DETECTION OF ANOMALIES IN TEXTURES BASED ON MULTI-RESOLUTION FEATURES

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ABSTRACT

Multi-resolution decompositions, such as the wavelet transform, are often employed in anomaly detection algorithms for feature extraction. However, the extracted features may be unreliable for anomaly detection in textures due to inconsistencies between the assumed background model and the true data. In this paper, we present an anomaly detection scheme which relies on a statistical model of textures and is specifically designed for detection of anomalies in textures. Motivated by recent works on texture segmentation and texture classification, we introduce a multi-resolution feature space that facilitates anomaly detection with constant false alarm rate for a wide range of textures. Experimental results demonstrate that the proposed algorithm, when applied to images containing background texture, achieves improved detection results and lower false alarm rate than a competitive anomaly detection scheme.

1. INTRODUCTION

Anomaly target detection is the process of locating elements in a scene which are unlikely to be a part of it. This is generally done with respect to a predefined probabilistic model and an appropriate feature space, in which a clear segregation between the anomalous elements and the rest of the background clutter in the scene is possible. Multi-resolution decompositions, such as the wavelet transform, are often used for feature extraction. These decompositions use a set of multi-scale bandpass oriented filters for decomposing the image, a process which is effective at decoupling the high-order statistical features of natural images. In addition, it efficiently represents the visually relevant features of images [1]. Detection methods often use classifiers which employ a probabilistic model upon the derived feature space. Bayesian classifiers require knowledge of a-priori and posteriori statistics of both background clutter and anomalous targets and thus may be a source for loss of robustness. A wide variety of potential targets which do not necessarily conform to a uniform model or even a characterizing subspace, render detectors such as the Matched Signal Detector, the Matched Subspace Detector [2, 3, 4, 5] and the Adaptive Subspace Detector [6] ineffective. Furthermore, these detection methods use probabilistic models which are usually Gaussian based, due to the mathematical traceability of this distribution. Noibar and Cohen have argued in [7] that these methods are therefore not appropriate for modeling two common phenomena of multi-resolution feature spaces: heavy tails of the marginal probability density function of the features (known as excess kurtosis) and volatility clustering (a characteristics in which large changes tend to follow large changes and small changes tend to follow small changes); leading to potentially higher false alarm rates due to the inadequacy between the model and the data. These phenomena of a wavelet based feature space therefore call for an alternative scheme that utilizes suitable long-tailed distributions, such as the Gaussian Scale Mixture (GSM).

In this paper, we propose an unsupervised anomaly detection scheme which does not rely on the exhaustive statistical model of the targets, but rather on the local multi-resolution statistics of the background clutter and possibly on some a-priori information of the minimal expected size of the targets. Our choice of a feature space is motivated by previous work on the enhancement of texture segmentation by Unser and Eden [8] and texture classification by Mittelman and Porat [9]. We utilize the Redundant Discrete Wavelet Transform (RDWT) for the purpose of generating a multi-resolution feature space. We calculate from the resulting wavelet coefficients their local second moment estimates (corresponding to the GSM hidden multipliers maximum likelihood estimates), followed by the logarithmic transformation. The use of the logarithmic transformation was reported to have the effect of variance equalization which yields better segmentation results [8]. The resulting feature space, when derived from natural textures, was shown to follow the Gaussian distribution [10, 11], hence, making it suitable for use with Bayesian classifiers. We then employ additional smoothing to the resulting feature channels. This final averaging reduces the feature component variances and results in clusters in feature space that are more compact and easier to distinguish. We show that increasing the size of the averaging window yields better detection results, as long as the window size does not exceed the anomalous target size. Our choice of a classifier is motivated by previous work on anomaly detection by Goldman and Cohen [12], where the Single Hypothesis Test (SHT) [13] was used. Applying the SHT on the proposed feature space yields an anomaly detection algorithm with constant false alarm rate (CFAR) regardless of the background clutter and anomaly type, depending only on the feature space dimensionality. We have implemented the proposed algorithm and tested it with various scenes, containing Brodatz-like background textures and target anomalies [14]. The proposed algorithm mimics the detection mechanism of the human eye, achieving outstanding detection results even when using textures with significant visual resemblance between the anomalous target and the background clutter.

This paper is organized as follows. In Section 2 we formulate the problem. In Section 3, we present the proposed anomaly detection scheme. In Section 4, we present experimental results.

2. PROBLEM FORMULATION

Let Ω be the support lattice for $\{y(\mathbf{s})\}_{\mathbf{s}\in\Omega}$ - the observations of an image containing a background natural texture with rare anomalous target signals scattered around in the image, denoted as $x(\mathbf{s})$ and $n(\mathbf{s})$ respectively. The target signals are assumed to be much smaller than the support lattice of the background image and therefore can be regarded as transients. We define two possible hypotheses for each pixel $\mathbf{s} \in \Omega$:

$$\begin{array}{ll} H_0: & y(\mathbf{s}) = x(\mathbf{s}), \\ H_1: & y(\mathbf{s}) = n(\mathbf{s}), \end{array} \tag{1}$$

where H_0 and H_1 represent the absence and presence of an anomaly in the image respectively. The problem at hand is to define an anomaly detection algorithm that achieves:

$$\begin{array}{ll} P_D \triangleq \mathrm{p}(H_1|H_1), & P_D \geqslant 1-\epsilon_1, \\ P_{FA} \triangleq \mathrm{p}(H_1|H_0), & P_{FA} \leqslant \epsilon_2, \end{array} \tag{2}$$

for given values of ϵ_1 and ϵ_2.

3. ANOMALY DETECTION ALGORITHM

A block diagram of the proposed algorithm is presented in Figure 1. Let $\{y_j(\mathbf{s})\}_{j=1,\dots,m}$ denote the jth layer wavelet coefficients obtained from the mean normalized image observations $y(\mathbf{s})$ using a RDWT with $(m-1)/3$ levels. Let $\{t_j(\mathbf{s})\}_{j=1,\dots,m}$ denote the logarithm of the GSM hidden multipliers estimate, given by:

$$t_j(\mathbf{s}) = \log\left(\frac{\sum\limits_{\mathbf{r}\in\mathcal{R}_1} y_j^2(\mathbf{s}+\mathbf{r})}{|\mathcal{R}_1|}\right), \tag{3}$$

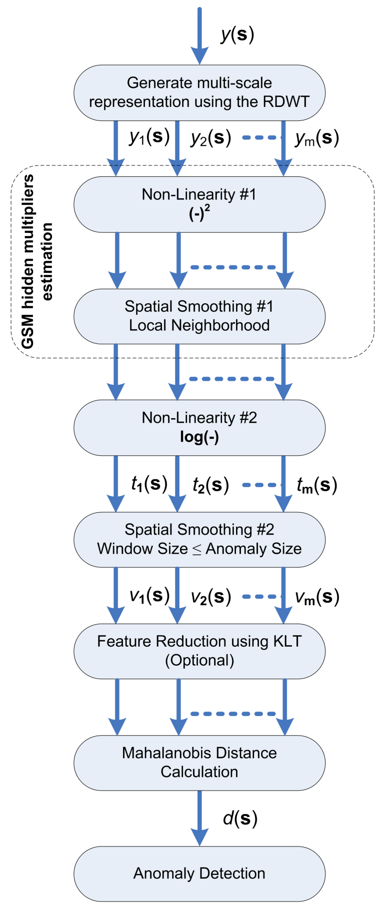


Fig. 1. Block diagram of the proposed anomaly detection algorithm.

where $\mathcal{R}_1$ denotes a given set of indices representing the $N \times N$ local neighborhood of a pixel. Let $\{v_j(\mathbf{s})\}_{j=1,\dots,m}$ denote the proposed feature space, given by:

$$v_j(\mathbf{s}) = \frac{\sum\limits_{\mathbf{r}\in\mathcal{R}_2} t_j(\mathbf{s}+\mathbf{r})}{|\mathcal{R}_2|}, \tag{4}$$

where $\mathcal{R}_2$ denotes a given set of indices representing the $M \times M$ local neighborhood of a pixel. Let $\mathbf{v}(\mathbf{s}) = [v_1(\mathbf{s}), v_2(\mathbf{s}), \dots, v_m(\mathbf{s})]^T$ denote the feature vector representing pixel $\mathbf{s} \in \Omega$. Let $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$ denote the expectancy of the random feature vector $\mathbf{v}(\mathbf{s})$ under hypotheses H_0 and H_1 respectively. Let $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}_1$ denote the covariance matrix of the random feature vector $\mathbf{v}(\mathbf{s})$ under hypotheses H_0 and H_1 respectively. Following the assumption that the anomalous targets are rare and can be regarded as transients:

$$\begin{array}{ll} \hat{\boldsymbol{\mu}}_0 & \approx E\left[\mathbf{v}(\mathbf{s})\right] \\ \hat{\boldsymbol{\Sigma}}_0 & \approx E\left[\left(\mathbf{v}(\mathbf{s})-\hat{\boldsymbol{\mu}}_0\right)\left(\mathbf{v}(\mathbf{s})-\hat{\boldsymbol{\mu}}_0\right)^T\right]. \end{array} \tag{5}$$

The Mahalanobis distance for pixel $\mathbf{s} \in \Omega$ is then given by:

$$d(\mathbf{s}) = (\mathbf{v}(\mathbf{s}) - \boldsymbol{\mu}_0)^T \boldsymbol{\Sigma}_0^{-1} (\mathbf{v}(\mathbf{s}) - \boldsymbol{\mu}_0). \tag{6}$$

Following the SHT scheme, the decision rule for pixel $\mathbf{s} \in \Omega$ is defined as follows:

$$d(\mathbf{s}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{7}$$

where η is the threshold that determines if a given pixel $\mathbf{s} \in \Omega$ is regarded as an anomaly or background clutter. This decision rule is based on the statistics of the background clutter alone. Some a-priori information regarding the minimum expected size of the targets can be utilized to determine M, thus improving the performance of the algorithm. The feature vector $\mathbf{v}(\mathbf{s})$ is a linear combination of Gaussian random vectors with dimension m [10, 11] and as such, it is also a Gaussian random vector. Since the covariance matrix $\boldsymbol{\Sigma}_0$ is a positive definite matrix, equation (6) can be formulated as follows:

$$d(\mathbf{s}) = \mathbf{z}(\mathbf{s})^T \mathbf{z}(\mathbf{s}), \tag{8}$$

where $\mathbf{z}(\mathbf{s}) \triangleq \boldsymbol{\Sigma}_0^{-1/2} (\mathbf{v}(\mathbf{s}) - \boldsymbol{\mu}_0)$. The random vector $\mathbf{z}(\mathbf{s})$ is distributed according to:

$$\begin{aligned} &\mathbf{z}(\mathbf{s})|_{H_0} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}), \\ &\mathbf{z}(\mathbf{s})|_{H_1} \sim \mathcal{N}\left(\boldsymbol{\Sigma}_0^{-1/2} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0), \boldsymbol{\Sigma}_0^{-1}\boldsymbol{\Sigma}_1\right). \end{aligned} \tag{9}$$

As such, the Mahalanobis distance under hypothesis H_0 is chi-square distributed with m degrees of freedom, regardless of the background clutter:

$$d(\mathbf{s})|_{H_0} \sim \chi_m^2(0). \tag{10}$$

The false alarm, as formulated in equation (2) is then given by:

$$P_{FA} = 1 - \mathrm{p}\left(\chi_m^2(0) \leqslant \eta\right). \tag{11}$$

A special case is when under hypothesis H_1 the observations are assumed to be a linear combination of the target signature and the background clutter. The statistical distributions of the two hypotheses are therefore Gaussian with common covariance matrices (i.e., $\boldsymbol{\Sigma}_0 = \boldsymbol{\Sigma}_1$) and different means. This yields a detection scheme similar to the RX algorithm [15]. In particular:

$$d(\mathbf{s})|_{H_1} \sim \chi_m^2(\lambda), \tag{12}$$

where $\lambda = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)^T \boldsymbol{\Sigma}_0^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$. The detection rate, as formulated in equation (2) is then given by:

$$P_D = 1 - \mathrm{p}\left(\chi_m^2(\lambda) \leqslant \eta\right). \tag{13}$$

When M is increased, the expectancies $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$ remain the same. However, the variances of the marginal distributions under both hypotheses are reduced. The reduction factor is approximately equal for both background clutter and target anomaly whenever the size of the averaging window is greater than the maximum distance over which pixels are significantly correlated [8]. As a result, we expect that the detection rate will be a monotone increasing function with the averaging window size M. Since a parametric form for P_D is not available for the general case, we have verified this on various Brodatz like textures [14]. An example is given in section 4.

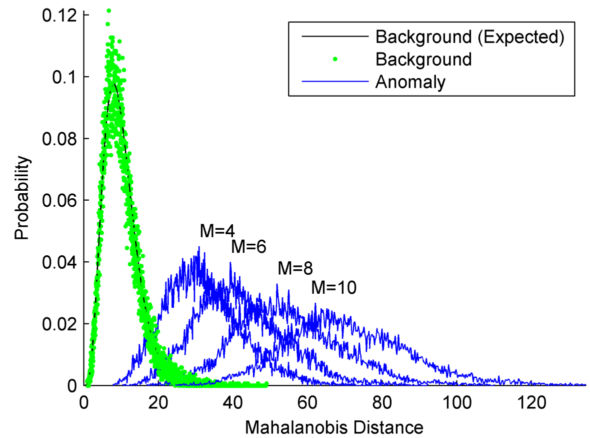
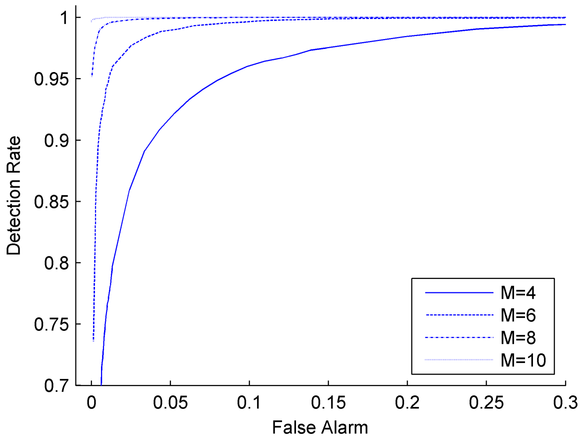


Fig. 2. Mahalanobis distance calculation using a background clutter image and an anomaly image, both Brodatz like, for various values of M.

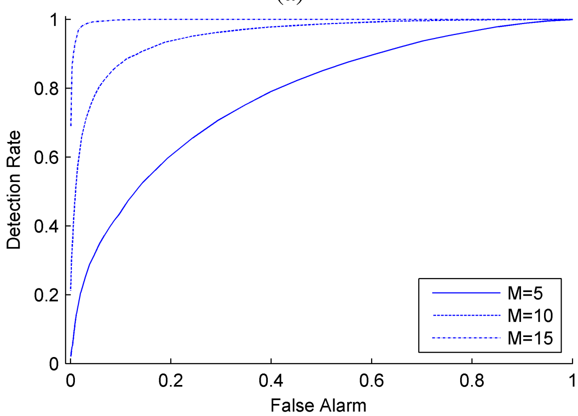
4. EXPERIMENTAL RESULTS

4.1. Performance Analysis

In order to verify the theory which states that the proposed feature space follows the Gaussian distribution and therefore conform to kurtosis value of 3, we have tested it on 40 Brodatz-like textures [14] using various averaging window sizes $1 \leqslant M \leqslant 40$. We have then examined the influence of the averaging window size M on the performance of the proposed algorithm. We have observed that given any two textures from the Brodatz-like database [14], higher values of M leads to a better detection rate at a pre-determined false alarm rate. This is shown in Figure 2, using background and anomaly textures which appear different. Concurrent Receiver Operation Characteristics (ROC) curves are shown in Figure 3 (a). Figure 2 shows that the Mahalanobis distance indeed follows the chi-square distribution under hypotheses H_0. Figure 3 (b) shows ROC curves for various window sizes, derived from background and anomaly textures having evident visual resemblance. This implies that M should be increased in order to achieve better performance, where the upper bound value is derived from the a-priori information of the minimal expected size of the targets.



(a)

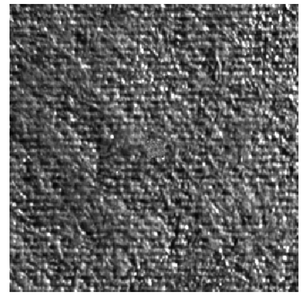


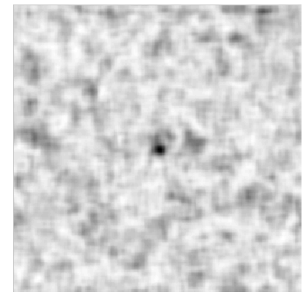
(b)

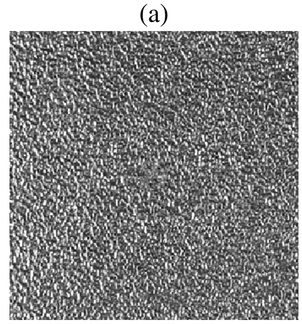
Fig. 3. Calculated ROC curves using various values of M for: (a) Background and anomaly textures which look different. (b) Background and anomaly textures having evident visual resemblance.

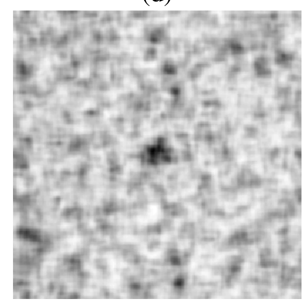
4.2. Anomaly Detection Examples

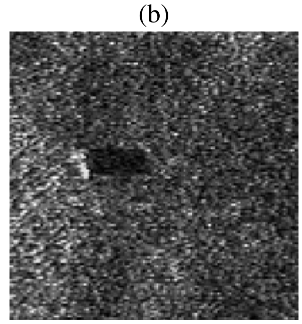
We have performed extensive testings of the proposed algorithm, using both real and synthesized images. Typical images are shown in Figure 4. The synthesized images were created using a set of Brodatz-like textures [14] that were used for both background clutter and anomalous targets. In particular, we have studied two interesting cases: anomalous targets taken from textures which appear different from the background clutter and anomalous targets taken from textures which have evident visual resemblance to the background clutter. These are shown in Figure 4 (a-b) with corresponding detection results in Figure 4 (d-e). It can be seen that the algorithm detects the anomalies in both cases, in spite of the difficulty to locate it when observing the images with the naked eye. For real image data, we have used the same set of sonar images that were used in [12]. The image shown in Figure 4 (c) is a good example for the volatility clustering phenomena that follows multi-resolution decompositions [7]. Nevertheless, it can be seen in Figure 4 (f) that this does not affect the detection of the sea-mine and produces no false alarms with proper thresholding of the calculated distance.


(a)


(d)


(b)


(e)


(c)

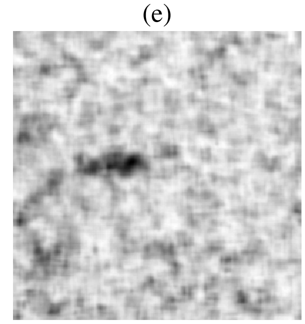

(f)

Fig. 4. (a) Synthesized image with anomaly in its center, using textures not having evident visual resemblance. (b) Synthesized image with anomaly in its center, using textures having evident visual resemblance. (c) Real sonar image containing a sea-mine. (d-f) Corresponding detection results using $m = 10$, $N = 3$, $M = 15$.

5. CONCLUSION

We have introduced a multi-resolution feature space and a corresponding anomaly detection method. The proposed feature space is well modeled by the Gaussian distribution and thus is appropriate for use with Bayesian classifiers. Our detection method is based on the Single Hypothesis Test, thus it is not restricted to targets which follow a uniform model or reside in a characterizing subspace. The proposed scheme yields a CFAR detection algorithm which achieves improved detection results.

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