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Analysis of White Light Speckle Imaging

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Abstract **- In this paper, we present a new approach for analyzing white light speckle patterns. The paper introduces an analytic model and heuristic explanations for the phenomena using the contrast and intensity statistics of the speckles. Relations between the coherence length, central wavelength and surface roughness are examined. It is shown that the speckle intensity is directly related to the autocorrelation function. We show that the new approach is consistent with previous models using simulation results and experimental data.**

Keywords—Speckle, incoherent light, surface roughness.

I. Introduction

The speckle phenomena became a popular object of interest since the 1960's when the first lasers came out [1]. Surprisingly, illuminated surfaces showed a great amount of "light granules" that their origins soon became topics of research. The phenomena's mechanisms lie in the light that is scattered from the surface itself. Speckle is no more than scatters' interference. This phenomenon is closely related to the invention of lasers which provided light sources with long coherence length. Since the speckle phenomena was revealed, a variety of applications were developed in diverse areas: non-destructive tests [2], remote vibration measurements [3], eavesdropping and roughness measurements [4-6]. Despite the strong advantages of the laser illumination, it has several drawbacks, such as appearance of multiple modes, creation of parasitic interferences, high cost, and difficulty to control the wavelength. The importance of these issues becomes significant as the speckle interference is being applied to authentication and detection of counterfeited objects.

The speckle pattern associated with a specific region on the object is individual to its 2-D roughness pattern and may serve as it's "fingerprint" [7-8] and therefore can be used for authentication and detection of counterfeited objects [9]. The existence of white light speckle interferometry was proven experimentally [10-11], but to the best of our knowledge has never been modeled heuristically before. Exploring the relations between coherence length, surface roughness (defined as height's surface RMS) and speckle distribution is our main goal. In our model, we propose a comprehensive analytical mechanism of speckle formation and present a mathematical model that is applied for incoherent speckle as well. It explains the relations between speckle contrast, coherence length and surface roughness for diversity of roughness values. Mathematical modeling and numerical simulation results supported by experimental results are presented to validate our analysis.

The paper is organized as follows: In Section II, we first introduce the theoretical background about light coherency and basic model of speckle. In Section III, we introduce our approach for exploring the relations of surface roughness and incoherent light speckle that are reinforced by simulations. Experimental results to support our theory are presented in Section IV.

II. Theoretical Background

Consider the wave function $U(h,t)$ to be a far-field light source, and define $\Gamma(\tau)$ to be the autocorrelation function of $U(h,t)$. Let us set the first variable of the wave function (space variable) as a constant so we can analyze the autocorrelation as a temporal function only. We can write:

$$\Gamma(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} U(t)U^*(t+\tau)dt = \left\langle U(t)U^*(t+\tau)\right\rangle_T, \quad (1)$$

where X^* denotes the complex conjugate of X, and T is the temporal duration of $U(t)$. Define also the normalized coherence function, $\gamma(\tau)$, to be

$$\gamma(\tau) = \Gamma(\tau)/\Gamma(0), \quad (2)$$

where $I = \left\langle U(t)U^*(t)\right\rangle_T = \Gamma(0)$ is the average intensity of the wave function $U(t)$.

The function $\gamma(\tau)$ is a measure for how similar the wave is to itself in different time intervals. For a pure sine, i.e., $U(t) = ae^{j\omega_0 t}$, the magnitude of $\gamma(\tau)$ is the unity function. Thus, the distance of $|\gamma(\tau)|$ from unity indicates how coherent the light wave is. The faster it decays, the less coherent it is.

It is common to define the time τ for which $|\gamma(\tau)| = e^{-1}$ as the coherence time τ_c. On the interval $[0,\tau_c]$ we can assume that the wave behaves very similar to a pure sine. The coherence length $L_c = c\tau_c$ is the distance that the wave travels during this interval, where c is the phase velocity which in our case is the speed of light. It can be shown that the coherence time is inversely proportional to the wave bandwidth, i.e.,

$$\tau_c = \frac{1}{\Delta\upsilon} \approx \frac{\lambda_0{}^2}{c\Delta\lambda}, \quad (3)$$

where λ_0 is the central wavelength, $\Delta\lambda$ is the spectral width of illumination [m] and $\Delta\upsilon$ is the illumination bandwidth [Hz].

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Our model is based on the assumption of a paraxial imaging system operating in the far field with unity magnification. We also assume that the scattering surface randomly changes the phase of the impinging light, but the scattering amplitude is the same for different locations on the surface. Light gathered by the objective, consisting of N scatters, is focused by projection lens onto a two-dimensional sensor array that has N_p elements. For a coherent light source (e.g., laser) with coherence length L_c, the intensity varies only due to phase changes (i.e., constructive and destructive interferences) and does not depend on the illumination bandwidth, $\Delta\upsilon$ (which is practically 0 for this kind of source making $\tau_c \rightarrow \infty \Rightarrow |\gamma(\tau)| \equiv 1$), in contrast to the incoherent case (e.g., white light). Therefore, we can drop the temporal dependency term of the source, i.e., $U(h,t) = U(h) = ae^{j\omega hc^{-1}} = ae^{j\phi}$, where ω is the angular velocity and h is the surface height. Consequentially, in the case of coherent light source, intensity in every pixel is determined only by the phase terms and is given by

$$\begin{aligned} I_p &= \left| \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a \exp(-j\phi_n) \right|^2 \\ &= \frac{a^2}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} \exp(j(\phi_m - \phi_n)) \\ &= a^2 \left[1 + \frac{2}{N} \sum_{n=1}^{N} \sum_{m=n+1}^{N} \cos(\phi_m - \phi_n) \right] \end{aligned} \quad (4)$$

where a is a constant amplitude.

The contrast of the image formed in the array of the detectors is defined as

$$\text{contrast} \triangleq \frac{\sigma}{\langle I \rangle}, \quad (5)$$

where $\langle I \rangle = \frac{1}{N_p} \sum_{n=1}^{N_p} I_p$, and $\sigma = \sqrt{\frac{1}{N_p - 1} \sum_{n=1}^{N_p} \left(I_p - \langle I \rangle \right)^2}$.

The numerator in (5) tells us how much the image intensity varies, and the denominator tells us about the total illumination. If variations are small relative to the total illumination, we get a poor contrast, and if it is high, we get a good contrast. Goodman [12] derived the distribution of the speckle for a uniformly distributed phase. Intensity probability density is given by an exponential probability density. For this speckle intensity distribution, the contrast equals one.

The derivation for uniformly distributed phase is correct only for extremely rough surfaces as will be shown below, but is not suitable for more practical situations. Accordingly, it cannot explain the image formed by a smooth surface with roughness significantly lower than the optical wavelength: The formed image is a blurred spot with low speckle contrast.

A physical interpretation of the phase uniform distribution implies that there is no preference for any phase value. We represent the scattered waves as phasors and the measured intensity as the resultant sum of vectors. In total, we get a drunkard's walk [13] that its probability density has a maximum value at zero (exponential distributed speckle) as illustrated in Fig. 1.

In our model, speckle pattern statistics are related to a specific surface height distribution. Phase differences between light waves that are scattered from different locations at the illuminated surface are dominated only by the surface height.

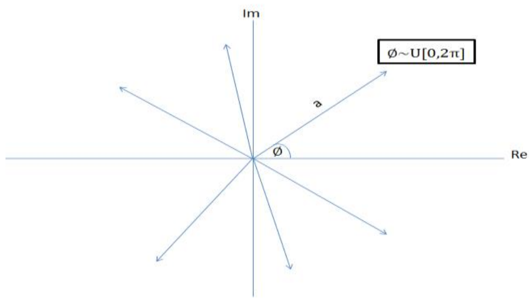


Figure 1: Six scattered waves with a constant amplitude a and uniformly distributed phase. The resultant magnitude is likely to be close to zero.

We define z as the surface height deviation from the nominal height. In many cases, surface roughness could be described as a Gaussian random vector [10]. We denote the height of the N scatters as $\overline{Z} = \left[z_1, z_2 \ldots z_N \right]^T$ where $z_i \sim N(0, \sigma_R^2)$ are i.i.d. Gaussian random variables. The phase of the light scattered from the microscopic surfaces (assuming single scattering events) depends linearly on the local surface height, i.e.

$$\phi_m - \phi_n = \frac{\omega}{c}\left(z_m - z_n \right). \quad (6)$$

The phases of the scattered waves are accumulated linearly with their optical paths (6). Thus, the phase has a Gaussian distribution, rather than a uniform distribution, and thus renders a more realistic situation. For a Gaussian distribution, we can think of a drunkard's walk again, where the azimuth of the phasor is somehow limited due to the distribution shape and variance as illustrated in Fig. 2.

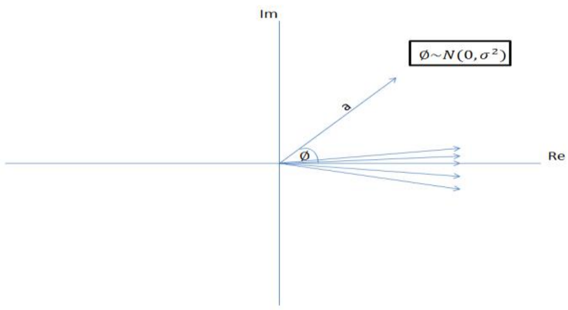


Figure 2: six scattered waves with a constant amplitude a and uniformly distributed phase. The resultant magnitude is likely to be close to its full magnitude which is $\left(6a\right)^2$.

The resultant magnitude keeps on growing for small values of σ_R , i.e., smooth surfaces. For greater roughness (i.e., higher values of σ_R) however, it is more likely for the intensity to converge into an exponential speckle distribution and a uniform phase distribution. This happens because now the azimuth (phase) of the phasor is not limited to the range of $\phi \in [0, 2\pi)$.

III. PROPOSED APPROACH

In this section, we introduce our generalization for incoherent light speckle under the assumption of a Gaussian distributed phase. The wave function that the detector (pixel) "senses" is given by $G(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} U(t+\tau_n)$, where $\tau_n = \frac{2(\tilde{z} - z_n)}{c}$ and $\tilde{z} = \max\left(\{z_i\}_{i=1}^{N}\right)$ to keep $\tau_n \geq 0$. Therefore, the average intensity of light gathered from N scatters over the exposure time T in a single pixel is:

$$
\begin{aligned}
I_p &= \frac{1}{T} \int_{-T/2}^{T/2} G(t) G^*(t) dt \\
&= \frac{1}{T} \int_{-T/2}^{T/2} \left\{ \frac{1}{\sqrt{N}} \sum_{n=1}^{N} U(t+\tau_n) \right\} \left\{ \frac{1}{\sqrt{N}} \sum_{m=1}^{N} U^*(t+\tau_m) \right\} dt \\
&\overset{\tilde{t}=t+\tau_n}{=} \frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{1}{T} \int_{-T/2}^{T/2} U(\tilde{t}) U^*(\tilde{t} + \tau_m - \tau_n) d\tilde{t} \\
&\overset{\tau_{nm}=\tau_m-\tau_n}{=} \frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{1}{T} \int_{-T/2}^{T/2} U(\tilde{t}) U^*(\tilde{t} + \tau_{nm}) d\tilde{t} \\
&= \frac{a^2}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} |\gamma(\tau_{nm})| \exp(j(\phi(\tau_{nm}))) . \qquad (7)
\end{aligned}
$$

Notice that the measured intensity is a sum of the sampled autocorrelation at times that are proportional to the surface height's variations $\tau_{nm} = \frac{2(z_n - z_m)}{c}$. Note also that for coherent light $(|\gamma(\tau_{nm})| = 1 \; \forall n, m)$ and when the phase term, $\phi(\tau_{nm})$, does not depend on time, (5) can be derived from (7). Generally, $\phi(\tau_{nm})$ depends on time and can have a complicated analytical representation. Hence, this term's contribution for the source incoherency remains unclear. For simplicity, the function that was chosen to describe a coherence limited light source is a Gaussian wave group centered around $\omega_0 = 2\pi c / \lambda_0$ where $\lambda_0 = 0.55[\mu m]$, i.e.,

$$
U(t) = e^{-j\omega_0 t} e^{-\frac{t^2}{2\sigma_g^2}} . \qquad (8)
$$

The phase term of this wave's autocorrelation is linear with time, i.e., $\phi(\tau_{nm}) = e^{j\omega_0 \tau_{nm}}$. It can be shown that in this case $\tau_c = 2\sigma_g$.

The normalized autocorrelation function of a Gaussian wave group is given by

$$
\gamma(\tau) = e^{-j\omega_0 \tau} e^{-\frac{\tau^2}{4\sigma_g^2}} . \qquad (9)
$$

Now, using the previous assumptions we introduce the following model for measured intensity in a pixel for an incoherent light source:

$$
\begin{aligned}
I_p &= \frac{a^2}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} |\gamma(\tau_{nm})| e^{j\omega_0 \tau_{nm}} \\
&= a^2 \left[1 + \frac{2}{N} \sum_{n=1}^{N} \sum_{m=n+1}^{N} |\gamma(\tau_{nm})| \cos(\omega_0 \tau_{nm}) \right] . \qquad (10)
\end{aligned}
$$

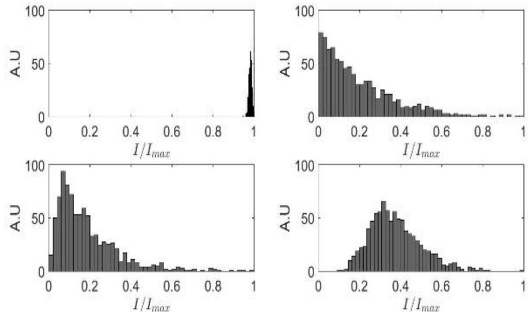
We can see that the main independent variables that control speckles are coherence length, wavelength and of course roughness that affects the delay time between pairs of scatters. The model is consistent with the derivations for the simple case of only two scatters (i.e., $N = 2$). In that case,

$$
I_p = a^2 \left[1 + |\gamma(\tau_{12})| \cos(\omega_0 \tau_{12}) \right] . \qquad (11)
$$

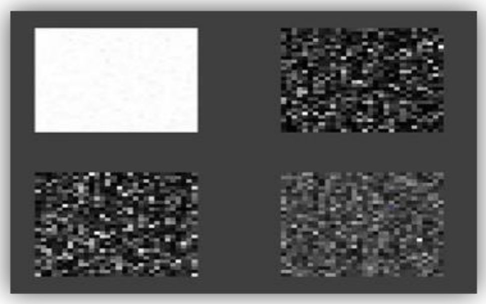
The proposed model describes both coherent and incoherent light speckles that are highly correlated with surface roughness.

Dependence of contrast and speckle distribution were examined using numerical simulations and are presented in Figs. 3 and 4. The simulations show that it is possible to have an informative speckle for incoherent light such as LED that we used in our experiments.

We can see in Figs. 3 and 4 two important results: The first one is that for both coherent light $(L_c / \lambda_0 \approx 50)$ and incoherent light $(L_c / \lambda_0 \approx 4)$ with parameters typical to LED we obtain the same trend at the aspect of distribution, which can be used as a proof of concept for our hypothesis that incoherent light speckle can be informative. The second one is that our simulations, based on our model, can explain the blurry spots that we get in very smooth surfaces in contrast to the assumption that the phase is uniformly distributed that results always in an exponential distribution of intensity as mentioned.

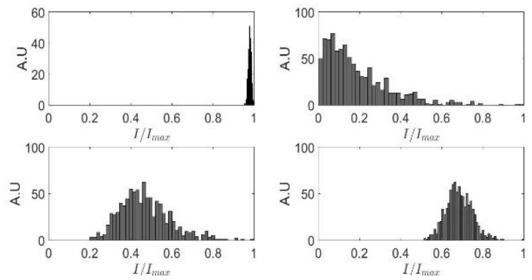


(a)

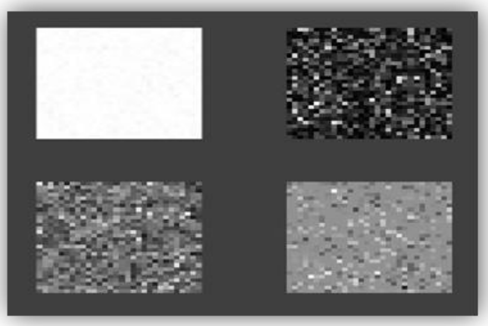


(b)

Figure 3: (a) Speckle distribution and (b) their corresponding images as a function of increasing roughness for $L_c = 25[\mu \mathrm{m}]$ (A.U stands for Arbitrary Units). We choose $\sigma_R = (0.01, 0.1, 5, 40)[\mu \mathrm{m}]$ from upper left to bottom right and number of scatters per pixel is 100.



(a)



(b)

Figure 4: (a) Speckle distribution and (b) their corresponding images as a function of increasing roughness for $L_c = 2[\mu \mathrm{m}]$. We choose $\sigma_R = (0.01, 0.1, 5, 40)[\mu \mathrm{m}]$ from upper left to bottom right and number of scatters per pixel is 100.

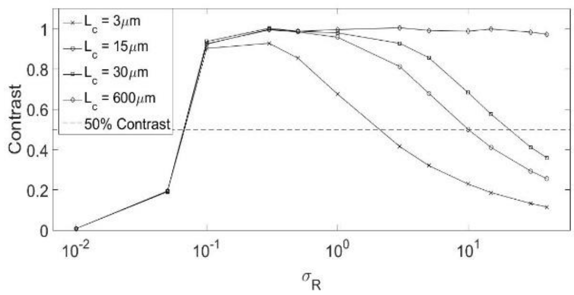


Figure 5: Contrast as a function of increasing roughness with different coherence lengths. Contrast at each roughness was averaged over 12 runs. Results are presented on a logarithmic scale of the x axis.

In Fig. 5, we can observe that for roughness much lower than the central wavelength the contrast is low. Then contrast keeps on rising until it gets a maximum value at $\sigma_R = \lambda_0 / 4$, i.e., phase of $\pi / 2$ for all coherence lengths. Then, for each coherence length, the contrast drops to 50% or less when $\sigma_R > L_c$. The contrast maintains a maximum value only for fully coherent light ($600[\mu \mathrm{m}]$ in our simulations) as expected.

IV. Experimental Results

In our experiments the objects were illuminated by a collimated light originated from a white light LED. The imaging was carried out with a 10-bit camera operating in a dark field mode so that the specular reflection was eliminated and only the scattered light was gathered. Three objects were examined in the experiment: 50 Shekel bill, 100 Dollar bill and a label ticket of a shirt. The results are shown in Fig. 6. In order to test the effects of light source with central wavelength of $0.55[\mu \mathrm{m}]$, we examined only the speckles in the second layer of the RGB matrix ("green matrix"). Furthermore, only the central area of the image was examined as being the best illuminated and focused area.

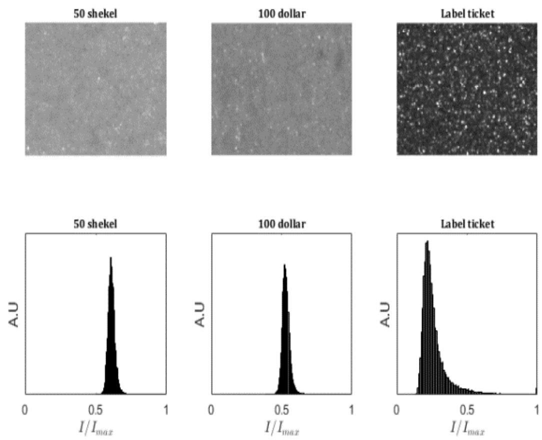


Figure 6: Experimental speckle patterns of salt and pepper like objects.

As can be seen in Fig. 6, in some cases an incoherent light source can form the speckle image. Despite previous studies, considering the interference between two light sources, our study covers a comprehensive set of surface parameters and illumination conditions. The predicted changes in distribution and visibility as a function of surface roughness and light coherency lead us to the thought that we could estimate σ_R only by using the speckle pattern, which to the best of our knowledge, was not addressed before.

V. CONCLUSIONS

Speckle intensity can be computed directly using the sampled autocorrelation function of the illuminating source. Our new model is consistent with both preliminary results obtained by different complex approaches and with experimental data. Accordingly, we can simulate and analyze white light speckle imaging using a simple model that can be easily implemented.

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