Beamforming With Small-Spacing Microphone Arrays Using Constrained/Generalized LASSO

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***Abstract*—In this letter, we develop an approach to the design of beamformers with small-spacing uniform linear microphone arrays by incorporating sparseness constraints for attenuating scattered interference incident from some pre-specified ranges of directions of arrival. The design process is formulated as a constrained LASSO problem. By adjusting the value of a tuning parameter, the proposed method can make compromises among three important yet conflicting (especially at low frequencies) performance measures of small-spacing microphone arrays, i.e., the directivity factor (DF), which quantifies the array spatial gain, the white noise gain (WNG), which evaluates the robustness of the beamformer, and the signal-to-interference-ratio (SIR) gain with respect to scattered interference. Simulation results illustrate the properties of the developed approach.**

***Index Terms*—Beamforming, white noise gain, directivity factor, signal-to-scattered-interference-ratio gain, LASSO.**

I. Introduction

Beamforming with microphone arrays has attracted much attention due to its broad range of applications [1]–[5], such as high-fidelity sound acquisition in human-machine interfaces and hands-free voice communications. The delay-and-sum (DAS) beamformer has been extensively studied [6]; but this beamformer is not well suited for processing broadband signals such as speech since its beampattern is frequency-dependent [2], which may lead to signal distortion. To overcome this drawback, many broadband beamforming methods have been developed, including the narrowband decomposition method [7]–[10], nested-array framework [11]–[13], modal beamforming method [14]–[18], constant beamwidth beamforming [19], [20], differential beamforming technique [21]–[28], etc. Among those, the differential beamforming technique (the associated microphone array is called differential microphone array, or DMA for short) is most suitable for processing speech signals as it exhibits frequency-invariant beampatterns and can achieve high directivity factors (DFs) with small apertures. As a result, DMAs have been used in a wide range of portable devices such as smart speakers, headphones, hearing aids, etc.

Traditionally, differential beamformers are designed in a cascaded subtraction way [22], [23], [25]. But this method is not very flexible in dealing with the white noise amplification problem, which is inherent to differential beamformers. Recently, the so-called null-constrained method was developed [24], [29], [30], which designs differential beamformers using only the null and distortionless constraints. A minimum-norm solution was derived based on this method, which maximizes the white noise gain (WNG) with the given null and distortionless constraints [24], [29]; so it is the best possible way to deal with the white noise amplification problem.

Basically, differential beamformers approximate the differential sound pressure field by finite difference between microphone sensors' outputs. A good approximation requires that the inter-element spacing of the array is much smaller than the smallest wavelength of the frequency band of interest. Due to small inter-element spacing, DMAs generally are small in size. In this case, interference sources such as big loudspeakers, air conditioners, and car engines can no longer be treated as point sources and interference signals emitted from those sources are incident to the array from certain angle ranges. Consequently, how to extract the signal of interest from the array observations corrupted by interference incident from a certain range of directions of arrival (DOA) become an important issue, which is addressed in this work. We develop a beamforming method with small-spacing uniform linear microphone arrays by incorporating sparseness constraints [31]–[33] using LASSO [34]–[37], which imposes a distortionless constraint in the look direction and a number of nulls and attenuation constraints in the pre-specified DOA ranges. In real applications, the interference DOA range needs to be known, which can be estimated using DOA estimation algorithms for scattered sources, e.g., the ones presented in [38]–[40]. The developed beamformer facilitates a good compromise among the DF, signal-to-scattered-interference-ratio (SSIR) gain, and WNG. Note that besides microphone arrays, the proposed method can also be extended to many other areas such as sonar in underwater multipath propagation environments.

II. Signal Model, Problem Formulation, and Performance Measures

We consider the farfield signal model in which a source signal (plane wave) propagates in an anechoic acoustic environment at

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the speed of sound, i.e., $c = 340$ m/s, and impinges on a uniform linear array (ULA) consisting of $M \geq 2$ omnidirectional microphones. Assume that the incidence angle is characterized by azimuthal angle θ. In this context, the steering vector (of length M) is given by

$$\mathbf{d}_\theta(\omega) = \left[1 \; e^{-\jmath\omega\tau_0 \cos\theta} \; \cdots \; e^{-\jmath(M-1)\omega\tau_0\cos\theta} \right]^T, \quad (1)$$

where the superscript ^{T} is the transpose operator, $\jmath^2 = -1$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, and $\tau_0 = \delta/c$ is the time delay between two successive sensors at the angle $\theta = 0$, with δ being the interelement spacing.

The main interest of this letter is beamforming with small-spacing microphone arrays like in differential beamforming [29], i.e., δ is very small ($\delta \ll \lambda_{\min}/2$, where $\lambda_{\min}$ is the minimum wavelength of the frequency band of interest) and the desired source propagates from the endfire, i.e., $\theta = 0$. This context corresponds to the optimal working conditions of differential beamformers or differential microphone arrays (DMAs) [41]. As a result, the observed signal vector is [3]

$$\begin{aligned} \mathbf{y}(\omega) &= \left[Y_1(\omega) \; Y_2(\omega) \; \cdots \; Y_M(\omega) \right]^T \\ &= \mathbf{x}(\omega) + \mathbf{v}(\omega) = \mathbf{d}_0(\omega) X(\omega) + \mathbf{v}(\omega), \end{aligned} \quad (2)$$

where $Y_m(\omega) = e^{-\jmath(m-1)\omega\tau_0} X(\omega) + V_m(\omega)$ is the signal picked up by the mth microphone, $X(\omega)$ is the zero-mean desired signal, $V_m(\omega)$ is the zero-mean additive noise at the mth microphone, $\mathbf{x}(\omega) = \mathbf{d}_0(\omega)X(\omega)$ with $\mathbf{d}_0(\omega)$ being the steering vector at $\theta = 0$, and $\mathbf{v}(\omega)$ is defined similarly to $\mathbf{y}(\omega)$. In the rest, in order to simplify the notation, we drop the dependence on the angular frequency, ω.

Conventional linear beamforming is performed by applying a complex-valued linear filter, $\mathbf{h}$ of length M, to the observation signal vector, $\mathbf{y}$, i.e.,

$$Z = \mathbf{h}^H \mathbf{y} = \mathbf{h}^H \mathbf{d}_0 X + \mathbf{h}^H \mathbf{v}, \quad (3)$$

where Z is an estimate of the desired signal, X, and the superscript ^{H} is the conjugate-transpose operator. In our context, the distortionless constraint is desired so that the signal of interest is passed through the beamformer without attenuation/distortion, i.e., $\mathbf{h}^H \mathbf{d}_0 = 1$.

In order to evaluate the performance of the developed beamformers, four fundamental performance measures will be used. They are:

- the beampattern: $\mathcal{B}_\theta(\mathbf{h}) = \mathbf{d}_\theta^H \mathbf{h}$,
- the WNG: $\mathcal{W}(\mathbf{h}) = \dfrac{|\mathbf{h}^H \mathbf{d}_0|^2}{\mathbf{h}^H\mathbf{h}}$,
- the DF: $\mathcal{D}(\mathbf{h}) = \dfrac{|\mathbf{h}^H \mathbf{d}_0|^2}{\mathbf{h}^H \boldsymbol{\Gamma} \mathbf{h}}$, where the elements of the $M \times M$ matrix $\boldsymbol{\Gamma}(\omega)$, which is the pseudo-coherence matrix corresponding to the spherically isotropic (diffuse) noise field, are $[\boldsymbol{\Gamma}(\omega)]_{ij} = \dfrac{\sin[\omega(j-i)\tau_0]}{\omega(j-i)\tau_0} = \mathrm{sinc}[\omega(j-i)\tau_0]$, with $i, j = 1, 2, \ldots, M$ and $[\boldsymbol{\Gamma}(\omega)]_{mm} = 1$, $m = 1, 2, \ldots, M$,
- and the signal-to-scattered-interference-ratio (SSIR) gain: $\mathcal{R}(\mathbf{h}) = \dfrac{N|\mathbf{h}^H \mathbf{d}_0|^2}{\mathbf{h}^H \mathbf{D}_\mathrm{c} \mathbf{D}_\mathrm{c}^H \mathbf{h}}$, where $\mathbf{D}_\mathrm{c}$ is the complex-valued version of $\mathbf{D}$, and $\mathbf{D}$ and N will be defined in Section IV.

The beampattern describes the response of a beamformer to planar acoustic waves that impinge on the sensor array from different directions. The WNG evaluates the robustness of a beamformer with respect to different kinds of imperfections in the array and sensors. Note that $\mathcal{W}(\mathbf{h}) < 1$ means white noise amplification. The smaller the value of $\mathcal{W}(\mathbf{h})$, the more serious is the white noise amplification. The DF quantifies the spatial gain of a beamformer. The SSIR gain defined here evaluates the performance of the beamformer in attenuating the scattered interference incident from a certain angle range. Detailed definition and meaning of these four measures can be found in [29]. Generally, at low frequencies, the WNG and DF are a pair of contradictory measures for small-spacing microphone arrays, i.e., a beamformer with high DF has low WNG, and vice versa. So, how to improve the WNG [42], [43] and control the compromise between a large DF and a reasonable level of WNG is an important issue [44]–[46].

III. Reformulation of the Problem With Real-Valued Variables

In order to fully exploit and better utilize the different LASSO optimization techniques, it is preferable to reformulate our problem in the real-valued domain. For that, any complex-valued vector $\mathbf{a}$ can be written as $\mathbf{a} = \overline{\mathbf{a}} + \jmath\widetilde{\mathbf{a}}$, where $\overline{\mathbf{a}}$ and $\widetilde{\mathbf{a}}$ are the real and imaginary parts of $\mathbf{a}$, respectively.

One can verify that the M-dimensional complex-valued vector $\mathbf{y}$ as given in (2) can, equivalently, be expressed as a $2M$-dimensional real-valued vector:

$$\begin{aligned} \overleftarrow{\mathbf{y}} &= \begin{bmatrix} \overline{\mathbf{y}} \\ \widetilde{\mathbf{y}} \end{bmatrix} = \overleftarrow{\mathbf{x}} + \overleftarrow{\mathbf{v}} \\ &= \begin{bmatrix} \overline{\mathbf{d}}_0 & -\widetilde{\mathbf{d}}_0 \\ \widetilde{\mathbf{d}}_0 & \overline{\mathbf{d}}_0 \end{bmatrix} \begin{bmatrix} \overline{X} \\ \widetilde{X} \end{bmatrix} + \begin{bmatrix} \overline{\mathbf{v}} \\ \widetilde{\mathbf{v}} \end{bmatrix} \\ &= \left[\overleftarrow{\mathbf{d}}_0 \; \mathbf{J}\overleftarrow{\mathbf{d}}_0 \right] \overrightarrow{\mathbf{x}} + \overleftarrow{\mathbf{v}}, \end{aligned} \quad (4)$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{0} & -\mathbf{I}_M \\ \mathbf{I}_M & \mathbf{0} \end{bmatrix},$$

with $\mathbf{J}^T\mathbf{J} = \mathbf{I}_{2M}$, and $\mathbf{I}_M$ and $\mathbf{I}_{2M}$ being the $M \times M$ and $2M \times 2M$ identity matrices, respectively. Note that we use $\overleftarrow{\mathbf{a}}$ to denote $[\, \overline{\mathbf{a}}^T \;\; \widetilde{\mathbf{a}}^T \,]^T$ where $\mathbf{a}$ is a complex-valued vector and use $\overrightarrow{\mathbf{a}}$ to denote $[\, \overline{A} \;\; \widetilde{A} \,]^T$ where A is a complex scalar.

As a consequence, by rewriting the filter $\mathbf{h} = \overline{\mathbf{h}} + \jmath\widetilde{\mathbf{h}}$ as

$$\overleftarrow{\mathbf{h}} = \begin{bmatrix} \overline{\mathbf{h}} \\ \widetilde{\mathbf{h}} \end{bmatrix},$$

it follows immediately that the estimate of the desired signal $Z = \overline{Z} + \jmath\widetilde{Z}$ can be rewritten as

$$\begin{aligned} \overrightarrow{\mathbf{z}} &= \begin{bmatrix} \overline{Z} \\ \widetilde{Z} \end{bmatrix} = \left[\overleftarrow{\mathbf{h}} \; \mathbf{J}\overleftarrow{\mathbf{h}} \right]^T \overleftarrow{\mathbf{y}} = \begin{bmatrix} \overleftarrow{\mathbf{h}}^T \overleftarrow{\mathbf{y}} \\ \overleftarrow{\mathbf{h}}^T \mathbf{J}^T \overleftarrow{\mathbf{y}} \end{bmatrix} \\ &= \begin{bmatrix} \overleftarrow{\mathbf{h}}^T \overleftarrow{\mathbf{d}}_0 & \overleftarrow{\mathbf{h}}^T \mathbf{J} \overleftarrow{\mathbf{d}}_0 \\ \overleftarrow{\mathbf{h}}^T \mathbf{J}^T \overleftarrow{\mathbf{d}}_0 & \overleftarrow{\mathbf{h}}^T \overleftarrow{\mathbf{d}}_0 \end{bmatrix} \overrightarrow{\mathbf{x}} + \begin{bmatrix} \overleftarrow{\mathbf{h}}^T \overleftarrow{\mathbf{v}} \\ \overleftarrow{\mathbf{h}}^T \mathbf{J}^T \overleftarrow{\mathbf{v}} \end{bmatrix}. \end{aligned} \quad (5)$$

Similarly, the distortionless constraint can be rearranged as

$$\overleftarrow{\mathbf{h}}^T \left[\overleftarrow{\mathbf{d}}_0 \;\; \mathbf{J}\overleftarrow{\mathbf{d}}_0 \right] = [1 \;\; 0], \quad (6)$$

which can also be equivalently written as

$$\overleftarrow{\mathbf{h}}^T \left[\mathbf{J}^T\overleftarrow{\mathbf{d}}_0 \;\; \overleftarrow{\mathbf{d}}_0 \right] = [0 \;\; 1] \quad (7)$$

by using the fact that $-\mathbf{J}^T = \mathbf{J}$.

In the same way, if we want to have a null in the direction $\theta_0 \neq 0$, the constraint on the $2M$-dimensional real-valued filter must be

$$\overleftarrow{\mathbf{h}}^T \left[\overleftarrow{\mathbf{d}}_{\theta_0} \quad \mathbf{J}\overleftarrow{\mathbf{d}}_{\theta_0} \right] = [0 \quad 0] \tag{8}$$

or

$$\overleftarrow{\mathbf{h}}^T \left[\mathbf{J}^T\overleftarrow{\mathbf{d}}_{\theta_0} \quad \overleftarrow{\mathbf{d}}_{\theta_0} \right] = [0 \quad 0], \tag{9}$$

where $\mathbf{d}_{\theta_0}$ is the steering vector at θ_0.

IV. Beamforming With LASSO

It has been shown that the shape of a DMA directivity pattern is uniquely determined by the number and the positions of the nulls. Such information was exploited in [24], [29], [30] to design differential beamformers. In this letter, we borrow similar ideas on the null information to design beamformers with LASSO. Let us start with a simple case with the distortionless constraint at $\theta = 0$ and one null constraint at $\theta_0 \in [90^\circ, 180^\circ]$. With these two particular angles (0 and θ_0), we can construct the constraint matrix of size $2M \times 4$ as

$$\mathbf{C}_{\theta_0} = \left[\overleftarrow{\mathbf{d}}_0 \quad \mathbf{J}\overleftarrow{\mathbf{d}}_0 \quad \overleftarrow{\mathbf{d}}_{\theta_0} \quad \mathbf{J}\overleftarrow{\mathbf{d}}_{\theta_0} \right], \tag{10}$$

whose column rank is equal to 4. Therefore, the important constraint applied to the beamforming filter should be $\mathbf{C}_{\theta_0}^T \overleftarrow{\mathbf{h}} = \mathbf{i}$, where $\mathbf{i} = [1\ 0\ 0\ 0]^T$.

Now, let us discretize the DOA range of the scattered interference into N directions, i.e., $\theta_n,\ n = 1, 2, \ldots, N$, with $N \geq M$. It is desired that the interferences indent from all the θ_n's are attenuated as much as possible. Define the matrix of size $2M \times 2N$:

$$\mathbf{D} = \left[\overleftarrow{\mathbf{d}}_{\theta_1}\ \mathbf{J}\overleftarrow{\mathbf{d}}_{\theta_1} \cdots \ \overleftarrow{\mathbf{d}}_{\theta_N}\ \mathbf{J}\overleftarrow{\mathbf{d}}_{\theta_N} \right]. \tag{11}$$

The column rank of $\mathbf{D}^T$ is equal to $2M$. One roughly equivalent way to express the previous statement is that the vector $\mathbf{D}^T\overleftarrow{\mathbf{h}}$ is as sparse as possible.

To make the designed beamformers robust to sensors' self noise and imperfection of microphone arrays, we consider in our optimization problem to improve the WNG. Then, the beamforming problem becomes

$$\min_{\overleftarrow{\mathbf{h}}} \left(\frac{1}{2} \left\| \overleftarrow{\mathbf{h}} \right\|_2^2 + \mu \left\| \mathbf{D}^T \overleftarrow{\mathbf{h}} \right\|_1 \right) \ \text{ s.t. } \mathbf{C}_{\theta_0}^T \overleftarrow{\mathbf{h}} = \mathbf{i}, \tag{12}$$

where $\|\cdot\|_2$ denotes the ℓ_2 norm, $\mu \geq 0$ is a tuning parameter, which allows us to compromise between the WNG and DF or interference attenuation, and $\|\cdot\|_1$ is the ℓ_1 norm. Because of the presence of the matrix $\mathbf{D}^T$ in the ℓ_1 norm, we have the generalized LASSO [36], [37], [47], [48]; and because of the constraint in (12), we have the constrained LASSO [34]–[37]. For $\mu = 0$, we get the classical minimum-norm (MN) solution developed in [29], which gives the best possible WNG given the constraints, i.e.,

$$\overleftarrow{\mathbf{h}}_{\mathrm{MN}} = \mathbf{C}_{\theta_0} \left(\mathbf{C}_{\theta_0}^T \mathbf{C}_{\theta_0} \right)^{-1} \mathbf{i}. \tag{13}$$

For $\mu = +\infty$, (12) degenerates to

$$\min_{\overleftarrow{\mathbf{h}}} \left\| \mathbf{D}^T \overleftarrow{\mathbf{h}} \right\|_1 \ \text{ s.t. } \mathbf{C}_{\theta_0}^T \overleftarrow{\mathbf{h}} = \mathbf{i}. \tag{14}$$

Let us first discuss a simplified and somewhat less ambitious version of (12), i.e.,

$$\min_{\overleftarrow{\mathbf{h}}} \left(\frac{1}{2} \left\| \mathbf{i} - \mathbf{C}_{\theta_0}^T \overleftarrow{\mathbf{h}} \right\|_2^2 + \mu \left\| \mathbf{D}^T \overleftarrow{\mathbf{h}} \right\|_1 \right), \tag{15}$$

where we recognize the generalized LASSO [36], [37], [47], [48]. With (15), we will never have exactly a one at the angle 0 and a zero at the angle θ_0; but we can get close to this objective. This generalized LASSO problem can be transformed to a constrained one as explained in [34]–[37]. As a matter of fact, the matrix $\mathbf{D}^T$ can be decomposed as $\mathbf{D}^T = \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix}$, where $\mathbf{D}_1$ is a full-rank square matrix of size $2M \times 2M$. Then,

$$\begin{aligned} &\frac{1}{2} \left\| \mathbf{i} - \mathbf{C}_{\theta_0}^T \overleftarrow{\mathbf{h}} \right\|_2^2 + \mu \left\| \mathbf{D}^T \overleftarrow{\mathbf{h}} \right\|_1 \\ &= \frac{1}{2} \left\| \mathbf{i} - \mathbf{C}_{\theta_0}^T \mathbf{D}_1^{-1} \mathbf{D}_1 \overleftarrow{\mathbf{h}} \right\|_2^2 + \mu \left\| \mathbf{D}_1 \overleftarrow{\mathbf{h}} \right\|_1 + \mu \left\| \mathbf{D}_2 \overleftarrow{\mathbf{h}} \right\|_1 \\ &= \frac{1}{2} \left\| \mathbf{i} - \left(\mathbf{C}_{\theta_0}^T \mathbf{D}_1^{-1} \right) \mathbf{D}_1 \overleftarrow{\mathbf{h}} \right\|_2^2 + \mu \left\| \mathbf{D}_1 \overleftarrow{\mathbf{h}} \right\|_1 \\ &\quad + \mu \left\| \mathbf{D}_2 \mathbf{D}_1^{-1} \mathbf{D}_1 \overleftarrow{\mathbf{h}} \right\|_1. \end{aligned} \tag{16}$$

Let us define the vector of length $2N$: $\mathbf{g} = [\, \mathbf{g}_1^T\ \mathbf{g}_2^T \,]$, where $\mathbf{g}_1$ and $\mathbf{g}_2$ are of lengths $2M$ and $2N - 2M$, respectively. With the change of variables:

$$\mathbf{g}_1 = \mathbf{D}_1 \overleftarrow{\mathbf{h}}, \tag{17}$$

$$\mathbf{g}_2 = \mathbf{D}_2 \mathbf{D}_1^{-1} \mathbf{D}_1 \overleftarrow{\mathbf{h}} = \mathbf{D}_2 \mathbf{D}_1^{-1} \mathbf{g}_1, \tag{18}$$

$$\mathbf{C}' = \left[\mathbf{C}_{\theta_0}^T \mathbf{D}_1^{-1} \quad \mathbf{0}_{4\times(2N-2M)} \right], \tag{19}$$

$$\mathbf{D}' = \left[\mathbf{D}_2 \mathbf{D}_1^{-1} \quad -\mathbf{I}_{2N-2M} \right], \tag{20}$$

where $\mathbf{I}_{2N-2M}$ is the $(2N - 2M) \times (2N - 2M)$ identity matrix and $\mathbf{0}_{Q_1\times Q_2}$ is a matrix of size $Q_1 \times Q_2$ with all its elements equal to zero, (15) becomes

$$\min_{\mathbf{g}} \left(\frac{1}{2} \left\| \mathbf{i} - \mathbf{C}'\mathbf{g} \right\|_2^2 + \mu \left\| \mathbf{g} \right\|_1 \right) \ \text{ s.t. } \mathbf{D}'\mathbf{g} = \mathbf{0}_{(2N-2M)\times 1}, \tag{21}$$

which can be solved with the constrained LASSO algorithm proposed in [34], [35]. When $\mathbf{g}$ is found, it is clear that the desired filter is $\overleftarrow{\mathbf{h}} = \mathbf{D}_1^{-1}\mathbf{g}_1$.

Going back to our original problem and following the previous steps, we can rewrite (12) as

$$\begin{aligned} &\min_{\mathbf{g}} \quad \left(\frac{1}{2} \left\| \mathbf{C}''\mathbf{g} \right\|_2^2 + \mu \left\| \mathbf{g} \right\|_1 \right) \\ &\text{s.t.} \quad \begin{bmatrix} \mathbf{C}' \\ \mathbf{D}' \end{bmatrix} \mathbf{g} = \begin{bmatrix} \mathbf{i} \\ \mathbf{0}_{(2N-2M)\times 1} \end{bmatrix}, \end{aligned} \tag{22}$$

where $\mathbf{C}'' = \left[\mathbf{D}_1^{-1}\ \mathbf{0}_{2M\times(2N-2M)} \right]$. As before, (22) can be solved with the constrained LASSO [34]–[37]. For $\mu = +\infty$, (22) becomes

$$\min_{\mathbf{g}} \left\| \mathbf{g} \right\|_1 \ \text{ s.t. } \begin{bmatrix} \mathbf{C}' \\ \mathbf{D}' \end{bmatrix} \mathbf{g} = \begin{bmatrix} \mathbf{i} \\ \mathbf{0}_{(2N-2M)\times 1} \end{bmatrix}, \tag{23}$$

which is a linear programming problem that can be solved with conventional algorithms [49]–[53]. The beamformer obtained

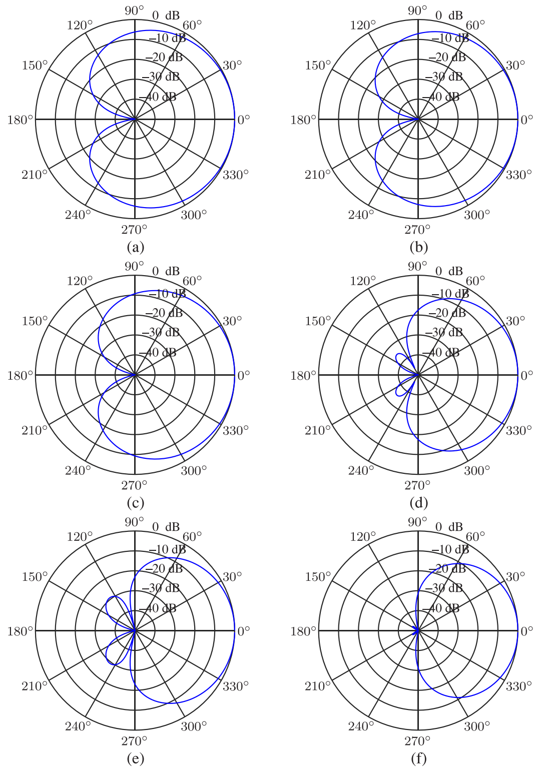

Fig. 1. Beampatterns of the proposed beamformer with different values of μ: (a) $\mu = 10^{-3}$, (b) $\mu = 10^{-1}$, (c) $\mu = 10^{-0.5}$, (d) $\mu = 10^{0}$, (e) $\mu = 10^{2}$ and (f) $\mu = 10^{4}$. Conditions: $M = 6$, $\delta = 1.0$ cm, $f = 1$ kHz, $\theta_0 = 180^\circ$, $\theta_n \in [90^\circ : 1^\circ : 120^\circ]$, $n = 1, 2, \ldots, N$, and $N = 31$.

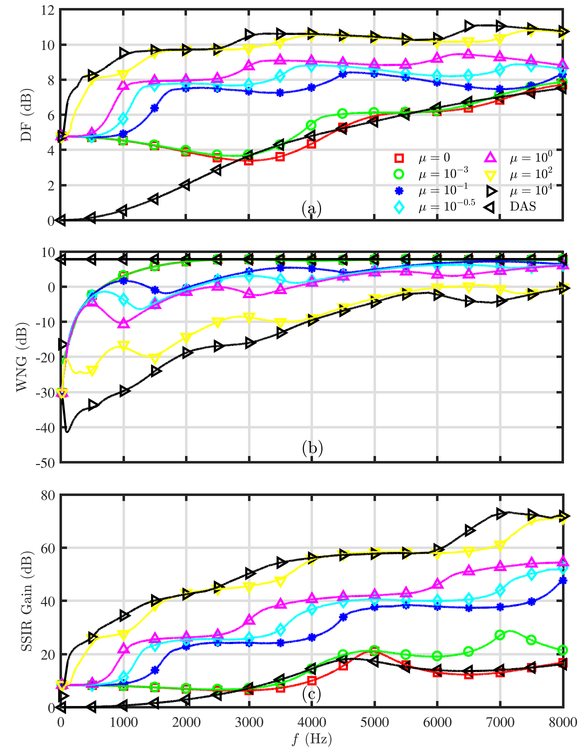

Fig. 2. Performance of the proposed beamformer versus frequency with different values of μ: (a) DF, (b) WNG, (c) SSIR gain. Conditions: $M = 6$, $\delta = 1.0$ cm, $\theta_0 = 180^\circ$, $\theta_n \in [90^\circ : 1^\circ : 120^\circ]$, $n = 1, 2, \ldots, N$, and $N = 31$.

from (23) has high DF and high SSIR gain for attenuating the scattered interference, and frequency-invariant beampattern, but at the price of white noise amplification.

Note that the above method is good for only one distortionless constraint and one null constraint. But it can be generalized to the case of multiple distortionless and null constraints. As an example, let us briefly discuss the design of the dipole. Since the directivity pattern of the dipole has also a one in the direction $\theta = 180^\circ$, we propose to replace the matrix $\mathbf{C}_{\theta_0}$ in (10) by

$$\mathbf{C}_{180,\theta_0} = \left[\overleftarrow{\mathbf{d}}_0 \quad \mathbf{J}\overleftarrow{\mathbf{d}}_0 \quad \overleftarrow{\mathbf{d}}_{180} \quad \mathbf{J}\overleftarrow{\mathbf{d}}_{180} \quad \overleftarrow{\mathbf{d}}_{\theta_0} \quad \mathbf{J}\overleftarrow{\mathbf{d}}_{\theta_0} \right] \quad (24)$$

of size $2M \times 6$ ($M \geq 3$), with $\theta_0 = 90^\circ$, whose column rank is equal to 6, and $\mathbf{d}_{180}$ is the steering vector at $\theta = 180^\circ$. As a result, the constraint on the filter is

$$\mathbf{C}_{180,\theta_0}^T \overleftarrow{\mathbf{h}} = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T. \quad (25)$$

As for the choice of the matrix $\mathbf{D}$, it is the same as the one defined in (11). It follows immediately that the previous optimization process can be used.

V. Design Example

In this section, we present a design example with a ULA of $M = 6$ and $\delta = 1.0$ cm. We set the distortionless constraint at 0° and a null at 180° and assume that the scattered interference is incident from the range between 90° and 120° with $\theta_n = 90^\circ + n^\circ - 1^\circ$, $n = 1, 2, \ldots, N$, and $N = 31$. The beampatterns of the designed beamformer with different values of μ at $f = 1$ kHz are shown in Fig. 1. As seen, the beampatterns of the proposed beamformer vary significantly with μ. When the value of μ is very small, the beampattern is close to a cardioid. If the value of μ is large, the beamformer can greatly attenuate the scattered interference incident from the range $[90^\circ, 120^\circ]$.

Now, let us change the value of μ and examine the SSIR gain, WNG and DF of the proposed beamformer. The results are plotted in Fig. 2. For comparison, the results of the DAS beamformer are also plotted, which achieves the maximum WNG, but low SSIR gain and DF. As expected, the SSIR gain and DF increase and the WNG decreases with the increase of the value of μ. This indicates that, through adjusting the tuning parameter μ, the proposed beamformer can flexibly make a compromise among the SSIR gain, DF and WNG.

VI. Conclusion

This letter addressed the problem of robust and high-directive beamforming with small-spacing ULAs, which have been used in a wide range of applications. We formulated the beamforming problem in the frequency domain as an optimization problem with sparseness constraints, solved by the constrained LASSO algorithm. The resulting beamformers can be made robust to sensors' self noise and array's imperfections, while achieving a high DF as well as good SSIR gain to attenuate scattered interference incident from a given *a priori* angle range. One design example was presented, which demonstrated the properties of the developed method.

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