ROBUST AND STEERABLE KRONECKER PRODUCT DIFFERENTIAL BEAMFORMING WITH RECTANGULAR MICROPHONE ARRAYS

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ABSTRACT

Differential microphone arrays (DMAs), a class of well-designed small-size arrays combined with differential beamforming, are very useful for processing broadband acoustic, audio, and speech signals in a wide range of applications. However, most efforts in the literature so far have been devoted to linear, circular, and spherical arrays. In this paper, we consider rectangular shapes of planar microphone arrays. Instead of adopting the traditional differential beamforming methods developed in the literature, we present a differential beamforming method based on the so-called Kronecker product. We first decompose the entire rectangular array into two virtual rectangular sub-arrays so that the steering vector of the entire array is the Kronecker product of the steering vectors of the two smaller virtual rectangular sub-arrays. We use the first virtual rectangular array, which is much smaller in size than the entire array but well satisfies the basic requirements for differential beamforming, to design a steerable differential beamformer. For the second virtual rectangular array, we can design either the delay-and-sum (DS) beamformer, which helps to improve the robustness of the global differential beamformer, or an adaptive beamformer, which makes the global differential beamformer adaptive. This method has many interesting properties, particularly the designed beamformer is fully steerable, and its robustness and the array gain can be easily controlled.

Index Terms— Microphone arrays, Kronecker product, differential beamforming, adaptive beamforming.

1. INTRODUCTION

Microphone arrays have been intensively studied and many beamforming methods have been developed over the last few decades [1,2, 4,23], such as the delay-and-sum (DS) beamformer, adaptive beamformers [5–7], the superdirective beamformer [8–10], and the differential beamformers [11–15]. Among those, the differential beamformers [the resulting microphone arrays are called differential microphone arrays (DMAs)] have the potential to achieve high directivity and almost frequency-invariant beampatterns; so, they have been used in a wide range of acoustic applications to process broadband acoustic, audio, and speech signals [11, 13]. As a result, the design of differential beamforming algorithms and the associated microphone arrays have attracted intensive attention over the last two decades [16–18].

Most efforts on differential beamforming in the literature so far have been devoted to linear, circular, and spherical array geometries. In this paper, we consider differential beamforming with rectangular shapes of planar microphone arrays. This type of DMAs can be used in many flat panel devices. One way to achieve differential beamforming with such arrays is based on the method developed in [19], which solves a linear system derived from the Jacobi-Anger expansion. Such an approach uses all the microphones of the array in one step to form the differential beamforming filter. However, if the number of sensors is large, this method lacks flexibility in controlling the array spatial gain and its robustness. The resulting array response may become more and more frequency dependent as the number of sensors increases.

Recently, the so-called Kronecker product beamformer has been developed, which decomposes the global array into sub-arrays, and optimizes each sub-array individually [2, 20]. The Kronecker beamformer not only offers flexibility to control the array performance and compromises among different performance metrics, but also makes it possible to flexibly combine different kinds of beamformers, such as fixed and adaptive beamformers. However, the existing work on Kronecker product beamforming does not consider the steering flexibility, which is also very important in practice. So, this paper extends the work in [2] to design flexible and robust steerable differential beamformers with rectangular arrays. We focus on designing steerable differential beamformers with rectangular arrays, so the two virtual arrays should also be planar arrays to ensure steering capabilities.

We decompose the entire rectangular array into two virtual rectangular sub-arrays, so that the steering vector of the entire array is the Kronecker product of the steering vectors of the two smaller virtual rectangular sub-arrays. We then use the first virtual rectangular array, which is small in size and can well satisfy the basic requirements for DMAs, to design a steerable differential beamformer. The second virtual rectangular array is subsequently used to design other beamformers to help better control the beamforming performance of the global array. In this way, the global beamforming filter is designed by optimizing the two sub-beamformers in a cascaded manner.

2. SIGNAL MODEL AND PROBLEM FORMULATION

We consider a two-dimensional rectangular array consisting of M omnidirectional microphones, as illustrated in Fig. 1. Assume that the center of the array coincides with the origin of the two-dimensional Cartesian coordinate system and the azimuthal angles are measured anti-clockwise from the x axis, the coordinates of the microphones can be written as

$$\mathbf{r}_m = r_m \left[\cos\psi_m \ \sin\psi_m\right]^T , \quad (1)$$

with $m = 1, 2, \ldots, M$, where the superscript ^{T} is the transpose operator, r_m is the distance from the mth microphone to the origin point, and ψ_m is the angular position of the mth array element. Assume that a plane wave in the farfield propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on the array from azimuthal angle θ. In this scenario, the

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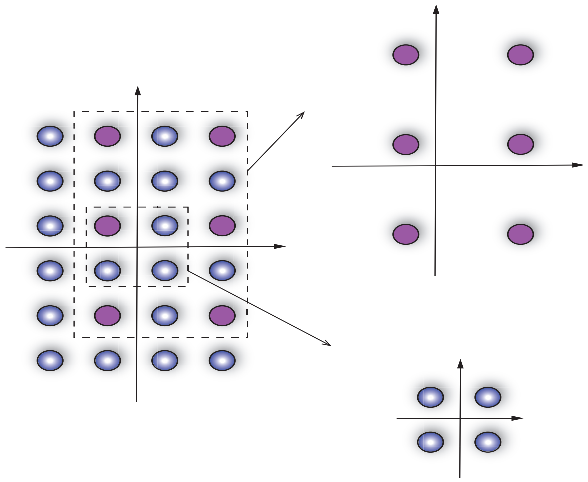

Fig. 1. Illustration of the decomposition of a uniform rectangular array with $M = 6 \times 4$ microphones and an interelement spacing of δ into two virtual uniform rectangular arrays, where the first virtual uniform rectangular array consists of $M_1 = 3 \times 2$ microphones with an interelement spacing of 2δ, and the second virtual uniform rectangular array consists of $M_2 = 2 \times 2$ microphones with an interelement spacing of δ.

steering vector of length M is defined as

$$\mathbf{d}_\theta(\omega) \triangleq \left[e^{\jmath \varpi_1(\theta)} \; e^{\jmath \varpi_2(\theta)} \; \cdots \; e^{\jmath \varpi_M(\theta)} \right]^T, \quad (2)$$

where $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\varpi_m(\theta) = (\omega r_m/c)\cos(\theta - \psi_m)$, $m = 1, 2, \ldots, M$, $\omega = 2\pi f$ is the angular frequency, and $f > 0$ is the temporal frequency.

The noisy observation vector of the rectangular sensor array of length M can be written as

$$\begin{aligned} \mathbf{y}(\omega) &= \left[\begin{array}{cccc} Y_1(\omega) & Y_2(\omega) & \cdots & Y_M(\omega) \end{array} \right]^T \\ &= \mathbf{d}_{\theta_s}(\omega) X(\omega) + \mathbf{v}(\omega), \end{aligned} \quad (3)$$

where $\mathbf{d}_{\theta_s}(\omega)$ is the signal propagation vector, which is equal to $\mathbf{d}_\theta(\omega)$ at $\theta = \theta_s$, and $\mathbf{v}(\omega)$ is the noise signal vector of length M, which is defined in a similar way to $\mathbf{y}(\omega)$.

Beamforming is the process of recovering the acoustic signal of interest that is corrupted by spatial acoustic noise through a spatial filter of length M. It is mathematically written as [8]

$$Z(\omega) = \mathbf{h}^H(\omega)\mathbf{y}(\omega), \quad (4)$$

where $Z(\omega)$ is an estimate of $X(\omega)$, the superscript ^{H} is the conjugate-transpose operator, and

$$\mathbf{h}(\omega) = \left[\begin{array}{cccc} H_1(\omega) & H_2(\omega) & \cdots & H_M(\omega) \end{array} \right]^T, \quad (5)$$

with $H_m(\omega)$, $m = 1, 2, \ldots, M$, being the complex weight at the mth microphone.

Without loss of generality, it is expected that the source signal of interest can pass through the beamformer without being changed (or distorted). So, the distortionless constraint in the look direction is desired, i.e.,

$$\mathbf{h}^H(\omega)\mathbf{d}_{\theta_s}(\omega) = 1. \quad (6)$$

Generally, the optimal beamforming filter is obtained by using all the microphones of the array under some criterion. Alternatively, this paper follows the idea of Kronecker beamforming in [2] to design flexible and robust steerable differential beamformers. To simplify the notation, we drop the dependence on the angular frequency, ω, in the rest of this paper.

3. KRONECKER PRODUCT BEAMFORMER

We decompose the steering vector in (2) as a Kronecker product of two steering vectors of smaller virtual rectangular arrays as

$$\mathbf{d}_\theta = \mathbf{d}_{1,\theta} \otimes \mathbf{d}_{2,\theta}, \quad (7)$$

where

$$\mathbf{d}_{1,\theta} = \left[e^{\jmath \varpi_{1,1}(\theta)} \; e^{\jmath \varpi_{1,2}(\theta)} \; \cdots \; e^{\jmath \varpi_{1,M_1}(\theta)} \right]^T, \quad (8)$$

$$\mathbf{d}_{2,\theta} = \left[e^{\jmath \varpi_{2,1}(\theta)} \; e^{\jmath \varpi_{2,2}(\theta)} \; \cdots \; e^{\jmath \varpi_{2,M_2}(\theta)} \right]^T \quad (9)$$

are the steering vectors of the two virtual arrays, with $\varpi_{1,m}(\theta) = (\omega r_{1,m}/c)\cos(\theta - \psi_{1,m})$, $m = 1, 2, \ldots, M_1$, and $\varpi_{2,m}(\theta) = (\omega r_{2,m}/c)\cos(\theta - \psi_{2,m})$, $m = 1, 2, \ldots, M_2$.

This paper focuses on designing steerable differential beamformers with rectangular arrays, so the two virtual arrays should also be planar arrays to ensure the steering capabilities. Generally, to make a beamformer fully steerable, i.e., achieving consistent beampatterns over different look directions, it is better to make the array structure symmetric with respect to the reference point. Clearly, how to choose the reference point is an important issue. A rule of thumb is to choose the center (or a point close to the center) of the two virtual arrays under the precondition of the Kronecker product and make both virtual arrays symmetric about the reference point.

We also write the beamforming filter, $\mathbf{h}$, corresponding to the global rectangular array as a Kronecker product of two filters:

$$\mathbf{h} = \mathbf{h}_1 \otimes \mathbf{h}_2, \quad (10)$$

where $\mathbf{h}_1$ and $\mathbf{h}_2$ are spatial filters of lengths M_1 and M_2, respectively, corresponding to the two virtual arrays.

The global beamformer's output can then be written as

$$\begin{aligned} Z &= (\mathbf{h}_1 \otimes \mathbf{h}_2)^H \mathbf{y} \\ &= (\mathbf{h}_1 \otimes \mathbf{h}_2)^H (\mathbf{d}_{1,\theta_s} \otimes \mathbf{d}_{2,\theta_s}) X + (\mathbf{h}_1 \otimes \mathbf{h}_2)^H \mathbf{v} \\ &= \left(\mathbf{h}_1^H \mathbf{d}_{1,\theta_s}\right)\left(\mathbf{h}_2^H \mathbf{d}_{2,\theta_s}\right) X + (\mathbf{h}_1 \otimes \mathbf{h}_2)^H \mathbf{v}. \end{aligned} \quad (11)$$

To satisfy the distortionless constraint as given in (6), we impose $\left(\mathbf{h}_1^H \mathbf{d}_{1,\theta_s}\right)\left(\mathbf{h}_2^H \mathbf{d}_{2,\theta_s}\right) = 1$. To simplify the developments, we choose

$$\mathbf{h}_1^H \mathbf{d}_{1,\theta_s} = \mathbf{h}_2^H \mathbf{d}_{2,\theta_s} = 1. \quad (12)$$

The beampattern, which describes the sensitivity of the beamformer to a plane wave impinging on the array from the direction θ, is defined as

$$\mathcal{B}_\theta(\mathbf{h}) \triangleq \mathbf{h}^H \mathbf{d}_\theta. \quad (13)$$

It follows immediately that

$$\begin{aligned} \mathcal{B}_\theta(\mathbf{h}) &= (\mathbf{h}_1 \otimes \mathbf{h}_2)^H (\mathbf{d}_{1,\theta} \otimes \mathbf{d}_{2,\theta}) \\ &= \left(\mathbf{h}_1^H \mathbf{d}_{1,\theta}\right)\left(\mathbf{h}_2^H \mathbf{d}_{2,\theta}\right) \\ &= \mathcal{B}_{1,\theta}(\mathbf{h}_1)\,\mathcal{B}_{2,\theta}(\mathbf{h}_2), \end{aligned} \quad (14)$$

where $\mathcal{B}_{1,\theta}(\mathbf{h}_1)$ and $\mathcal{B}_{2,\theta}(\mathbf{h}_2)$ are the beampatterns corresponding to the first and second virtual arrays, respectively. As seen, the global beampattern, $\mathcal{B}(\mathbf{h})$, is a product of beampatterns of the two virtual arrays.

With the Kronecker product decomposition, the white noise gain (WNG) of the global beamformer, which is the signal-to-noise-ratio (SNR) gain in spatially white noise, can be written as

$$\mathcal{W}(\mathbf{h}) = \frac{\left|\mathbf{h}^H \mathbf{d}_{\theta_s}\right|^2}{\mathbf{h}^H \mathbf{h}} = \mathcal{W}_1(\mathbf{h}_1)\,\mathcal{W}_2(\mathbf{h}_2), \quad (15)$$

where $\mathcal{W}_1(\mathbf{h}_1)$ and $\mathcal{W}_2(\mathbf{h}_2)$ denote, respectively, the WNGs of the first and second virtual arrays. In other words, the WNG of the global beamformer is the product of the WNGs of the two virtual arrays.

The directivity factor (DF) of the global beamformer, which is the array SNR gain in diffuse noise [23], is written as

$$\mathcal{D}(\mathbf{h}) = \frac{\left|\mathbf{h}^H \mathbf{d}_{\theta_\mathrm{s}}\right|^2}{\mathbf{h}^H \boldsymbol{\Gamma} \mathbf{h}}, \tag{16}$$

where the elements of the fixed diffuse noise matrix $\boldsymbol{\Gamma}$ are given in [2]. If we denote by $\mathcal{D}_1(\mathbf{h}_1)$ and $\mathcal{D}_2(\mathbf{h}_2)$ the DFs of, respectively, the first and second virtual arrays, one can check that

$$\mathcal{D}(\mathbf{h}) \neq \mathcal{D}_1(\mathbf{h}_1)\,\mathcal{D}_2(\mathbf{h}_2). \tag{17}$$

4. DESIGN OF STEERABLE BEAMFORMERS

Let us use the first virtual array to design the basic DMA beamformer. Generally, an Nth-order symmetric DMA directivity pattern with its main beam pointing to the direction θ_s is given by [14]

$$\begin{aligned}\mathcal{B}_{\theta-\theta_\mathrm{s}}(\mathbf{b}_{2N}) &= \sum_{n=-N}^{N} b_{2N,n} e^{\jmath n(\theta-\theta_\mathrm{s})} \\ &= \left(\boldsymbol{\Upsilon}_{\theta_\mathrm{s}} \mathbf{b}_{2N}\right)^T \mathbf{p}_\theta,\end{aligned} \tag{18}$$

where

$$\begin{aligned}\boldsymbol{\Upsilon}_{\theta_\mathrm{s}} &= \mathrm{diag}\left(e^{\jmath N\theta_\mathrm{s}}, \ldots, 1, \ldots, e^{-\jmath N\theta_\mathrm{s}}\right), \\ \mathbf{b}_{2N} &= \left[\begin{array}{ccccc} b_{2N,-N} & \cdots & b_{2N,0} & \cdots & b_{2N,N}\end{array}\right]^T, \\ \mathbf{p}_\theta &= \left[\begin{array}{ccccc} e^{-\jmath N\theta} & \cdots & 1 & \cdots & e^{\jmath N\theta}\end{array}\right]^T.\end{aligned}$$

Now, we need to find a proper beamforming filter, $\mathbf{h}_1$, so that the resulting beampattern, i.e., $\mathcal{B}_{1,\theta}(\mathbf{h}_1) = \mathbf{h}_1^H \mathbf{d}_{1,\theta}$, is as close as possible to $\mathcal{B}_{\theta-\theta_\mathrm{s}}$.

Using the Jacobi-Anger expansion, we get

$$e^{\jmath \frac{\omega r_m}{c} \cos(\theta-\psi_m)} = \sum_{n=-\infty}^{\infty} \jmath^n J_n\left(\frac{\omega r_m}{c}\right) e^{\jmath n(\theta-\psi_m)}, \tag{19}$$

where $J_n(x)$ is the nth-order Bessel function of the first kind. Substituting (19) into $\mathcal{B}_{1,\theta}(\mathbf{h}_1)$ and limiting the Jacobi-Anger series to the order N, we obtain

$$\mathcal{B}_{1,\theta,N}(\mathbf{h}_1) = \sum_{n=-N}^{N} e^{\jmath n\theta} \jmath^n \boldsymbol{\psi}_{1,n}^T \mathbf{h}_1^*, \tag{20}$$

where

$$\begin{aligned}\boldsymbol{\psi}_{1,n} = \Big[&J_n(\omega r_{1,1}/c)\, e^{-\jmath n\psi_{1,1}} \quad J_n(\omega r_{1,2}/c)\, e^{-\jmath n\psi_{1,2}} \\ &\cdots \quad J_n(\omega r_{1,M_1}/c)\, e^{-\jmath n\psi_{1,M_1}}\Big]^T\end{aligned} \tag{21}$$

is a vector of length M_1. Forcing $\mathcal{B}_{1,\theta,N}(\mathbf{h}_1)$ to be equal to $\mathcal{B}_{\theta-\theta_\mathrm{s}}$, we have the following relation:

$$\boldsymbol{\Psi}_1 \mathbf{h}_1 = \boldsymbol{\Upsilon}_{\theta_\mathrm{s}}^* \mathbf{b}_{2N}, \tag{22}$$

where

$$\boldsymbol{\Psi}_1 = \left[\begin{array}{c} (-\jmath)^{-N} \boldsymbol{\psi}_{1,-N}^H \\ \vdots \\ \boldsymbol{\psi}_{1,0}^H \\ \vdots \\ (-\jmath)^{N} \boldsymbol{\psi}_{1,N}^H \end{array}\right] \tag{23}$$

is a $(2N+1)\times M_1$ matrix. The minimum-norm solution of (22) gives the DMA beamforming filter of the first virtual array:

$$\mathbf{h}_{1,\mathrm{DMA}} = \boldsymbol{\Psi}_1^H \left(\boldsymbol{\Psi}_1 \boldsymbol{\Psi}_1^H\right)^{-1} \boldsymbol{\Upsilon}_{\theta_\mathrm{s}}^* \mathbf{b}_{2N}. \tag{24}$$

It is seen from (24) that steering of the proposed beamformer corresponds to a simple multiplication of its coefficients by $\boldsymbol{\Upsilon}_{\theta_\mathrm{s}}^*$, and it can be steered to any direction in the sensor plane.

The DMA beamformer may suffer from white noise amplification, which is particularly serious at low frequencies. To improve its WNG, we use the second virtual array to design a beamformer with maximum WNG, which happens to be the DS beamformer, i.e.,

$$\mathbf{h}_{2,\mathrm{DS}} = \frac{1}{M_2} \mathbf{d}_{2,\theta_\mathrm{s}}. \tag{25}$$

The corresponding WNG, which is the maximum WNG that the second virtual array can possibly obtain, is

$$\mathcal{W}_2(\mathbf{h}_2) = M_2. \tag{26}$$

The robust steerable global DMA beamformer is subsequently obtained as

$$\mathbf{h}_\mathrm{RDMA} = \mathbf{h}_{1,\mathrm{DMA}} \otimes \mathbf{h}_{2,\mathrm{DS}} \tag{27}$$

and the corresponding WNG is

$$\mathcal{W}(\mathbf{h}) = \mathcal{W}_1(\mathbf{h}_1)\,\mathcal{W}_2(\mathbf{h}_2) = M_2 \mathcal{W}_1(\mathbf{h}_1). \tag{28}$$

This means that the WNG of the DMA beamformer has been improved by $10\log M_2$ dB.

With the Kronecker product approach, we can also design adaptive DMA beamformers in a more flexible way. For instance, we can use the second virtual array to design a minimum variance distortionless response (MVDR) beamformer [21]:

$$\mathbf{h}_{2,\mathrm{MVDR}} = \frac{\boldsymbol{\Phi}_{\mathbf{v}_2}^{-1} \mathbf{d}_{2,\theta_\mathrm{s}}}{\mathbf{d}_{2,\theta_\mathrm{s}}^H \boldsymbol{\Phi}_{\mathbf{v}_2}^{-1} \mathbf{d}_{2,\theta_\mathrm{s}}}, \tag{29}$$

where $\boldsymbol{\Phi}_\mathbf{v} = E\left(\mathbf{v}_2 \mathbf{v}_2^H\right)$ is the correlation matrix of $\mathbf{v}_2$, which is the noise signal vector of length M_2. The adaptive steerable global DMA beamformer is obtained as

$$\mathbf{h}_\mathrm{ADMA} = \mathbf{h}_{1,\mathrm{DMA}} \otimes \mathbf{h}_{2,\mathrm{MVDR}}. \tag{30}$$

5. SIMULATIONS

In this section, we show an example on how to use the proposed Kronecker product differential beamforming method to design robust steerable DMA beamformers.

We consider a uniform rectangular microphone array (uniform means that both the interelement spacing along the x and y axes are the same) as illustrated in Fig. 1, consisting of $4 \times 6 = 24$ microphones, with an interelement spacing of 1 cm both along the x and y axes. We decompose it into two virtual rectangular arrays, the first virtual rectangular array consists of $2 \times 2 = 4$ microphones, with an interelement spacing of 1 cm, and the second virtual rectangular array consists of $2 \times 3 = 6$ microphones, with an interelement spacing of 3 cm. For the first virtual rectangular array, we design the first-order hypercardioid according to (24), where $\mathbf{b}_{2N}$ is given by [13]

$$\mathbf{b}_{2N} = \left[\begin{array}{ccc} 1/3 & 1/3 & 1/3 \end{array}\right]^T.$$

For the second virtual rectangular array, we design the DS beamformer. For both virtual rectangular arrays, the desired direction is chosen as $\theta_\mathrm{s} = 30^\circ$ (arbitrarily selected).

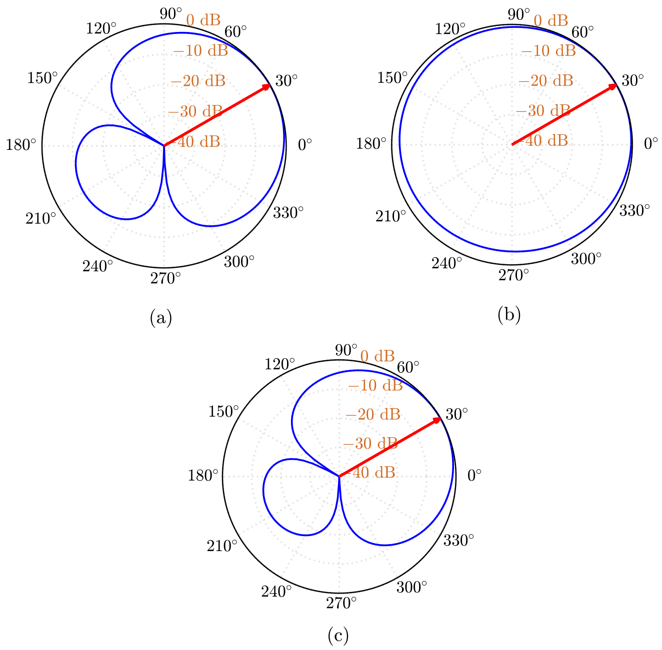

Fig. 2. Beampatterns of the two virtual sub-arrays and global rectangular array designed with the Kronecker product differential beamformer: (a) first sub-array, (b) second sub-array, and (c) global rectangular array. Conditions: $M = 24$, $M_1 = 4$, $M_2 = 6$, $\delta = 1$cm, $f = 1000$ Hz, and $\theta_s = 30°$. For the first rectangular array, we designed the DMA beamformer and for the second rectangular array, we designed the DS beamformer.

Figure 2 plots the beampatterns of the two virtual rectangular arrays as well as that of the global rectangular array at $f = 1000$ Hz, where red arrows point to the look direction. It is clearly seen that all the designed beampatterns' main beams point to the desired direction. The global beampattern is the product of the two virtual rectangular arrays' beampatterns. So, it can be steered to any direction, just as the beampatterns of the two virtual arrays.

Figure 3 plots the DFs and the WNGs of the two virtual and global rectangular arrays, all as a function of frequency. As seen, both the WNG and the DF of the global DMA are higher than those of the first virtual array. Comparing the WNG of the DMA filter of the first virtual array and that of the global one, we can observe that the WNG improvement is approximately $10 \log_{10} M_2 = 7.7$ dB over the entire frequency range, which is consistent with the theoretical analysis in (15). Due to limited space, we do not present here the results for the adaptive steerable DMA beamformer, but the observations are the same as with the previous simulation. As a matter of fact, the two previous simulations can be viewed as a particular case of (30).

6. CONCLUSIONS

This paper presented a method to design robust and steerable differential beamformers with rectangular arrays. Instead of designing the beamforming filter directly with the entire rectangular array, we decompose the array into two virtual rectangular sub-arrays so that the steering vector of the entire rectangular array is a Kronecker product of the steering vectors of the two smaller virtual rectangular sub-arrays. We use the first virtual rectangular array, which is small in size and can well satisfy the basic requirements for DMA beamforming, to design steerable differential beamformers, and use the second virtual sub-array to design other beamformers to help control the performance of the global beamformer. The developed method is rather flexible and can be useful to design robust and steerable differential beamformers for practical applications.

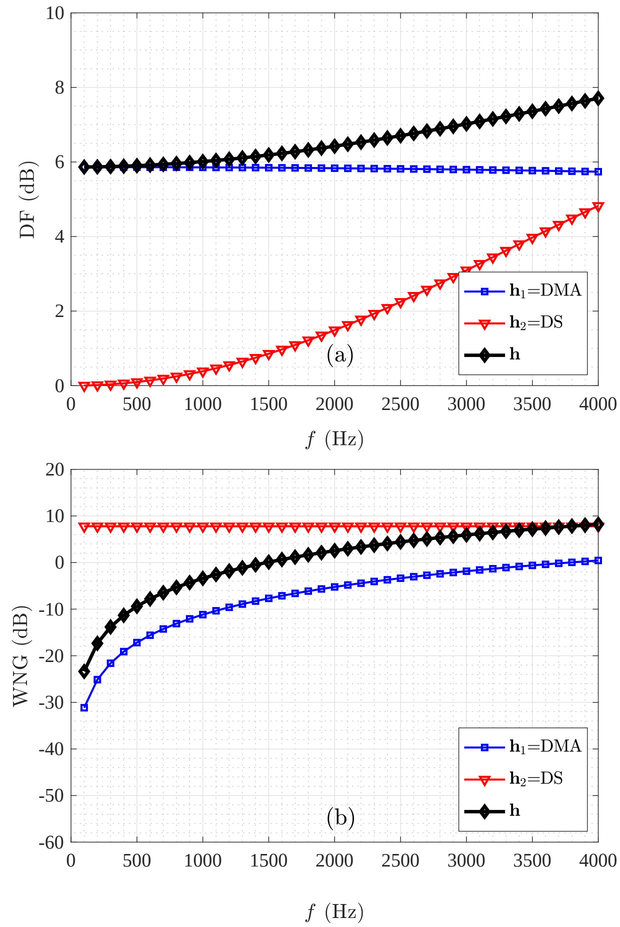

Fig. 3. DFs and WNGs of the two virtual rectangular arrays and global rectangular array designed with the Kronecker product differential beamformer: (a) DFs and (b) WNGs. Conditions: $M = 24$, $M_1 = 4$, $M_2 = 6$, $\delta = 1$cm, and $\theta_s = 30°$. For the first rectangular array, we designed the DMA beamformer and for the second rectangular array, we designed the DS beamformer.

7. RELATION TO PRIOR WORK

Differential beamformers have a wide range of applications. A large number of methods have been developed for the design of differential beamformers in the literature [11–16, 22–27]. Among those, the recently developed approach base on the use of the Kronecker product is of great interest since it has the flexibility in combining different kinds of differential beamformers for better performance or better tradeoff [2, 20]. The fundamentals of Kronecker product beamforming were discussed in [2, 20], where principles were presented on how to decompose the original microphone array into two sub-arrays and how to design two sub-beamformers, each corresponding to a sub-array, so that the global beampattern is the product of the beampatterns of the two sub-beamformers. One important issue left not discussed in [2, 20] is about steering ability, which is important for practical applications. So, this paper attempts to address the issue of steering in Kronecker product differential beamforming and we extend the work in [2, 20] to the design of robust and steerable Kronecker product differential beamformers.

8. REFERENCES

[1] J. Benesty, I. Cohen, and J. Chen, *Fundamentals of Signal Enhancement and Array Signal Processing*. Singapore: Wiley-IEEE Press, 2018.

[2] J. Benesty, I. Cohen, and J. Chen, *Array Processing–Kronecker Product Beamforming*. Switzerland: Springer Nature, 2019.

[3] G. W. Elko and J. Meyer, "Microphone arrays," in *Springer Handbook of Speech Processing* (J. Benesty, M. M. Sondhi, and Y. Huang, eds.), ch. 48, pp. 1021–1041, Berlin, Germany: Springer-Verlag, 2008.

[4] Y. Peled and B. Rafaely, "Linearly-constrained minimum-variance method for spherical microphone arrays based on plane-wave decomposition of the sound field," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 21, pp. 2532–2540, Dec. 2013.

[5] B. Rafaely, *Fundamentals of Spherical Array Processing*. Berlin, Germany: Springer-Verlag, 2015.

[6] S. Yan, "Optimal design of modal beamformers for circular arrays," *J. Acoust. Soc. Am.*, vol. 138, no. 4, pp. 2140–2151, 2015.

[7] S. Markovich-Golan, S. Gannot, and W. Kellermann, "Combined LCMV-trinicon beamforming for separating multiple speech sources in noisy and reverberant environments," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 25, pp. 320–332, 2016.

[8] J. Benesty, J. Chen, and Y. Huang, *Microphone Array Signal Processing*. Berlin, Germany: Springer-Verlag, 2008.

[9] M. Crocco and A. Trucco, "Stochastic and analytic optimization of sparse aperiodic arrays and broadband beamformers with robust superdirective patterns," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 20, pp. 2433–2447, 2012.

[10] M. Crocco and A. Trucco, "Design of superdirective planar arrays with sparse aperiodic layouts for processing broadband signals via 3-d beamforming," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 22, pp. 800–815, 2014.

[11] G. W. Elko, "Differential microphone arrays," in *Audio Signal Processing for Next-Generation Multimedia Communication Systems*, pp. 11–65, Springer, 2004.

[12] G. W. Elko, "Microphone array systems for hands-free telecommunication," *Speech Commun.*, vol. 20, no. 3, pp. 229–240, 1996.

[13] J. Benesty and J. Chen, *Study and Design of Differential Microphone Arrays*. Berlin, Germany: Springer-Verlag, 2012.

[14] G. Huang, J. Benesty, and J. Chen, "On the design of frequency-invariant beampatterns with uniform circular microphone arrays," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 25, no. 5, pp. 1140–1153, 2017.

[15] A. Bernardini, M. D. Aria, R. Sannino, and A. Sarti, "Efficient continuous beam steering for planar arrays of differential microphones," *IEEE Signal Process. Lett.*, vol. 24, no. 6, pp. 794–798, 2017.

[16] G. Huang, J. Chen, and J. Benesty, "Insights into frequency-invariant beamforming with concentric circular microphone arrays," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 26, no. 12, pp. 2305–2318, 2018.

[17] A. Bernardini, F. Antonacci, and A. Sarti, "Wave digital implementation of robust first-order differential microphone arrays," *IEEE Signal Process. Lett.*, vol. 25, no. 2, pp. 253–257, 2018.

[18] X. Wu and H. Chen, "Directivity factors of the first-order steerable differential array with microphone mismatches: deterministic and worst-case analysis," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 24, no. 2, pp. 300–315, 2016.

[19] G. Huang, J. Chen, and J. Benesty, "On the design of differential beamformers with arbitrary planar microphone array," *J. Acoust. Soc. Am.*, vol. 144, no. 1, pp. 3024–3035, 2018.

[20] I. Cohen, J. Benesty, and J. Chen, "Differential Kronecker product beamforming," *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, 2019.

[21] O. L. Frost III, "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, vol. 60, pp. 926–935, Aug. 1972.

[22] T. D. Abhayapala and A. Gupta, "Higher order differential-integral microphone arrays," *J. Acoust. Soc. Am.*, vol. 136, pp. 227–233, May 2010.

[23] G. W. Elko, "Superdirectional microphone arrays," in *Acoustic Signal Processing for Telecommunication*, pp. 181–237, Springer, 2000.

[24] J. Chen, J. Benesty, and C. Pan, "On the design and implementation of linear differential microphone arrays," *J. Acoust. Soc. Am.*, vol. 136, pp. 3097–3113, Dec. 2014.

[25] E. D. Sena, H. Hacihabiboglu, and Z. Cvetkovic, "On the design and implementation of higher-order differential microphones," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 20, pp. 162–174, Jan. 2012.

[26] J. Byun, Y. C. Park, and S. W. Park, "Continuously steerable second-order differential microphone arrays," *J. Acoust. Soc. Am.*, vol. 143, no. 3, pp. EL225–EL230, 2018.