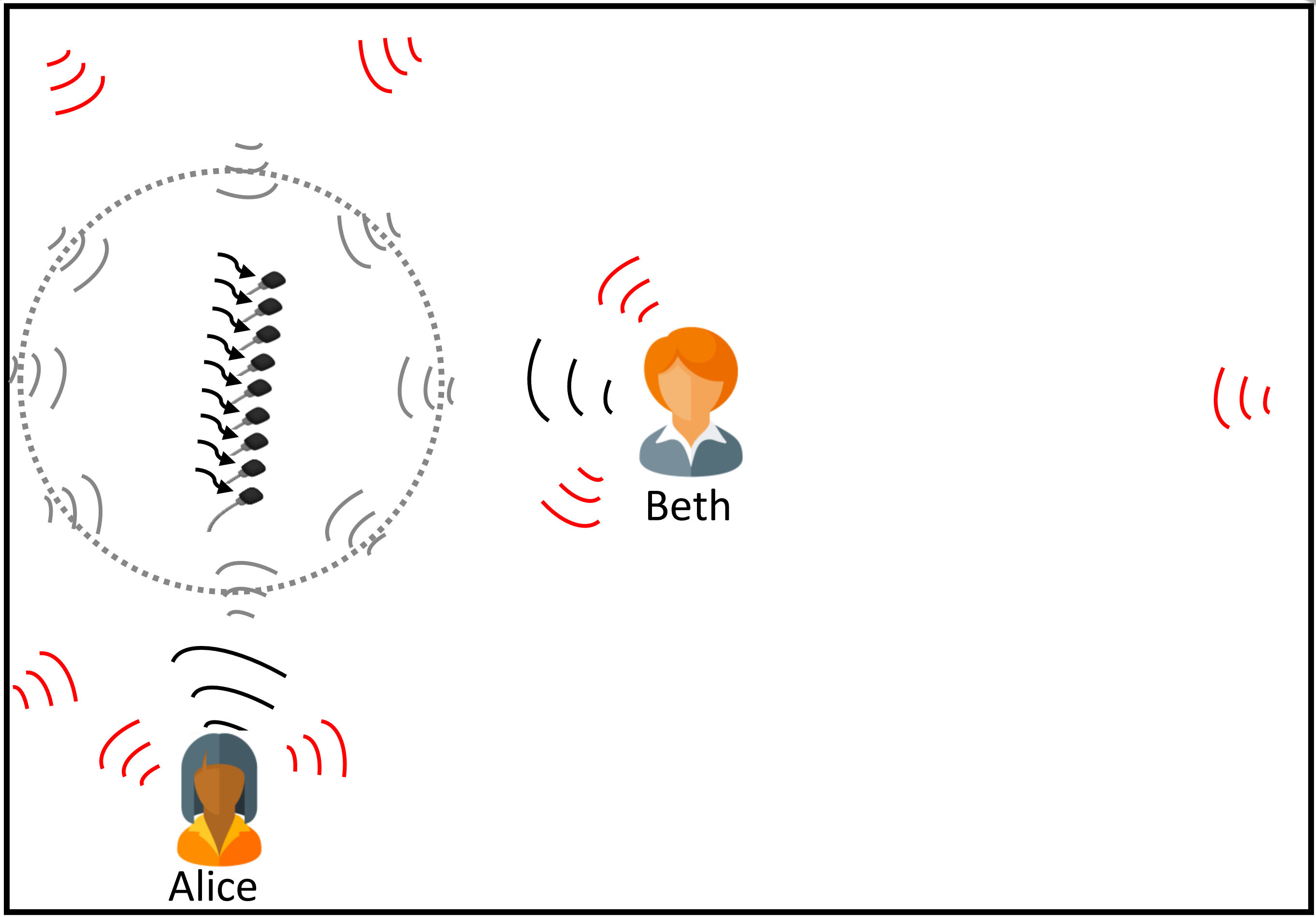
WNG and DF Analysis

MWNG Beamformer

MDF Beamformer



Experimental Setup



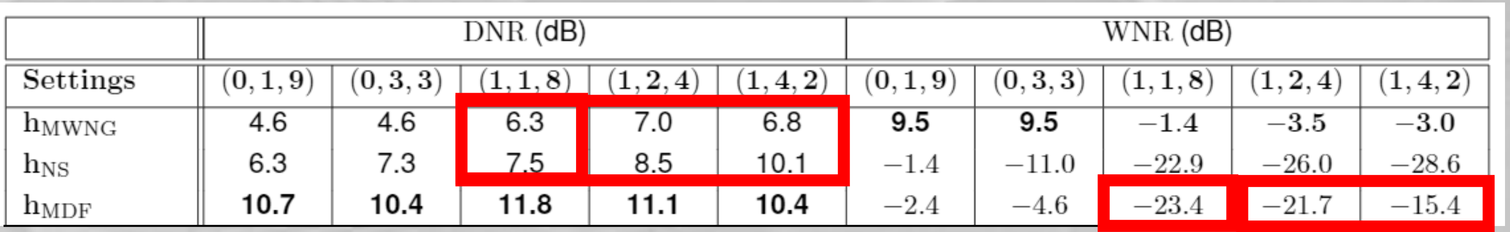
Noise Reduction Factors

We define four factors which quantify the reduction for each type of noise:

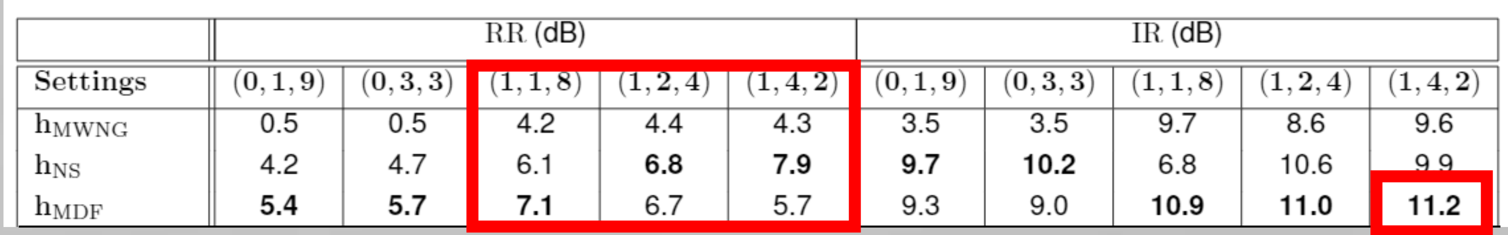
- White noise reduction (WNR) factor.
- Diffuse noise reduction (DNR) factor.
- Desired-signal reverberations reduction (RR) factor.
- Interference reduction (IR) factor.

Noise Reduction Factors

Control by setting the three design parameters $(P, M_{\mathbf{a}}, M_{\mathbf{b}})$:

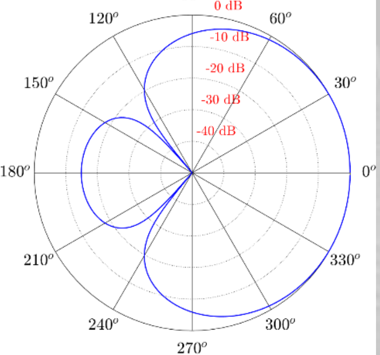
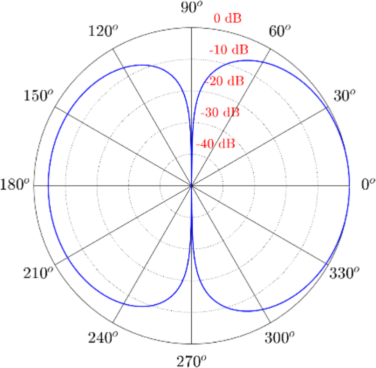
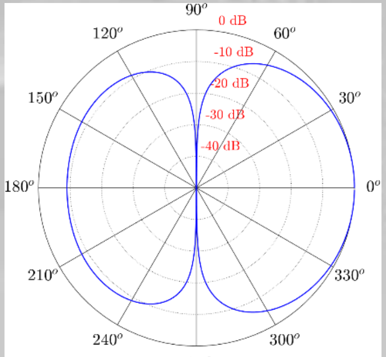
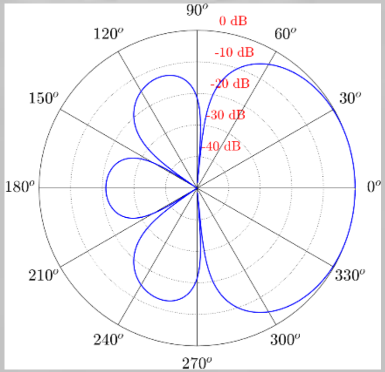
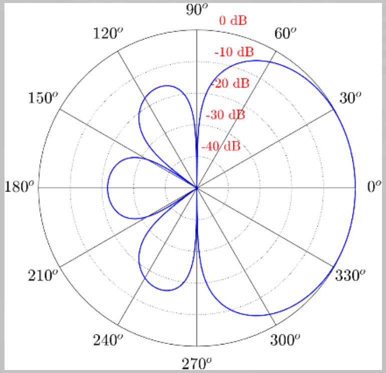
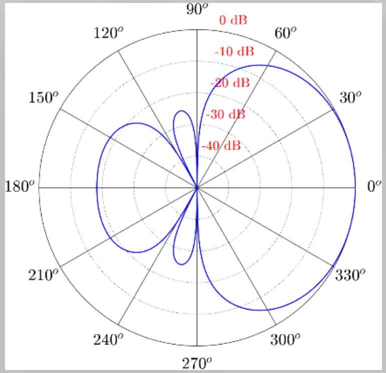


	DNR (dB)					WNR (dB)				
Settings	$(0,1,9)$	$(0,3,3)$	$(1,1,8)$	$(1,2,4)$	$(1,4,2)$	$(0,1,9)$	$(0,3,3)$	$(1,1,8)$	$(1,2,4)$	$(1,4,2)$
$\mathrm{h}_{\mathrm{MWNG}}$	4.6	4.6	6.3	7.0	6.8	**9.5**	**9.5**	-1.4	-3.5	-3.0
h_{NS}	6.3	7.3	7.5	8.5	10.1	-1.4	-11.0	-22.9	-26.0	-28.6
$\mathrm{h}_{\mathrm{MDF}}$	**10.7**	**10.4**	**11.8**	**11.1**	**10.4**	-2.4	-4.6	-23.4	-21.7	-15.4



	RR (dB)					IR (dB)				
Settings	$(0,1,9)$	$(0,3,3)$	$(1,1,8)$	$(1,2,4)$	$(1,4,2)$	$(0,1,9)$	$(0,3,3)$	$(1,1,8)$	$(1,2,4)$	$(1,4,2)$
$\mathrm{h}_{\mathrm{MWNG}}$	0.5	0.5	4.2	4.4	4.3	3.5	3.5	9.7	8.6	9.6
h_{NS}	4.2	4.7	6.1	**6.8**	**7.9**	**9.7**	**10.2**	6.8	10.6	9.9
$\mathrm{h}_{\mathrm{MDF}}$	**5.4**	**5.7**	**7.1**	6.7	5.7	9.3	9.0	**10.9**	**11.0**	**11.2**

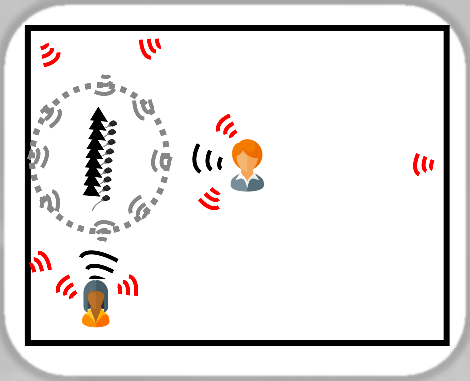
MWNG Beampatterns

	$P = 0$	$P = 1$	$P = 1$
$M = 5$	$M_{\mathbf{a}} = 1, M_{\mathbf{b}} = 5$ 	$M_{\mathbf{a}} = 1, M_{\mathbf{b}} = 4$ 	$M_{\mathbf{a}} = 2, M_{\mathbf{b}} = 2$ 
$M = 9$	$M_{\mathbf{a}} = 1, M_{\mathbf{b}} = 9$ 	$M_{\mathbf{a}} = 1, M_{\mathbf{b}} = 8$ 	$M_{\mathbf{a}} = 4, M_{\mathbf{b}} = 2$ 

MDF Enhanced Speech Samples

Noisy: 🔊

Clean: 🔊



$(P, M_{\mathbf{a}}, M_{\mathbf{b}})$:	(0,1,9)	(0,3,3)	(1,1,8)	(1,4,2)
	🔊	🔊	🔊	🔊

MDF PESQ and STOI Scores

PESQ

$(P, M_{\mathbf{a}}, M_{\mathbf{b}})$:	Noisy	$(0,1,9\ )$	$(0,3,3\ )$	$(1,1,8\ )$	$(1,4,2\ )$
	1.39	2.4	2.3	2.4	**3.0**

STOI

$(P, M_{\mathbf{a}}, M_{\mathbf{b}})$:	Noisy	$(0,1,9\ )$	$(0,3,3\ )$	$(1,1,8\ )$	$(1,4,2\ )$
	0.58	0.85	0.83	0.88	**0.93**

Conclusions

- We have proposed a generalized differential beamforming approach which is controlled by three design parameters.
- The design parameters allow a great flexibility; depending on their selection previous approaches may be obtained.

Conclusions

- Alternatively, this flexibility enables to mitigate the white noise amplification with the differential MDF and beamformer or improve the array directivity with the differential MWNG and NS beamformers.

- Finally, we demonstrated preferable PESQ and STOI scores of the time-domain enhanced signals.

Quadratic Approach for Multichannel Noise Reduction

Signal Model

Consider an array of M omnidirectional microphones:

$$Y_m(f) = X_m(f) + V_m(f), m = 1, 2, \ldots, M$$

In a vector form:

$$\begin{aligned} \mathbf{y}(f) &= \mathbf{x}(f) + \mathbf{v}(f) \\ &= \mathbf{d}(f) X_1(f) + \mathbf{v}(f) \end{aligned}$$

Linear Filtering

Apply a complex-valued **linear** filter $\mathbf{h}(f)$ to $\mathbf{y}(f)$:

$$\begin{aligned}\widehat{X}(f) &= \mathbf{h}^H(f)\mathbf{y}(f)\\ &= X_{\mathrm{fd}}(f) + V_{\mathrm{rn}}(f)\end{aligned}$$

where

$$X_{\mathrm{fd}}(f) = X_1(f)\mathbf{h}^H(f)\mathbf{d}(f)$$

$$V_{\mathrm{rn}}(f) = \mathbf{h}^H(f)\mathbf{v}(f)$$

Linear Filtering

The variance of $\hat{X}(f)$ is given by:

$$\begin{aligned} \phi_{\widehat{X}}(f) &= \mathbf{h}^H(f)\mathbf{\Phi}_{\mathbf{y}}(f)\mathbf{h}(f) \\ &= \phi_{X_{\mathrm{fd}}}(f) + \phi_{V_{\mathrm{rn}}}(f) \end{aligned}$$

where

$$\phi_{X_{\mathrm{fd}}}(f) = \phi_{X_1}(f)\left|\mathbf{h}^H(f)\mathbf{d}(f)\right|^2$$

$$\phi_{V_{\mathrm{rn}}}(f) = \mathbf{h}^H(f)\mathbf{\Phi}_{\mathbf{v}}(f)\mathbf{h}(f)$$

Linear Filtering

Narrowband output SNR:

$$\mathrm{oSNR}\left[\mathbf{h}(f)\right] = \frac{\phi_{X_1}(f)\left|\mathbf{h}^H(f)\mathbf{d}(f)\right|^2}{\mathbf{h}^H(f)\mathbf{\Phi}_{\mathbf{v}}(f)\mathbf{h}(f)}$$

Narrowband output SNR gain:

$$\mathcal{G}\left[\mathbf{h}(f)\right] = \frac{\mathrm{oSNR}\left[\mathbf{h}(f)\right]}{\mathrm{iSNR}(f)}$$

Linear MVDR and LCMV

The distortionless constraint:

$$\mathbf{h}^H(f)\mathbf{d}(f) = 1$$

Linear MVDR:

$$\mathbf{h}_{\mathrm{MVDR}}(f) = \frac{\mathbf{\Phi}_{\mathbf{y}}^{-1}(f)\mathbf{d}(f)}{\mathbf{d}^H(f)\mathbf{\Phi}_{\mathbf{y}}^{-1}(f)\mathbf{d}(f)}$$

Linear LCMV:

$$\mathbf{h}_{\mathrm{LCMV}}(f) = \mathbf{\Phi}_{\mathbf{y}}^{-1}(f)\mathbf{C}(f)[\mathbf{C}^H(f)\mathbf{\Phi}_{\mathbf{y}}^{-1}(f)\mathbf{C}(f)]^{-1}\boldsymbol{\beta}$$

where $\mathbf{C}(f)$ is an $M \times L$ matrix whose columns are the steering vectors in the directions of constraints

Quadratic Filtering

Main idea

Estimate $|X_1(f)|^2$, the **spectral power** of the desired signal

Motivation

The spectral power is known to be more prominent than the spectral phase

Quadratic Filtering

$$\begin{aligned}
\left|\widehat{X}(f)\right|^2 &= \mathbf{h}^H(f)\mathbf{y}(f)\mathbf{y}^H(f)\mathbf{h}(f) \\
&= \mathrm{tr}\left[\mathbf{h}(f)\mathbf{h}^H(f)\mathbf{y}(f)\mathbf{y}^H(f)\right] \\
&= \mathrm{vec}^H\left[\mathbf{h}(f)\mathbf{h}^H(f)\right]\mathrm{vec}\left[\mathbf{y}(f)\mathbf{y}^H(f)\right] \\
&= \left[\mathbf{h}^*(f)\otimes\mathbf{h}(f)\right]^H\left[\mathbf{y}^*(f)\otimes\mathbf{y}(f)\right] \\
&= \left[\mathbf{h}^*(f)\otimes\mathbf{h}(f)\right]^H\widetilde{\mathbf{y}}(f)
\end{aligned}$$

Quadratic Filtering

Let $\tilde{\mathbf{h}}(f)$ be a complex-valued filter of length M^2 which is **not necessarily** of the form $\tilde{\mathbf{h}}(f) = \mathbf{h}^*(f) \otimes \mathbf{h}(f)$.

Obtain $Z(f)$, an estimate $|X_1(f)|^2$:

$$Z(f) = \widetilde{\mathbf{h}}^H(f)\widetilde{\mathbf{y}}(f)$$

The estimate of $X_1(f)$ is given by:

$$\widehat{X}(f) = e^{\jmath\psi(f)}\sqrt{|Z(f)|}$$

Quadratic Filtering

Noting that

$$\begin{aligned}\widetilde{\mathbf{y}}(f) &= \mathbf{y}^*(f) \otimes \mathbf{y}(f) \\ &= \left[\mathbf{x}^*(f) + \mathbf{v}^*(f)\right] \otimes [\mathbf{x}(f) + \mathbf{v}(f)] \\ &= \left|X_1(f)\right|^2 \widetilde{\mathbf{d}}(f) + \mathbf{x}^*(f) \otimes \mathbf{v}(f) \\ &\quad + \mathbf{v}^*(f) \otimes \mathbf{x}(f) + \widetilde{\mathbf{v}}(f)\end{aligned}$$

where

$$\widetilde{\mathbf{d}}(f) = \mathbf{d}^*(f) \otimes \mathbf{d}(f)$$

$$\widetilde{\mathbf{v}}(f) = \mathbf{v}^*(f) \otimes \mathbf{v}(f).$$

Quadratic Filtering

The variance of $\hat{X}(f)$ is given by:

$$\begin{aligned}
\phi_{\hat{X}}(f) &= E[|Z(f)|] \\
&\approx |E[Z(f)]| \\
&= \left|\widetilde{\mathbf{h}}^H(f) E\left[\widetilde{\mathbf{y}}(f)\right]\right| \\
&= \left|\phi_{X_1}(f)\widetilde{\mathbf{h}}^H(f)\widetilde{\mathbf{d}}(f) + \widetilde{\mathbf{h}}^H(f) E\left[\widetilde{\mathbf{v}}(f)\right]\right| \\
&= \left|\widetilde{\mathbf{h}}^H(f)\mathrm{vec}\left[\mathbf{\Phi}_{\mathbf{x}}(f)\right] + \widetilde{\mathbf{h}}^H(f)\mathrm{vec}\left[\mathbf{\Phi}_{\mathbf{v}}(f)\right]\right|
\end{aligned}$$

where the second-row approximation assumes $Z(f)$ to be **real and positive**, that is, $|X_1(f)|^2$ is dominant.

Quadratic Filtering

Narrowband output SNR:

$$\mathrm{oSNR}\left[\widetilde{\mathbf{h}}(f)\right] = \frac{\left|\widetilde{\mathbf{h}}^H(f)\mathrm{vec}\left[\mathbf{\Phi}_{\mathbf{x}}(f)\right]\right|}{\left|\widetilde{\mathbf{h}}^H(f)\mathrm{vec}\left[\mathbf{\Phi}_{\mathbf{v}}(f)\right]\right|}$$

Narrowband output SNR gain:

$$\mathcal{G}\left[\widetilde{\mathbf{h}}(f)\right] = \frac{\mathrm{oSNR}\left[\widetilde{\mathbf{h}}(f)\right]}{\mathrm{iSNR}(f)}$$

Quadratic MVDR

Consider the following criterion:

$$\begin{aligned}\mathcal{J}\left[\widetilde{\mathbf{h}}(f)\right] &= E\left[\left|Z(f)\right|^2\right] \\ &= \widetilde{\mathbf{h}}^H(f)\mathbf{\Phi}_{\widetilde{\mathbf{y}}}(f)\widetilde{\mathbf{h}}(f)\end{aligned}$$

where we spot the 4th-order moment matrix of the observations:

$$\mathbf{\Phi}_{\widetilde{\mathbf{y}}}(f) = E\left[\widetilde{\mathbf{y}}(f)\widetilde{\mathbf{y}}^H(f)\right]$$

Quadratic MVDR

We would like to minimize the optimization criterion **subject to the distortionless constraint:**

$$\widetilde{\mathbf{h}}^H(f)\widetilde{\mathbf{d}}(f) = 1$$

The solution yields the **quadratic MVDR**:

$$\widetilde{\mathbf{h}}_{\mathrm{MVDR}}(f) = \frac{\mathbf{\Phi}_{\widetilde{\mathbf{y}}}^{-1}(f)\widetilde{\mathbf{d}}(f)}{\widetilde{\mathbf{d}}^H(f)\mathbf{\Phi}_{\widetilde{\mathbf{y}}}^{-1}(f)\widetilde{\mathbf{d}}(f)}$$

Quadratic LCMV

Generalize the quadratic MVDR:

Impose a set of linear constraints w.r.t. certain directions in space:

$$\widetilde{\mathbf{h}}^H(f)\widetilde{\mathbf{C}}(f) = \boldsymbol{\beta}^H$$

The solution (w.r.t the same optimization criterion) yields the **quadratic LCMV**:

$$\widetilde{\mathbf{h}}_{\mathrm{LCMV}}(f) = \boldsymbol{\Phi}_{\widetilde{\mathbf{y}}}^{-1}(f)\widetilde{\mathbf{C}}(f)[\widetilde{\mathbf{C}}^H(f)\boldsymbol{\Phi}_{\widetilde{\mathbf{y}}}^{-1}(f)\widetilde{\mathbf{C}}(f)]^{-1}\boldsymbol{\beta}.$$

Quadratic Filtering – Pros and Cons

Pros:

- **Estimation focuses on the spectral power**
  - Known to be more prominent than the phase
- **Exploits higher-order statistics**
  - Further information is used: better noise reduction performance *
- **Generalized approach**
  - Linear filtering is obtained as a special case
  - In general: squared number of filter taps (degrees of freedom)

Cons:

- **Derivation requires inversion of larger correlation matrices**
  - More sensitive to estimation errors
  - Higher computational complexity
- **Spectral power estimate is not guaranteed**
  - Possible distortion in very silent frames (if $Z(f)$ not real and positive)
- **Spectral Phase is not estimated**
  - May be taken from a linear filter

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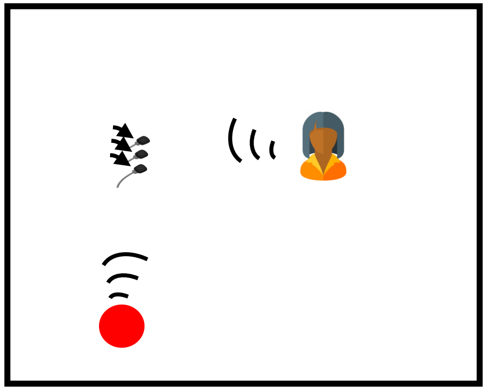
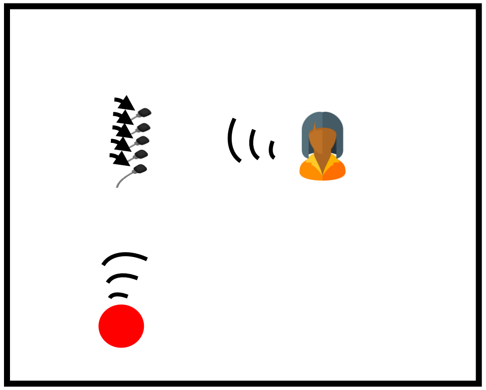
* For more information, see our work: G. Itzhak, J. Benesty, and I. Cohen, ``Quadratic Beamforming for Magnitude Estimation," in *Proc. 29th European Signal Processing Conference (EUSIPCO),* August 2021

Scenario Settings

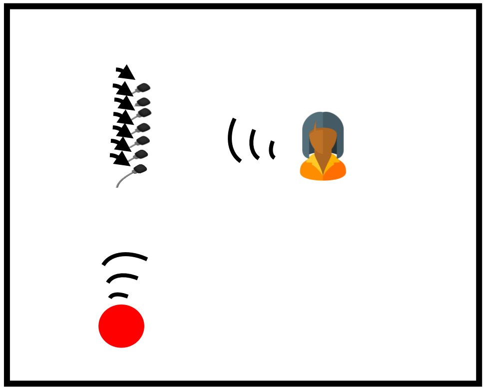
$M = 3$



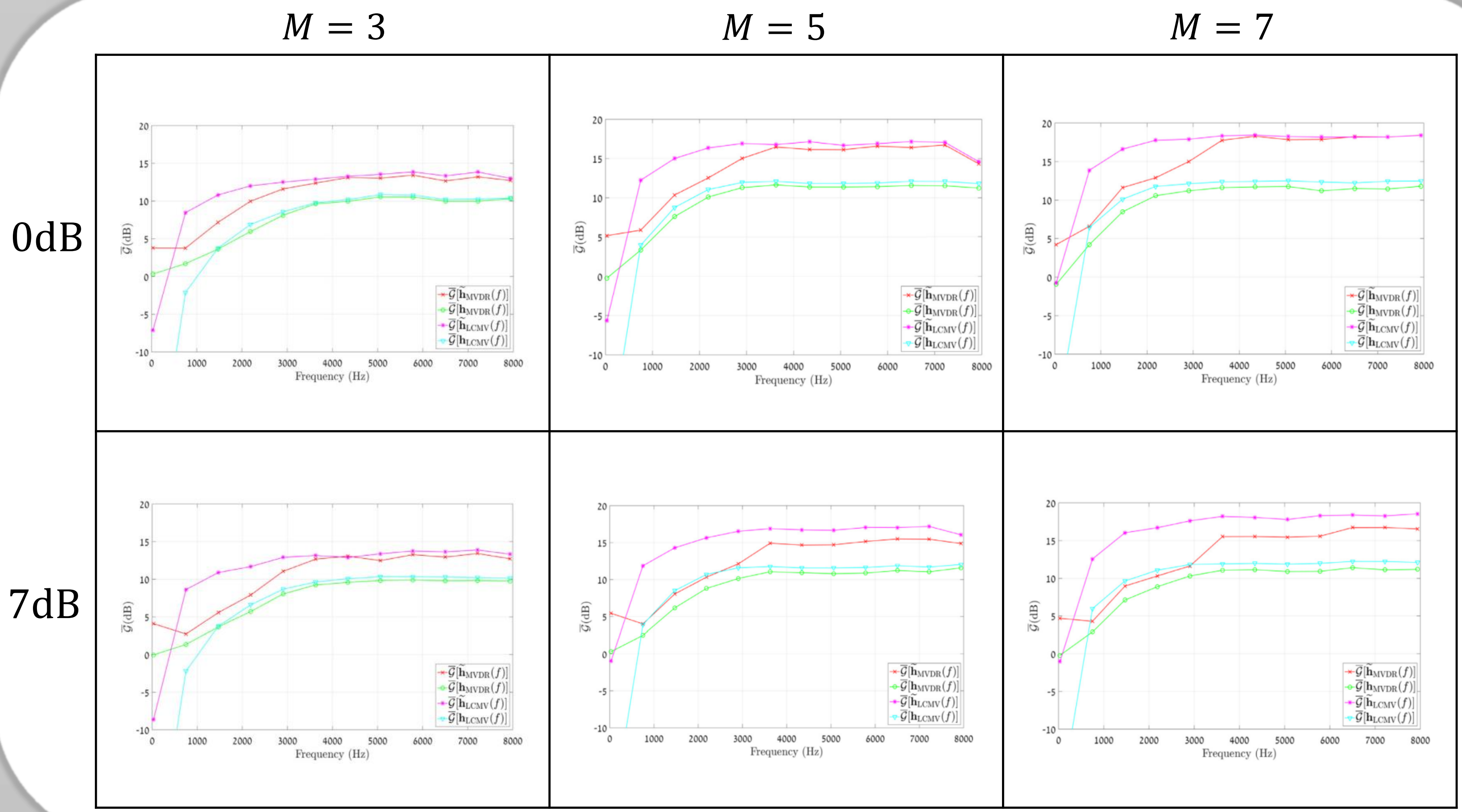
$M = 5$



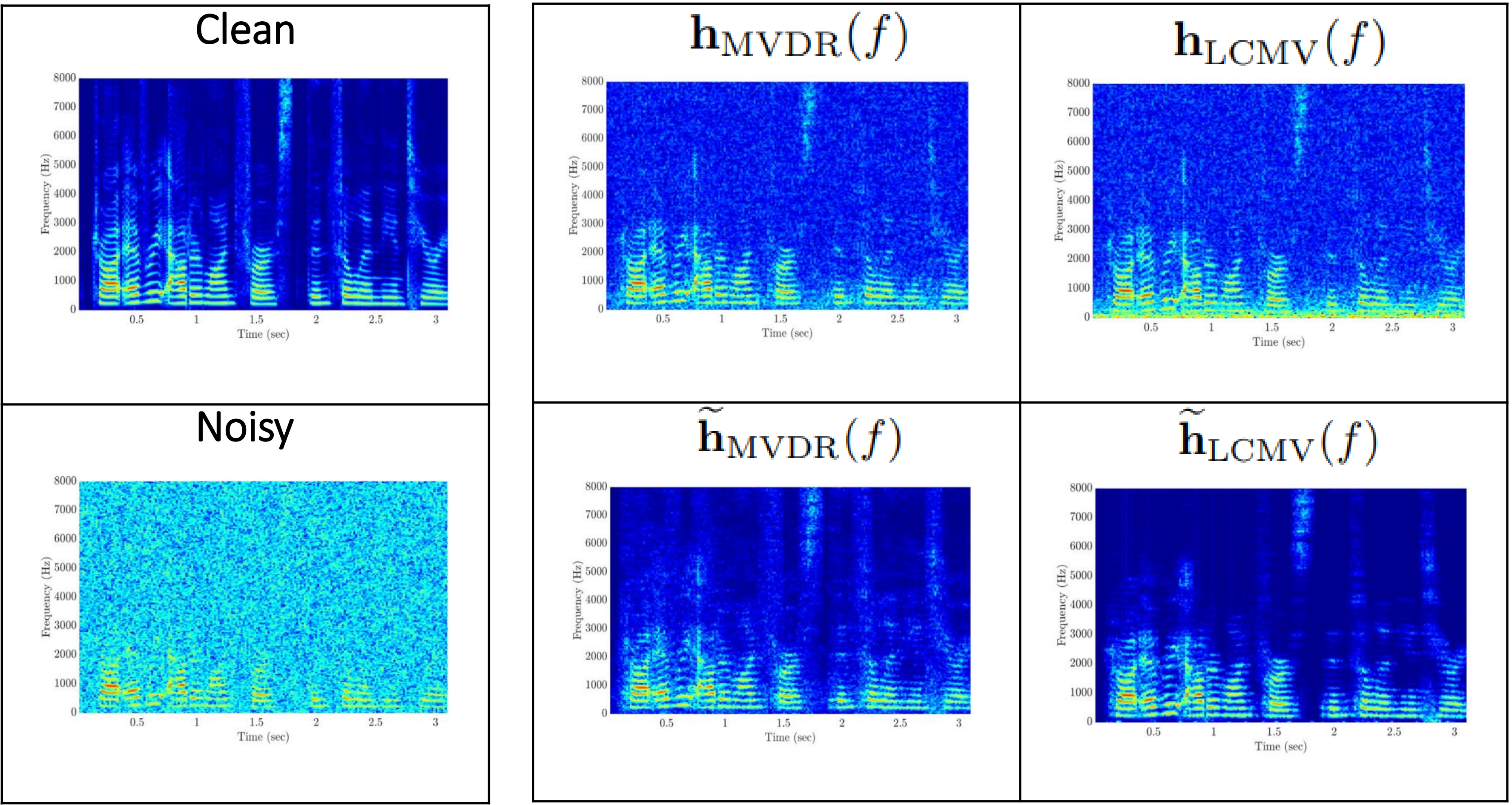
$M = 7$



SNR Gain


$M = 3$
$M = 5$
$M = 7$
0dB
7dB
$\overline{\mathcal{G}}$ (dB)
Frequency (Hz)
20
15
10
5
0
-5
-10
0
1000
2000
3000
4000
5000
6000
7000
8000
$\overline{\mathcal{G}}[\widetilde{\mathbf{h}}_{\text{MVDR}}(f)]$
$\overline{\mathcal{G}}[\mathbf{h}_{\text{MVDR}}(f)]$
$\overline{\mathcal{G}}[\widetilde{\mathbf{h}}_{\text{LCMV}}(f)]$
$\overline{\mathcal{G}}[\mathbf{h}_{\text{LCMV}}(f)]$

Spectrograms



Audio Samples

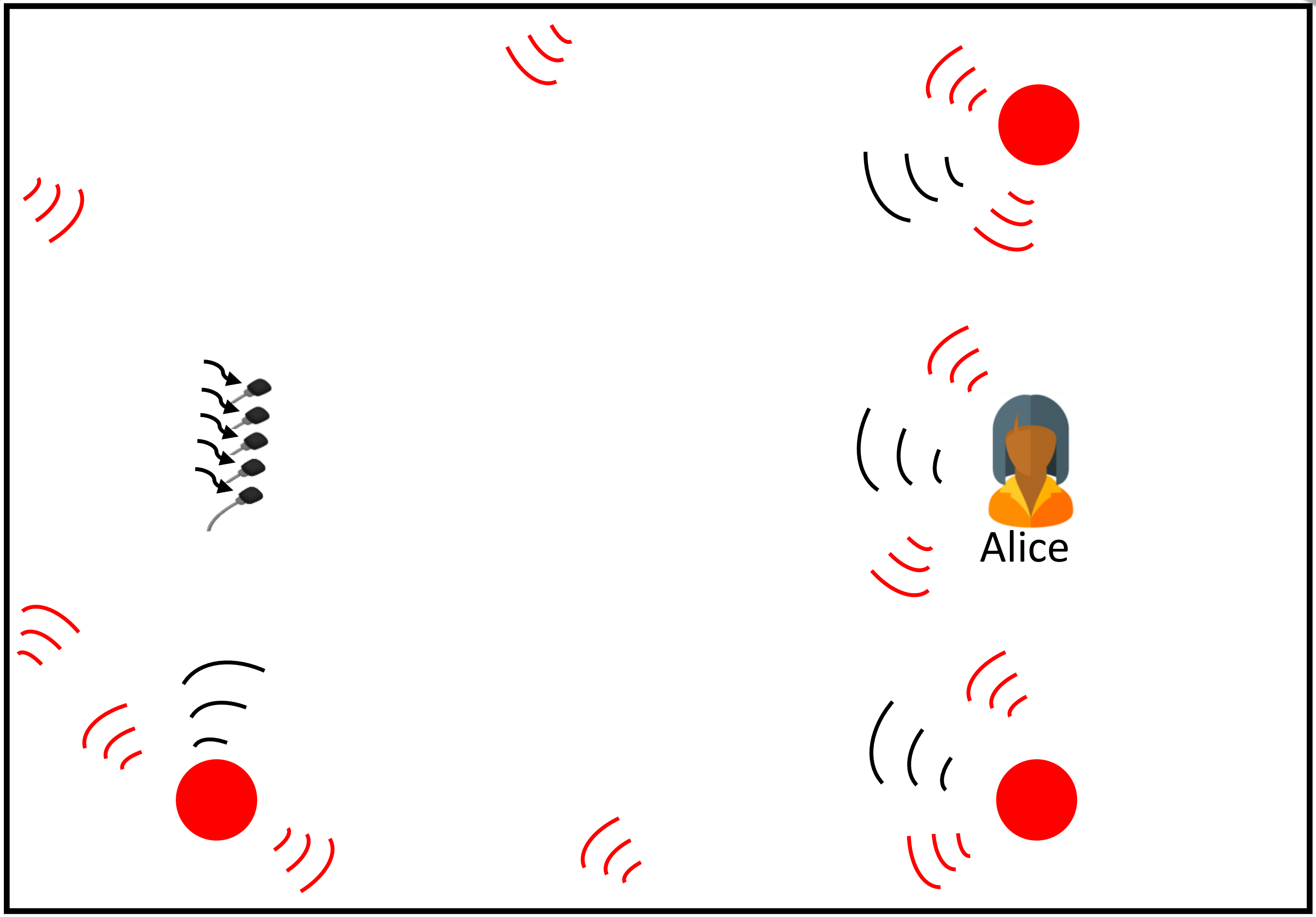
Noisy: 🔊

Clean: 🔊

iSNR = 0dB

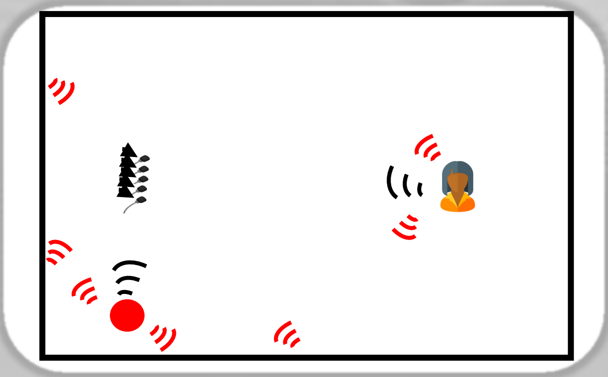
	$\mathbf{h}_{\mathrm{MVDR}}(f)$	$\widetilde{\mathbf{h}}_{\mathrm{MVDR}}(f)$	$\mathbf{h}_{\mathrm{LCMV}}(f)$	$\widetilde{\mathbf{h}}_{\mathrm{LCMV}}(f)$
$M = 3$	🔊	🔊	🔊	🔊
$M = 5$	🔊	🔊	🔊	🔊

Simulations in Reverberant Scenarios

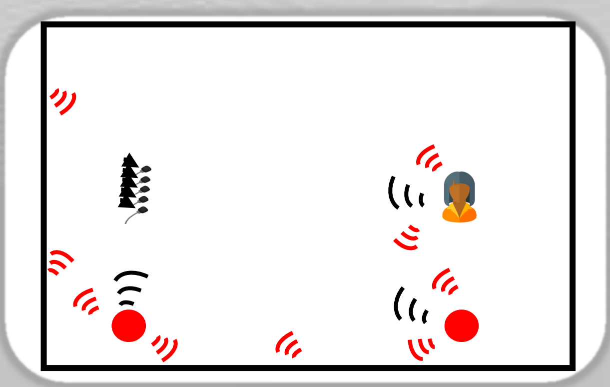


Simulations in Reverberant Scenarios

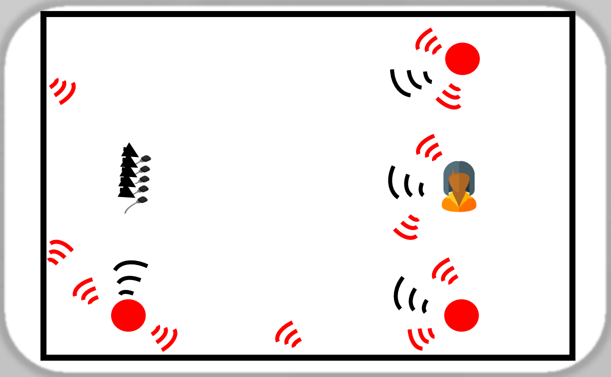
We simulate three scenarios:



Scen. (a)

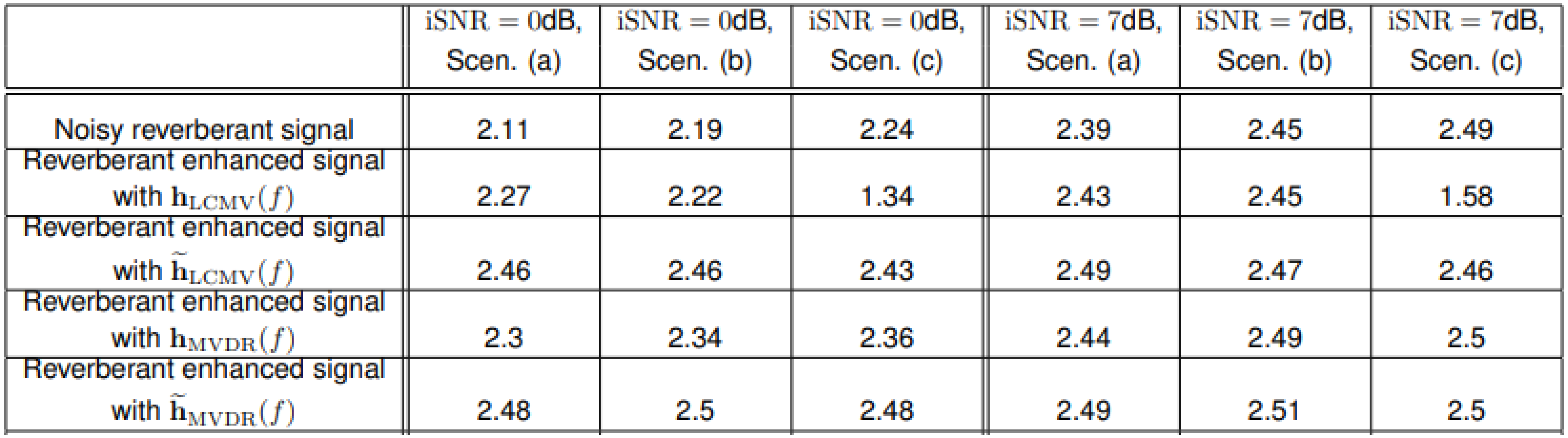


Scen. (b)



Scen. (c)

Average PESQ Scores



	$\mathrm{iSNR} = 0$dB, Scen. (a)	$\mathrm{iSNR} = 0$dB, Scen. (b)	$\mathrm{iSNR} = 0$dB, Scen. (c)	$\mathrm{iSNR} = 7$dB, Scen. (a)	$\mathrm{iSNR} = 7$dB, Scen. (b)	$\mathrm{iSNR} = 7$dB, Scen. (c)
Noisy reverberant signal	2.11	2.19	2.24	2.39	2.45	2.49
Reverberant enhanced signal with $\mathbf{h}_{\mathrm{LCMV}}(f)$	2.27	2.22	1.34	2.43	2.45	1.58
Reverberant enhanced signal with $\widetilde{\mathbf{h}}_{\mathrm{LCMV}}(f)$	2.46	2.46	2.43	2.49	2.47	2.46
Reverberant enhanced signal with $\mathbf{h}_{\mathrm{MVDR}}(f)$	2.3	2.34	2.36	2.44	2.49	2.5
Reverberant enhanced signal with $\widetilde{\mathbf{h}}_{\mathrm{MVDR}}(f)$	2.48	2.5	2.48	2.49	2.51	2.5

Settings:

$$M = 5$$

$$T_{60} = 250\mathrm{msec}$$

Conclusions

- We have introduced a **quadratic filtering approach** to estimate the **spectral power** of a desired signal by exploiting the **higher-order statistics** of its noisy observations
- In some level, this approach may be considered as a generalization of the linear approach, which is obtained as a special case
- We have demonstrated that in both anechoic and reverberant environments the quadratic MVDR and LCMV outperform their common linear versions
- This is emphasized in particular when the **number of sensors is small or when the input SNR is low**.

Future Research Leads

Higher-order Beamforming

- In low SNR or with small arrays:

  quadratic beamforming outperforms the traditional linear beamforming

- We suggest to extend this concept one step further and perform **higher-order beamforming** ,i.e., cubic beamforming, fourth-order beamforming and so on

- We believe that this would enable further reduction of background noise depending on its properties, by taking advantage of its higher-order statistics.

Quadratic Multistage Differential Beamforming

- **Multistage differential beamforming**:

  improves the array directivity but increases sensitivity to white noise

- **Quadratic beamforming**:

  further reduction of the background noise with respect to linear beamforming

- We propose to combine them both: at first apply the spatial difference operator; then, apply a quadratic beamformer
- Consequently, taking advantage of the two concepts, quadratic multistage differential beamforming is likely to benefit from both: **achieve high array directivity** and **mitigate white noise amplification**, in particular in low frequencies

Differential Constant-Beamwidth Beamforming with URAs

- Differential beamformers typically **poor WNG performance** and are not designed to achieve a **constant-beamwidth property**
- On the contrary, they tend to exhibit the appealing properties of a **small physical size** and **high array directivity**
- **Constant-Beamwidth beamformers** are usually designed with ULAs. They exhibit **high WNG performance** and **inferior array directivity** with respect to differential beamformers

Differential Constant-Beamwidth Beamforming with URAs

- I propose to combine a **differential sub-beamformer along the x-axis** with a **constant-beamwidth sub-beamformer along the y-axis.**
- The combination of the two linear sub-beamformers into the global rectangular beamformer will be performed by the KP operator.
- I expect the global beamformer to exhibit **high directivity**, **constant-beamwidth** (to some extent) and an **improved robustness to white noise** in low frequencies, with respect to traditional differential beamformers.
- Nevertheless, I **do not expect this approach to allow convincing beam steering** capabilities.

Highly-Directive Steerable Differential Beamforming

- The former approach is **not steerable**: the **directivity** of the array will **significantly degrade** if the incident angle is not in endfire with respect to the differential sub-beamformer.
- The steering performance may improve if the **constant-beamwidth** sub-beamformer is replaced by a **circular** sub-beamformer.
- In this case, I expect the global beamformer to exhibit **high directivity** for every **azimuth angle.**
- Nevertheless, such a global beamformer is unlikely to achieve the **constant-beamwidth** property.

Highly-Directive and Steerable Constant-Beamwidth Beamforming

- We would like to achieve both **constant-beamwidth** and **steerable** beamforming.
- One possible solution: replace the uniform circular array by a concentric circular array.
- Based on recent advancements, this would potentially allow constant beamwidth with respect to both the azimuth and elevation angles.
- The array directivity may improve to an even greater extent by applying a multistage differentiation scheme across adjacent concentric rings

Thank You!

- I would like to express my sincere gratitude to my principal supervisor, **Prof. Israel Cohen**, for his guidance. His great advice, consistent support and helpful ideas had a huge impact on my work.

- I would also like to thank **Prof. Jacob Benesty**, for his meaningful assistance, for sharing his innovative ideas and expertise in the field.

Questions