KRONECKER PRODUCT BEAMFORMING WITH MULTIPLE DIFFERENTIAL MICROPHONE ARRAYS

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ABSTRACT

Differential microphone arrays (DMAs) are very attractive because of their high directional gains and frequency-invariant beampatterns. However, it is generally required that the array aperture is small, such that the DMA can respond to acoustic pressure differentials. In this paper, we propose a method to design differential beamformers with larger arrays consisting of multiple DMAs. In our study, conventional DMAs are considered as elementary units. The beamforming process consists of elementary differential beamformers and an additional beamformer that combines the multiple DMAs' outputs. The steering vector of the global array is written as a Kronecker product of the steering vectors of an elementary DMA unit and the virtual array constructed from all the DMA units. This enables to design the global beamformer as a Kronecker product of the differential beamformer and the beamformer that corresponds to the virtual array. With the proposed method, one can take advantage of the good properties of DMAs for the design of beamformers with any size of microphone array.

Index Terms— Microphone arrays, differential beamforming, Kronecker product, superdirectivity, multiple arrays.

1. INTRODUCTION

Microphone arrays and beamforming techniques are widely used in acoustic environments with audio and speech applications to recover the desired signal from noisy observations, and many algorithms have been proposed [1], such as superdirective beamforming [2, 3], adaptive beamforming [4–8], differential beamforming [9], etc.. Among them, differential microphone arrays (DMAs) with small array apertures are very attractive because of their high directional gains and frequency-invariant beampatterns [10–16]. The early DMA is implemented in a multistage manner, which measures the sound pressure gradient instead of the sound pressure [17, 18]. This kind of DMA mainly focuses on the linear array and often suffers from inflexibility to form different beampatterns and the so-called white noise amplification problem [10]. In contrast, the recently developed null-constraints based DMAs in the short-time Fourier transform (STFT) domain can better deal with these problems, leading to significant improvements in performance [9,19,20].

Generally, one fundamental assumption in the design of differential beamformers is that the sensor spacing is much smaller than the acoustic wavelength so that the true acoustic pressure differentials can be approximated by finite differences of the microphones' outputs [9,18,21]. In applications with larger array apertures, where this fundamental assumption does not hold, it is difficult to preserve the good properties of DMAs. The Kronecker product decomposition, which has been used in various kinds of signal processing applications [1, 23, 24], can be used. In [1, 22], Kronecker product beamformers are developed, which decompose global arrays into virtual ones, and optimize virtual arrays individually. The control of the performance of this family of beamformers is very flexible and different kinds of beamformers can be combined. However, the Kronecker product beamformers are limited to particular linear and rectangular arrays, which can be perfectly decomposed into smaller sub-arrays.

In this paper, we propose to design differential beamformers with larger arrays consisting of multiple DMAs, where conventional DMAs are considered as elementary units. The beamforming process consists of two stages: the first one is the fundamental differential beamforming, and the second stage further improves the beamforming performance by combining the multiple DMAs' outputs. By writing the steering vector of the global array as a Kronecker product of the steering vectors of the DMA unit and a virtual array constructed from all the DMA units, the global beamformer is designed as a Kronecker product of the fundamental differential beamformer and a beamformer that corresponds to the virtual array. Consequently, the global beampattern is a product of the two beamformers' beampatterns. With the proposed method, we can take advantage of the good properties of DMAs for the design of beamformers with any microphone array.

2. SIGNAL MODEL AND PROBLEM FORMULATION

In our study, we consider a two-dimensional planar array consisting of K DMA units, which are distributed with an arbitrary geometry within a specified area on a plane as illustrated in Fig. 1, and each

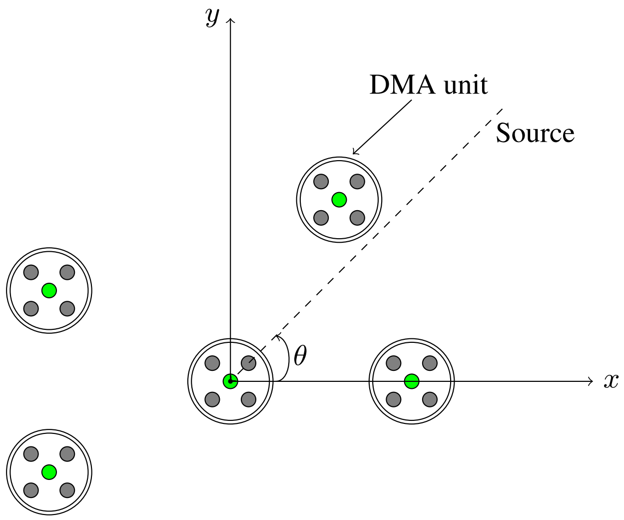

Fig. 1. Illustration of a global microphone array with K small DMA units. Each DMA unit consists of M microphones. The reference points (marked as green) in the DMA units constitute the virtual array (the reference points are arbitrarily chosen in the DMA units).

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DMA unit consists of M microphones. (DMA units can be different, but we consider the same units for simplicity.) The reference points of the DMA units constitute the virtual array. Given a DMA unit, we assume that its center coincides with the origin of the two-dimensional Cartesian coordinate system and the azimuthal angles are measured anti-clockwise from the x axis.

Assuming that a plane wave, in the far-field, propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on a DMA unit from the azimuth angle θ, the corresponding steering vector is defined as

$$\mathbf{d}(\omega, \theta) = \left[e^{\jmath \frac{\omega r_1}{c} \cos(\theta - \psi_1)} \cdots e^{\jmath \frac{\omega r_M}{c} \cos(\theta - \psi_M)} \right]^T, \tag{1}$$

where $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, r_m is the distance from the mth microphone to the origin point, and ψ_m is the angular position of the mth element of the DMA unit.

The signal vector in the STFT domain corresponding to the kth DMA unit can be written as

$$\begin{aligned} \mathbf{y}_k(\omega) &= \left[\begin{array}{cccc} Y_{k,1}(\omega) & Y_{k,2}(\omega) & \cdots & Y_{k,M}(\omega) \end{array} \right]^T \\ &= \mathbf{d}(\omega, \theta_\mathrm{s}) X_k(\omega) + \mathbf{v}_k(\omega),\ k = 1, 2, \ldots, K, \end{aligned} \tag{2}$$

where θ_s is the direction of the desired signal, $X_k(\omega)$ is the desired signal at the reference point of the kth DMA unit, and $\mathbf{v}_k(\omega)$ is the noise vector.

Assume that the differential beamforming filter of the DMA unit is $\mathbf{w}(\omega)$, which will be discussed in Section 4. Its beampattern is

$$\begin{aligned} \mathcal{B}[\mathbf{w}(\omega), \theta] &= \mathbf{w}^H(\omega) \mathbf{d}(\omega, \theta) \\ &= \sum_{m=1}^{M} W_m^*(\omega) e^{\jmath \omega r_m \cos(\theta - \psi_m)/c}, \end{aligned} \tag{3}$$

where the superscripts ^{H} and * are the conjugate-transpose and complex-conjugate operators, respectively. The output of the kth DMA unit is

$$Z_k(\omega) = \mathbf{w}^H(\omega) \mathbf{y}_k(\omega),\ k = 1, 2, \ldots, K. \tag{4}$$

The K DMA units outputs can be written in a vector form as

$$\mathbf{z}(\omega) = \left[\begin{array}{cccc} Z_1(\omega) & Z_2(\omega) & \cdots & Z_K(\omega) \end{array} \right]^T. \tag{5}$$

Now, the objective is to further improve the beamforming performance by combining the K DMA units' outputs with a filter, i.e.,

$$\overline{Z}(\omega) = \mathbf{h}^H(\omega) \mathbf{z}(\omega), \tag{6}$$

where

$$\mathbf{h}(\omega) = \left[\begin{array}{cccc} H_1(\omega) & H_2(\omega) & \cdots & H_K(\omega) \end{array} \right]^T \tag{7}$$

is a beamforming filter of length K.

To better derive the signal model, we also choose a reference point of the global array. Clearly, this reference point can also be arbitrary. Denoting by $X(\omega)$ the desired signal at the reference point of the global array, then we have

$$X_k(\omega) = \alpha_k(\omega, \theta_\mathrm{s}) X(\omega),\ k = 1, 2, \ldots, K, \tag{8}$$

where $\alpha_k(\omega, \theta_\mathrm{s}) = e^{\jmath \omega \tau(\theta_\mathrm{s})}$ and $\tau(\theta_\mathrm{s})$ is the time delay between the reference point of the kth DMA unit and the reference point of the global array. Consequently, the steering vector of length K corresponding to the virtual array can be written as

$$\boldsymbol{\alpha}(\omega, \theta) = \left[\begin{array}{cccc} \alpha_1(\omega, \theta) & \alpha_2(\omega, \theta) & \cdots & \alpha_K(\omega, \theta) \end{array} \right]^T, \tag{9}$$

where $\alpha_k(\omega, \theta) = e^{\jmath \omega \tau(\theta)}$.

The K observation signal vectors, $\mathbf{y}_k(\omega)$, can be concatenated into a vector as

$$\begin{aligned} \underline{\mathbf{y}}(\omega) &= \left[\begin{array}{ccc} \mathbf{y}_1^T(\omega) & \cdots & \mathbf{y}_K^T(\omega) \end{array} \right]^T \\ &= \left[\mathbf{d}^T(\omega, \theta_\mathrm{s}) X_1(\omega) \ \cdots \ \mathbf{d}^T(\omega, \theta_\mathrm{s}) X_K(\omega) \right]^T + \underline{\mathbf{v}}(\omega) \\ &= \left[\boldsymbol{\alpha}(\omega, \theta_\mathrm{s}) \otimes \mathbf{d}(\omega, \theta_\mathrm{s}) \right] X(\omega) + \underline{\mathbf{v}}(\omega), \end{aligned} \tag{10}$$

where $\otimes$ is the Kronecker product and $\underline{\mathbf{v}}(\omega)$ is defined in a similar way to $\underline{\mathbf{y}}(\omega)$. This means that the steering vector of the global array can be written as a Kronecker product of the steering vectors of the DMA unit and the virtual array constructed from the reference points of all the DMA units.

3. KRONECKER PRODUCT BEAMFORMING

Substituting (5) into (6), considering (4), and using (10), the output of the global array can be written as

$$\begin{aligned} \overline{Z}(\omega) &= \mathbf{h}^H(\omega) \left[\begin{array}{ccc} \mathbf{w}^H(\omega) \mathbf{y}_1(\omega) & \cdots & \mathbf{w}^H(\omega) \mathbf{y}_K(\omega) \end{array} \right]^T \\ &= \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \underline{\mathbf{y}}(\omega) \\ &= \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \left[\boldsymbol{\alpha}(\omega, \theta_\mathrm{s}) \otimes \mathbf{d}(\omega, \theta_\mathrm{s}) \right] X(\omega) \\ &\quad + \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \underline{\mathbf{v}}(\omega). \end{aligned} \tag{11}$$

It is seen that the two stage beamforming approach corresponds to a global beamforming filter:

$$\mathbf{g}(\omega) = \mathbf{h}(\omega) \otimes \mathbf{w}(\omega). \tag{12}$$

Consequently, the beampattern of the global array can be written as [1]

$$\begin{aligned} \mathcal{B}[\mathbf{g}(\omega), \theta] &= \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \left[\boldsymbol{\alpha}(\omega, \theta) \otimes \mathbf{d}(\omega, \theta) \right] \\ &= \mathcal{B}[\mathbf{w}(\omega), \theta] \, \mathcal{B}[\mathbf{h}(\omega), \theta], \end{aligned} \tag{13}$$

where $\mathcal{B}[\mathbf{w}(\omega), \theta]$ and $\mathcal{B}[\mathbf{h}(\omega), \theta]$ are the beampatterns of the DMA unit and virtual array, respectively. Accordingly, the global beampattern is the product of the DMA unit beampattern and the virtual array beampattern, which indicates that one can design the beampatterns (or beamformers) separately as in a cascaded manner.

The white noise gain (WNG) evaluates the robustness of a beamformer. The WNG of the global beamformer is the product of WNGs of the DMA unit and the virtual array [1], i.e.,

$$\begin{aligned} \mathcal{W}[\mathbf{g}(\omega)] &= \frac{\left| \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \left[\boldsymbol{\alpha}(\omega, \theta_\mathrm{s}) \otimes \mathbf{d}(\omega, \theta_\mathrm{s}) \right] \right|^2}{\left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]} \\ &= \mathcal{W}[\mathbf{w}(\omega)] \, \mathcal{W}[\mathbf{h}(\omega)]. \end{aligned} \tag{14}$$

The directivity factor (DF) quantifies the performance of the beamformer in a spherically isotropic noise field [18]. The DF of the global beamformer is written as

$$\mathcal{D}[\mathbf{g}(\omega)] = \frac{\left| \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \left[\boldsymbol{\alpha}(\omega, \theta_\mathrm{s}) \otimes \mathbf{d}(\omega, \theta_\mathrm{s}) \right] \right|^2}{\left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]^H \boldsymbol{\Gamma}(\omega) \left[\mathbf{h}(\omega) \otimes \mathbf{w}(\omega) \right]}, \tag{15}$$

where $\boldsymbol{\Gamma}(\omega)$ is the pseudo-coherence matrix of the noise signal in a spherically isotropic noise field, and the (i, j)th element of $\boldsymbol{\Gamma}(\omega)$ is $\mathrm{sinc}(\omega \delta_{ij}/c)$, with δ_{ij} being the distance between microphones i and j. The DFs of the DMA unit and the virtual array are

$$\mathcal{D}[\mathbf{w}(\omega)] = \frac{\left| \mathbf{w}^H(\omega) \mathbf{d}(\omega, \theta_\mathrm{s}) \right|^2}{\mathbf{w}^H(\omega) \boldsymbol{\Gamma}_\mathrm{d}(\omega) \mathbf{w}(\omega)},$$

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{\left| \mathbf{h}^H(\omega) \boldsymbol{\alpha}(\omega, \theta_\mathrm{s}) \right|^2}{\mathbf{h}^H(\omega) \boldsymbol{\Gamma}_{\boldsymbol{\alpha}}(\omega) \mathbf{h}(\omega)},$$

where $\mathbf{\Gamma}_{\mathrm{d}}(\omega)$ and $\mathbf{\Gamma}_{\boldsymbol{\alpha}}(\omega)$ are the pseudo-coherence matrices of the noise signal in a spherically isotropic noise field, corresponding to the DMA unit and the virtual array [1]. Note that $\mathcal{D}[\mathbf{g}(\omega)] \neq \mathcal{D}[\mathbf{w}(\omega)]\,\mathcal{D}[\mathbf{h}(\omega)]$.

To satisfy the distortionless constraint in the desired direction, we choose

$$\mathbf{w}^H(\omega)\,\mathbf{d}(\omega,\theta_{\mathrm{s}}) = \mathbf{h}^H(\omega)\,\boldsymbol{\alpha}(\omega,\theta_{\mathrm{s}}) = 1. \tag{16}$$

4. DIFFERENTIAL BEAMFORMING

The frequency-independent beampattern of an Nth-order DMA with its main beam pointing to θ_{s} is given by [9, 19]

$$\mathcal{B}_N(\theta-\theta_{\mathrm{s}}) = \sum_{n=-N}^{N} b_{2N,n} e^{\jmath n(\theta-\theta_{\mathrm{s}})}, \tag{17}$$

where $b_{2N,n}$, $n = 0, \pm 1, \cdots, \pm N$ are real coefficients that determine the different directivity patterns. To satisfy the distortionless constraint in the desired direction, it is generally required that

$$\sum_{n=-N}^{N} b_{2N,n} = 1.$$

The design of a differential beamformer entails finding a proper beamforming filter, $\mathbf{w}(\omega)$, so that the designed beampattern is close to the desired frequency-invariant beampattern, i.e.,

$$\mathcal{B}[\mathbf{w}(\omega),\theta] \approx \mathcal{B}_N(\theta-\theta_{\mathrm{s}}). \tag{18}$$

There are many ways to find the optimal DMA beamforming filter, such as null constraints and series expansion methods [9, 20, 21]. Here, we use the Jacobi-Anger expansion method [25], where the exponential function that appears in the beamformer beampattern, $\mathcal{B}[\mathbf{w}(\omega),\theta]$, can be approximated as [19]

$$e^{\jmath \frac{\omega r_m}{c}\cos(\theta-\psi_m)} = \sum_{n=-N}^{N} \jmath^n J_n\left(\frac{\omega r_m}{c}\right) e^{\jmath n(\theta-\psi_m)}, \tag{19}$$

with $J_n(\cdot)$ being the nth-order Bessel function of the first kind with $J_{-n}(x) = (-1)^n J_n(x)$ [25].

Substituting (19) into the definition of the beampattern with a DMA from (3), the relation between the desired directivity pattern and the designed beampattern can be found and the beamforming filter is finally solved as [21]

$$\mathbf{w}_{\mathrm{DMA}}(\omega) = \mathbf{\Psi}^H(\omega)\left[\mathbf{\Psi}(\omega)\mathbf{\Psi}^H(\omega)\right]^{-1}\mathbf{\Upsilon}^*(\theta_{\mathrm{s}})\,\mathbf{b}_{2N}, \tag{20}$$

where

$$\mathbf{\Psi}(\omega) = \begin{bmatrix} (-\jmath)^{-N}\boldsymbol{\psi}_{-N}^H(\omega) \\ \vdots \\ \boldsymbol{\psi}_0^H(\omega) \\ \vdots \\ (-\jmath)^{N}\boldsymbol{\psi}_N^H(\omega) \end{bmatrix},$$

with

$$\boldsymbol{\psi}_n(\omega) = \left[J_n\left(\frac{\omega r_1}{c}\right)e^{-\jmath n\psi_1} \ \cdots \ J_n\left(\frac{\omega r_M}{c}\right)e^{-\jmath n\psi_M}\right]^T,$$

$n = 0, \pm 1, \cdots, \pm N$, and

$$\mathbf{\Upsilon}(\theta_{\mathrm{s}}) = \mathrm{diag}\left(e^{\jmath N\theta_{\mathrm{s}}}, \ldots, 1, \ldots, e^{-\jmath N\theta_{\mathrm{s}}}\right),$$
$$\mathbf{b}_{2N} = [\begin{array}{ccccc} b_{2N,-N} & \cdots & b_{2N,0} & \cdots & b_{2N,N} \end{array}]^T.$$

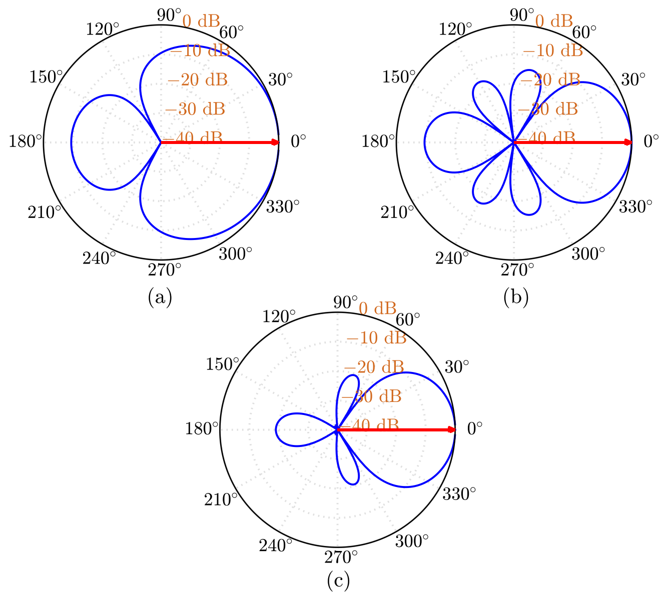

Fig. 2. Beampatterns: (a) DMA unit (designed as the first-order hypercardioid), (b) virtual array (designed with the maximum DF beamformer), and (c) multiple DMAs. Conditions: $M = 4$, $r = 1$ cm, $d = 4$ cm, $f = 1$ kHz, and $\theta_{\mathrm{s}} = 0^\circ$.

Therefore, by changing the value of θ_{s}, we can control the steering of the DMA beampattern. By changing the value of $\mathbf{b}_{2N}$, we can change the shape of the DMA beampattern.

For the virtual array, there are many ways to design the beamformer. Here, we consider maximizing the DF, which leads to the maximum DF beamformer [26]:

$$\mathbf{h}(\omega) = \mathbf{h}_{\mathrm{MDF}}(\omega) = \frac{\mathbf{\Gamma}_{\boldsymbol{\alpha}}^{-1}(\omega)\,\boldsymbol{\alpha}(\omega,\theta_{\mathrm{s}})}{\boldsymbol{\alpha}^H(\omega,\theta_{\mathrm{s}})\,\mathbf{\Gamma}_{\boldsymbol{\alpha}}^{-1}(\omega)\,\boldsymbol{\alpha}(\omega,\theta_{\mathrm{s}})}. \tag{21}$$

Another widely-used beamformer is the delay-and-sum (DS) beamformer:

$$\mathbf{h}(\omega) = \mathbf{h}_{\mathrm{DS}}(\omega) = \frac{1}{K}\boldsymbol{\alpha}(\omega,\theta_{\mathrm{s}}), \tag{22}$$

which maximizes the WNG. Finally, taking into account (20) and (21) or (22), the global beamformer is obtained by (12).

5. SIMULATIONS AND DISCUSSION

In this section, we study the performance of the proposed multiple DMAs (MDMAs) beamformer through simulations. The DMA unit is a circular differential microphone array (CDMA), which consists of 4 microphones uniformly distributed on a circle of 1 cm radius. The fundamental differential beamformer, i.e., $\mathbf{w}_{\mathrm{DMA}}(\omega)$, is the first-order hypercardioid [9], where $\mathbf{b}_{2N}$ is given by

$$\mathbf{b}_{2N} = [\begin{array}{ccc} 1/3 & 1/3 & 1/3 \end{array}]^T. \tag{23}$$

The K DMA units are linearly distributed to form the global array. The desired direction is chosen as $\theta_{\mathrm{s}} = 0^\circ$.

In the first experiment, the distance between two adjacent DMA units is $d = 4$ cm. The second stage beamformer, i.e., $\mathbf{h}(\omega)$, is designed as the maximum DF beamformer according to (21). Figure 2 plots the beampatterns of the DMA unit, virtual array, and multiple DMAs, with $K = 4$ at $f = 1$ kHz. Notice that the beampattern of the DMA unit [Fig. 2(a)] is the first-order hypercardioid, the beampattern of the maximum DF beamformer [Fig. 2(b)] is the second-order hypercardioid (the maximum DF beamformer corresponds to

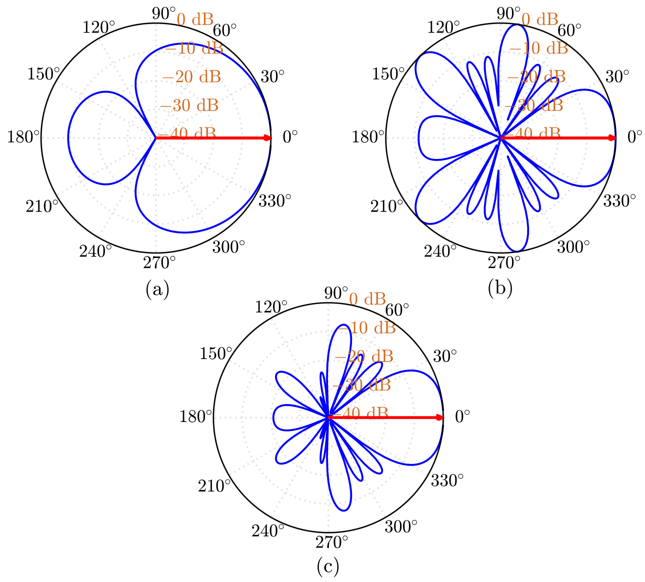

Fig. 3. Beampatterns: (a) DMA unit (designed as the first-order hypercardioid), (b) virtual array (designed with the maximum DF beamformer), and (c) multiple DMAs. Conditions: $M = 4$, $r = 1$ cm, $d = 20$ cm, $f = 1$ kHz, and $\theta_s = 0°$.

the hypercardioid when the array is small), the global beampattern of the multiple DMAs [Fig. 2(c)] is a product of the DMA unit beampattern and the maximum DF beampattern.

In the second experiment, we consider a larger array aperture with a distance between two adjacent DMA units being $d = 20$ cm and the second stage beamformer is designed as the maximum WNG beamformer according to (22). Figure 3 plots the beampatterns of the DMA unit, virtual array, and multiple DMAs, with $K = 4$ at $f = 1$ kHz. It can be seen that the DS beamformer with the virtual array have grating lobes. This is because larger array apertures inevitably result in grating lobes at high frequencies. However, the global beampattern of the multiple DMAs, as a product of the DMA unit beampattern and the DS beampattern can be designed with low grating lobes. Figure 4 plots the DFs and the WNGs of the proposed multiple DMAs beamformers as a function of frequency, f, with $K = 1, 2, 4, 6$ ($K = 1$ is the conventional DMA). Clearly, all the beamformers achieve high directivity even at low frequencies. It is also observed that both the DF and the WNG can be improved by using multiple DMAs, i.e., increasing K. This demonstrates that the proposed method enables to maintain the good properties of DMAs, not only reducing the grating lobes, but also achieving high directivity over a wide frequency range.

So far, we have only considered using a first-order DMA as a fundamental unit. This can be extended to the general case with any other higher order and different structures. For instance:

- **Different beamformers.** The DMA unit can be any kind of differential beamformer, such as the dipole, cardioid, supercardioid, etc. The number of microphones can be arbitrary. (Note that at least $2N + 1$ microphones are needed to design a fully steerable Nth-order differential beamformer.) The DMA beamformers can also be adaptive.
- **Structures.** The DMA unit can be designed in widely-used structure, such as a linear array, circular array, concentric circular array, and three-dimensional array. The DMA units can also share some sensors, in such a way that there will be more freedom to satisfy further constraints.
- **Directional microphones.** The DMA unit can be replaced by fixed directional microphones. In this case, we may not be able to design the two stages beamformer as a Kronecker product. However, we can optimize the global performance in a two-stage beamforming from the beampattern perspective.

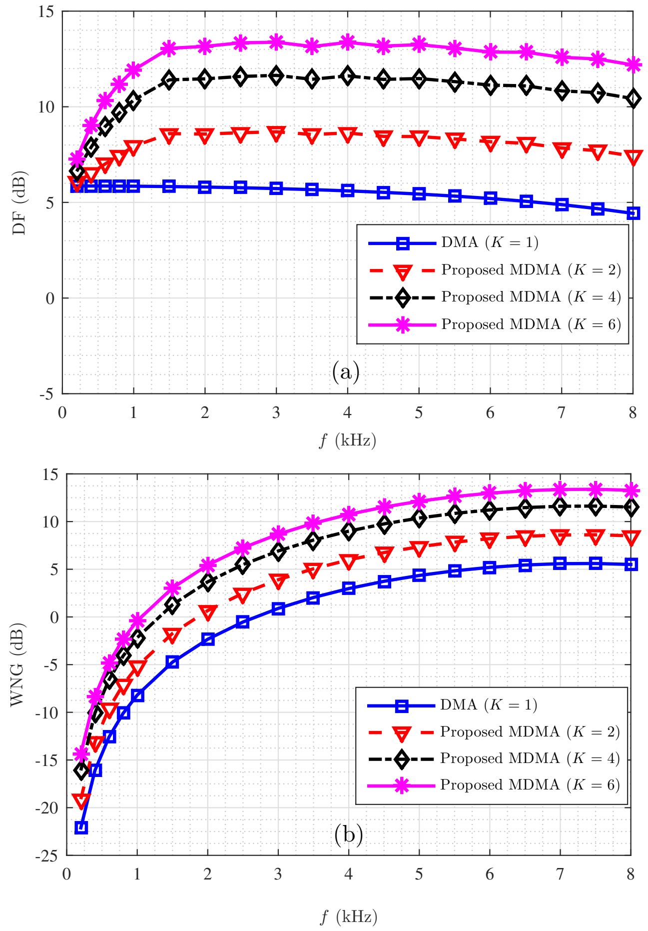

Fig. 4. DFs and WNGs: (a) DFs and (b) WNGs. Conditions: $M = 4$, $r = 1$ cm, $d = 20$ cm, and $\theta_s = 0°$.

6. CONCLUSIONS

We proposed a method for designing differential beamformers with multiple DMAs. We considered a two-dimensional planar array consisting of multiple DMA units, which are distributed in an arbitrary specified area. The beamforming process is carried out in two stages: the first stage includes the design of the elementary unit of differential beamforming, and the second stage further improves the beamforming performance by combining the multiple DMAs outputs. In our study, we designed the two-stage beamformers as a Kronecker product of the elementary differential beamformer and the beamformer that corresponds to a virtual array. Consequently, the global beamformer can be optimized in a cascaded manner and the global beampattern is a product of the two beampatterns. Simulations demonstrated that the proposed method is useful for the design of broadband beamformers with large arrays, while retaining the good properties of DMAs. Note that this basic idea can be extended to the design of DMAs with any kind of differential beamformer and array geometry.

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