Source Localization With Feedback Beamforming

Itay Yehezkel Karo

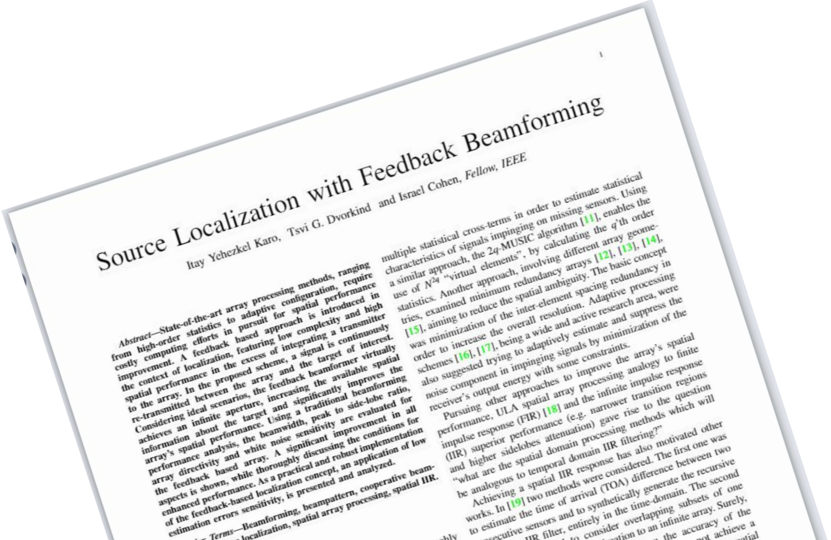
Supervisors:

Dr. Tsvi Dvorkind

Prof. Israel Cohen

Publication info

Itay Yehezkel Karo, Tsvi Gregory Dvorkind, and Israel Cohen.
Source localization with feedback beamforming.
IEEE Transactions on Signal Processing, 69:631–640, 2021.



Source Localization with Feedback Beamforming

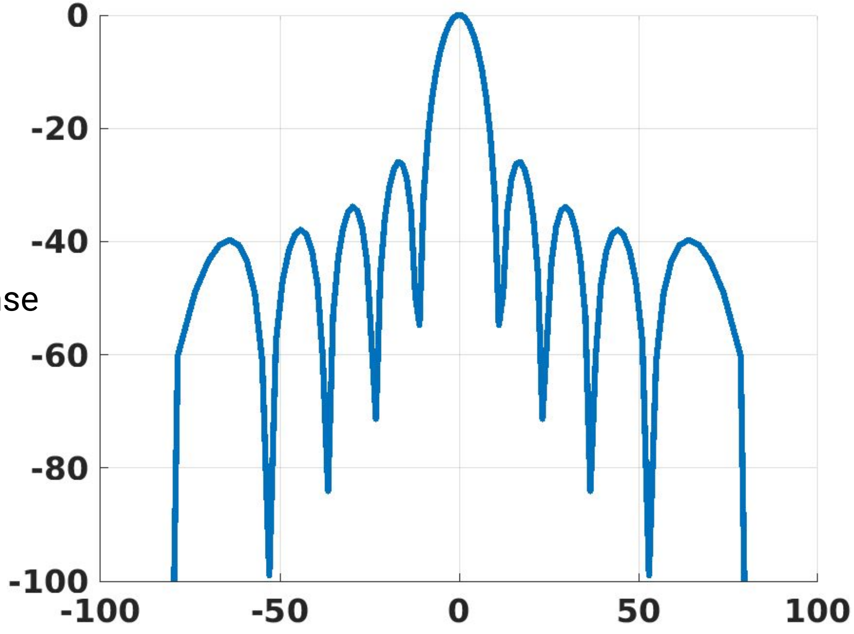
Itay Yehezkel Karo, Tsvi G. Dvorkind and Israel Cohen, *Fellow, IEEE*

Our contribution – in a nutshell

- We successfully integrated **spatial feedback** to a **simple phase-shift beamformer**
    - **Infinite virtual aperture** gain
    - Spatial response was analogous to FIR - **Now IIR**
    - **Controllable main lobe width**
    - **High performance** in low SNR scenarios (*localization/detection*)

Our contribution – in a nutshell

A conventional phase shift beamformer of 10 elements:



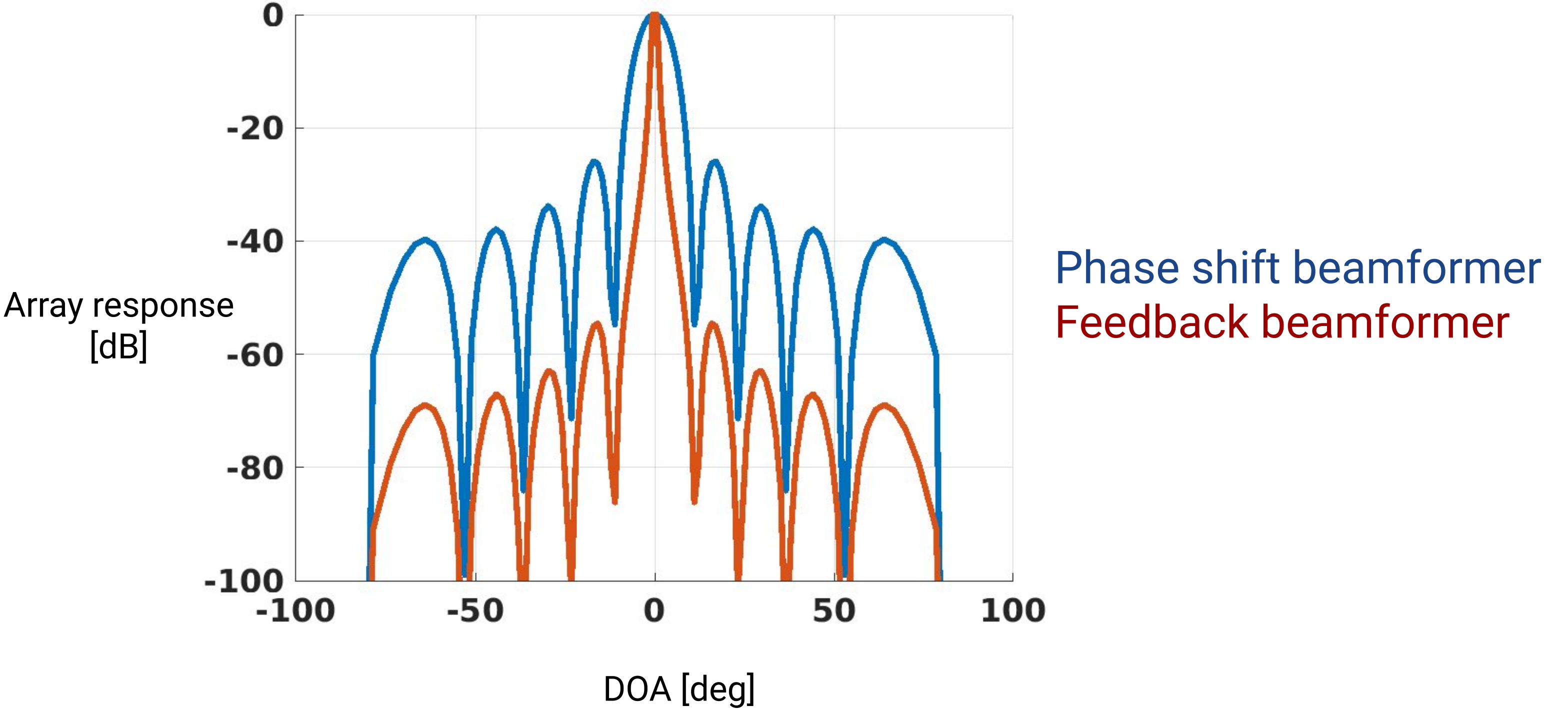
Array response [dB]

DOA [deg]

Phase shift beamformer

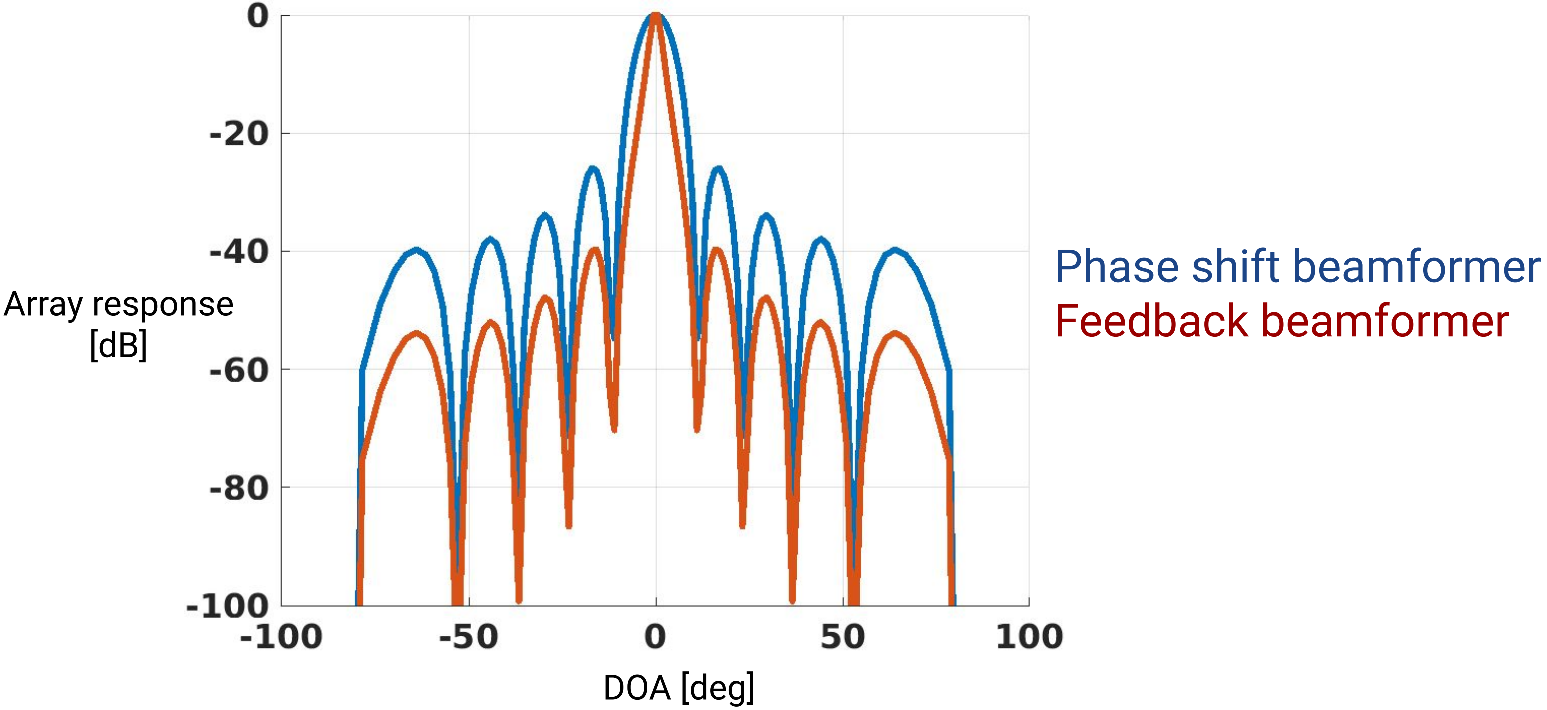
Our contribution – in a nutshell

The feedback beamformer



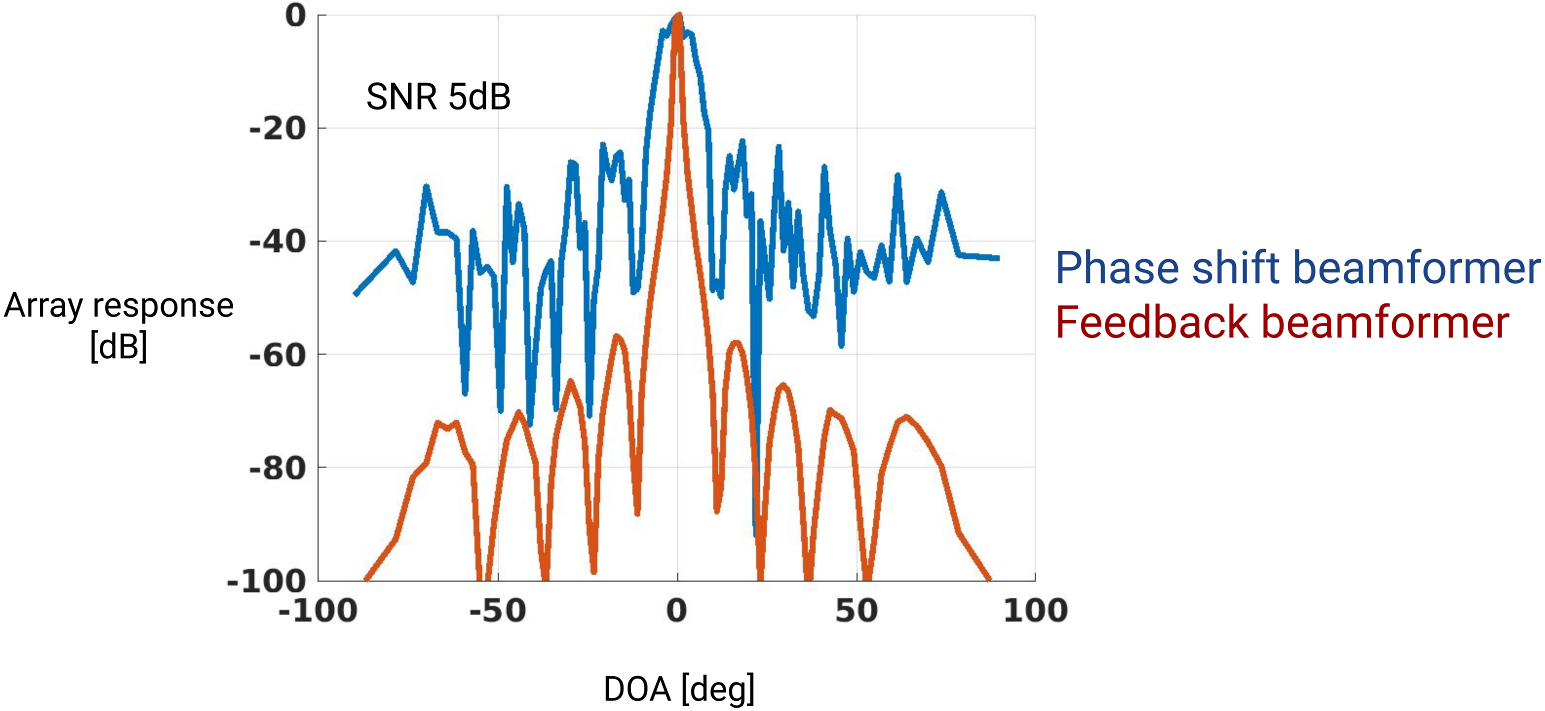
Our contribution – in a nutshell

Controllable main lobe width



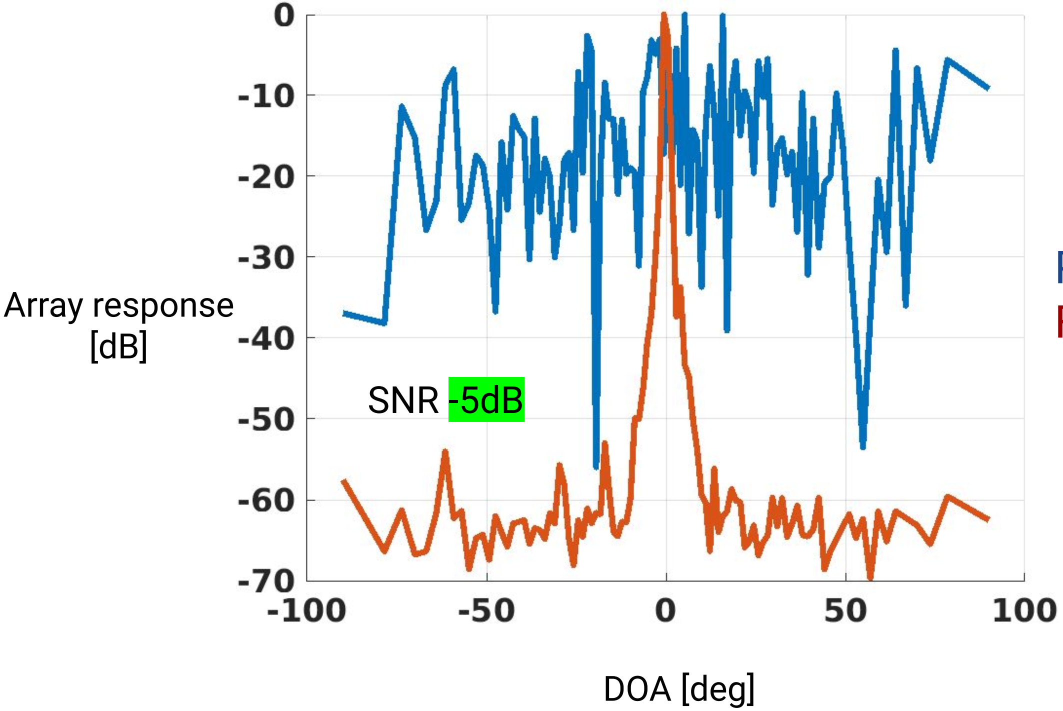
Our contribution – in a nutshell

High performance in low signal to noise ratio (SNR)



Our contribution – in a nutshell

Also in very low SNR



Phase shift beamformer
Feedback beamformer

Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

Sensor arrays – applications

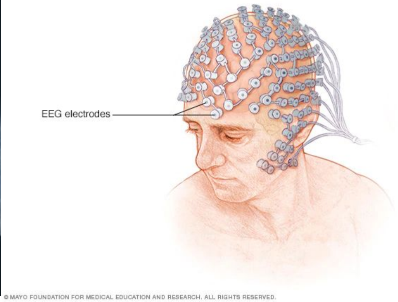
Sensor arrays - applications

Astronomy

Sensor arrays - applications

Astronomy

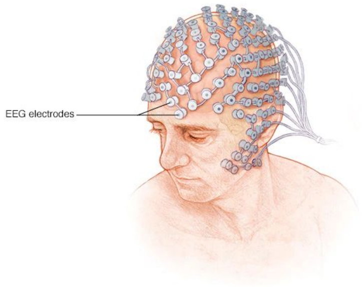
Medical imaging



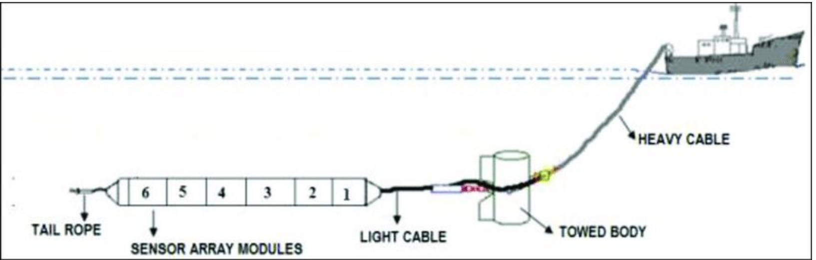
Sensor arrays - applications

Astronomy

Medical imaging

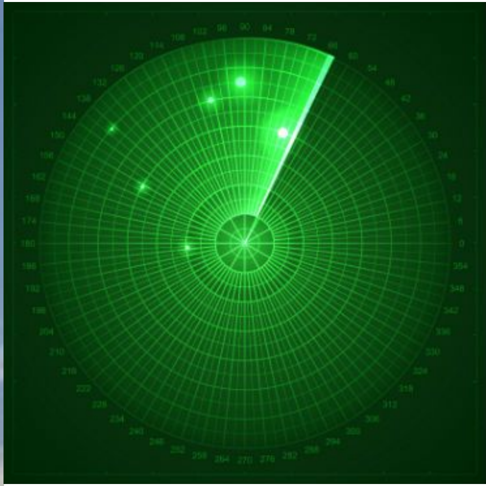


Sonar



Sensor arrays - applications

Localization



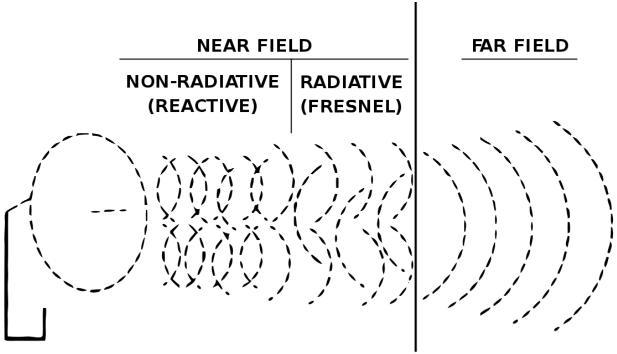
Sensor arrays – the basics

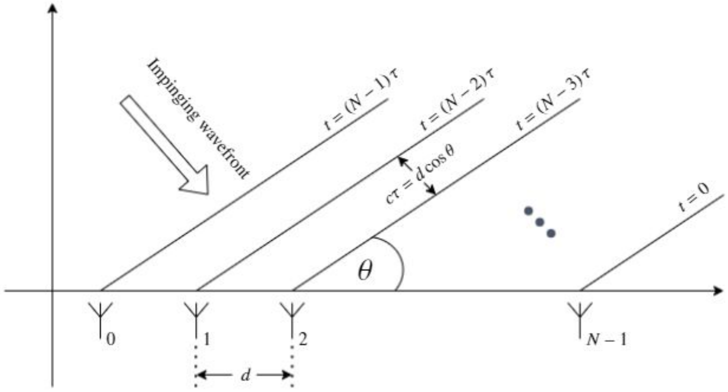
Wave propagation

- Signals propagate according to the wave equation

$$\nabla^2 f(t,x,y,z) = \frac{1}{c^2}\frac{\partial^2 f(t,x,y,z)}{\partial t^2}$$

- We consider far-field sources - plane waves





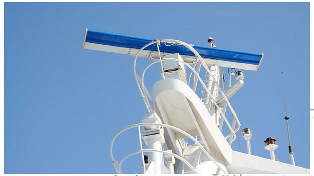
Sensor arrays – the basics

Beamforming

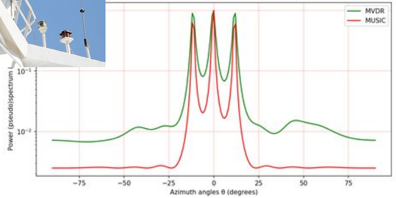
- Enhance signals from wanted sources
- Suppress unwanted signals

Localization

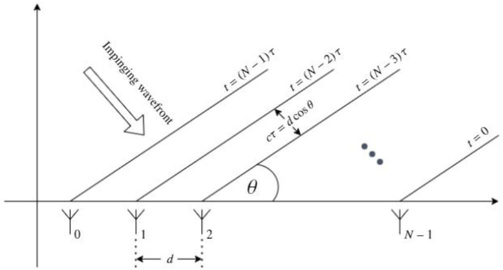
- Estimation of target's spatial characteristics:
  - direction of arrival (DOA)
  - Range
- Exploiting the spatial information of the array's output samples:
  - Beamforming



  - Subspace decomposition



  - Stochastic approach; Maximum likelihood (ML) etc.



Also neural networks and sparsity based approaches exist...

Localization

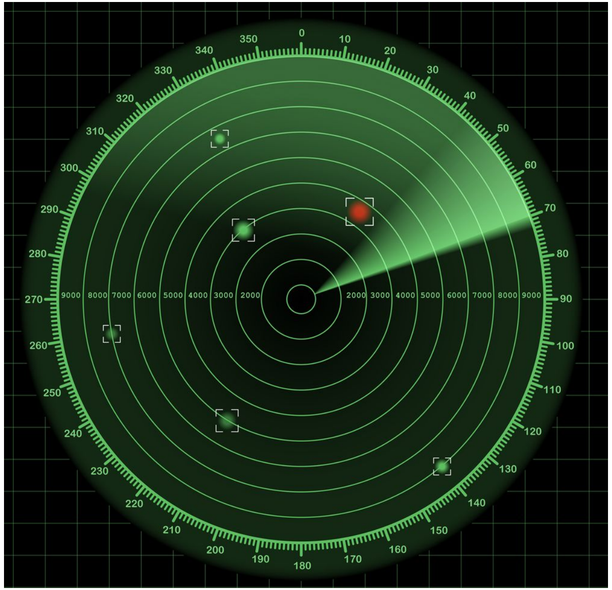
Beamforming

- Search for energy peaks - targets of interest

- Two types:

    - Non adaptive:
      conventional beamformer
      Use phase shifters to “align” the impinging plane waves to the array.

    - Adaptive:
      Capon’s MVDR -
      Minimizing noise energy according to continuously computed covariance matrices.

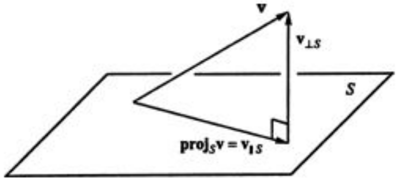


Localization

Subspace decomposition

- Plane waves excite the sensor array according to its sensors positioning.
  - Define the steering vector
  $$d(\theta) = \left[1, e^{-j\omega\tau}, \ldots, e^{-j\omega\tau(N-1)}\right]^T$$
  - The array manifold is a set, which contains all of the array's steering vectors.
  - Each valid input signal resides in the array manifold.

Orthogonal projection



- Assuming noise subspace is orthogonal to the array manifold.
- Projecting the array's output onto the array manifold.

Known Algorithms

- Multiple signal classification (MUSIC)
- ROOT-MUSIC
- etc.

Spatial performance

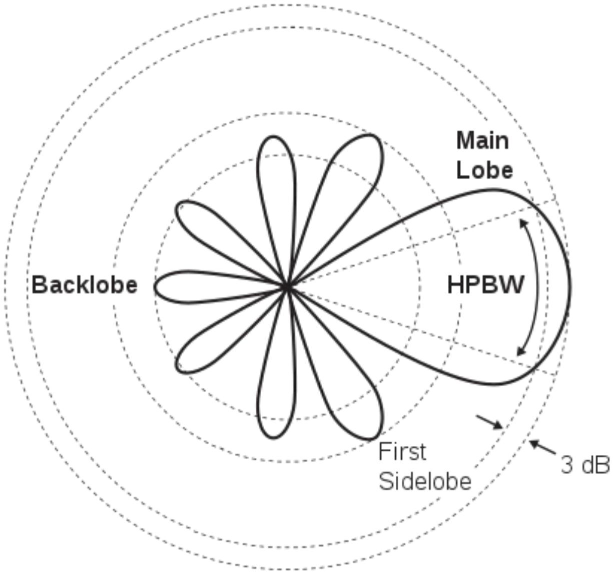
- We follow classic performance analysis [1]
    - Beamwidth
    - Sidelobe level
    - Directivity

[1] Van Trees, Harry L. Optimum array processing: Part IV of detection, estimation, and modulation theory. John Wiley & Sons, 2004.

Half power beamwidth (HPBW)

- Half of the angular width between the points $|B(\theta)|^2 = 0.5$
- Lower HPBW = Higher spatial resolution
- Phase shift beamformer (**ULA**, N>>1)

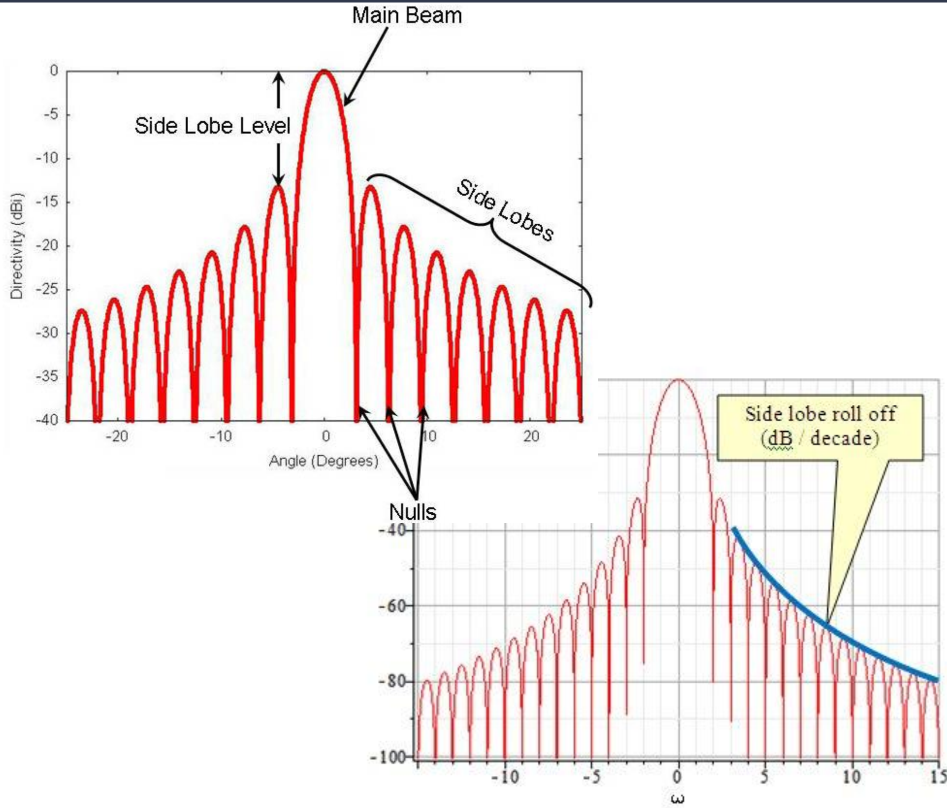
$$\Delta\theta_{\mathrm{HPBW}}/2 = 1.4/N.$$



Sidelobe attenuation

- Side lobes are usually unwanted
- Higher attenuation = less interference
- Phase shift beamformer (**ULA**, N>>1)

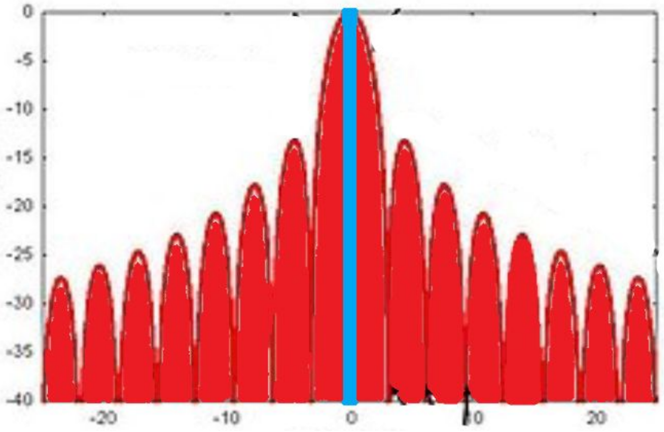
$$B_\psi\left(\pm\frac{3\pi}{N}\right) \cong \frac{2}{3\pi}$$



Directivity

- Maximum radiation intensity (power at DOA of interest)
  Divided by
  The average radiation intensity (averaged over all DOAs)

$$\mathcal{D}(\theta_T) = \frac{P(\theta_T)}{\frac{1}{2\pi}\int_0^{2\pi} P(\theta)\, d\theta}$$



- Phase shift beamformer (**ULA**): $D = N$

- Interpretation:
  - <u>Transmitters</u>
    Represents the efficiency of the transmission focussing.
  - <u>Receivers</u>
    The array gain against isotropic noise (noise from all directions).

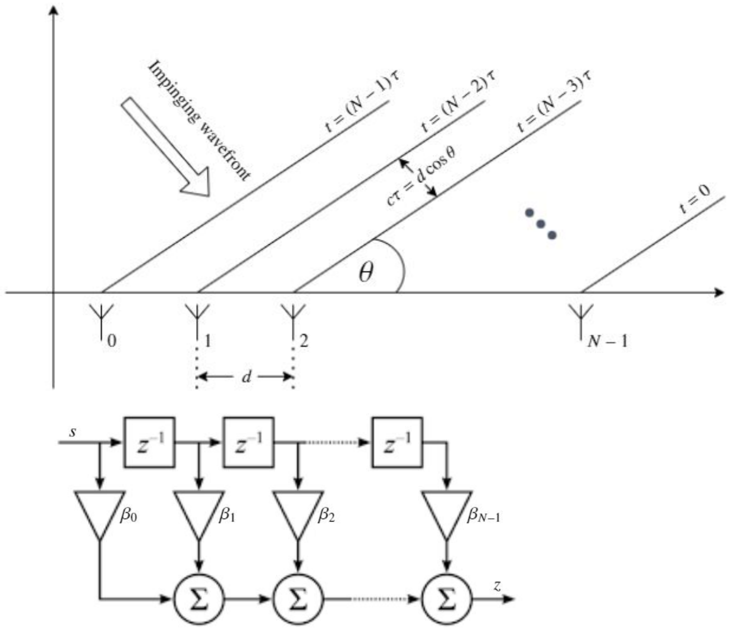
Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

Digital Filtering

The analogy between Uniform linear array (ULA) and Finite impulse response (FIR)

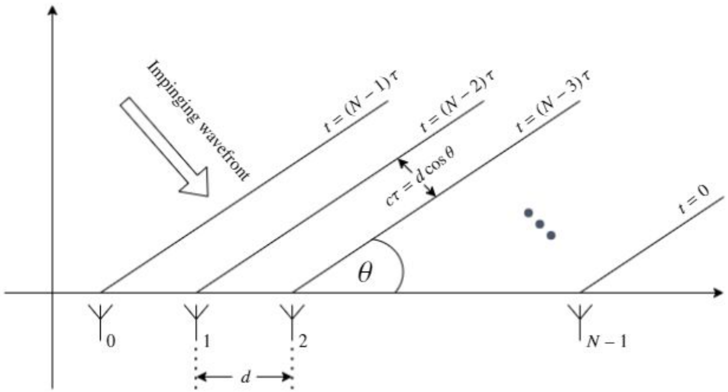
- Sample the signal on a uniformly spaced grid
- Considering narrowband signal, time shift is equivalent to phase shift.
- A desired spatial beampattern's spec is similar to frequency response spec.
- DOA is the spatial equivalent to frequency.

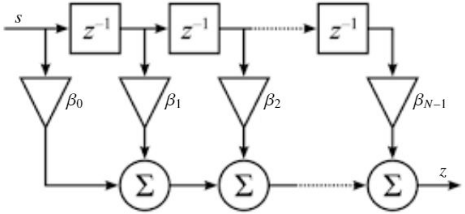


Digital Filtering

The analogy between Uniform linear array (ULA) and Finite impulse response (FIR)

Response: $$\sum_{n=0}^{N-1} \beta_n \exp(-jn\theta)$$



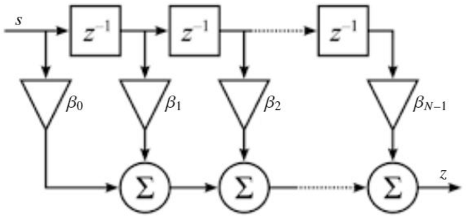


Digital Filtering

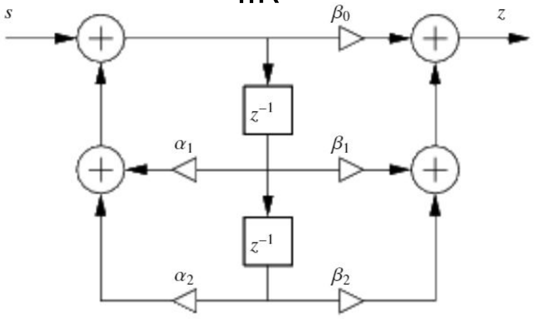
FIR vs IIR (infinite impulse response)

- IIR’s beamwidth is sharper - higher resolution
- In most cases, IIR needs less taps (sensors) for the same frequency response spec
- FIR is always Bounded Input Bounded Output (BIBO) stable

FIR



IIR



Research question

*“What are the **spatial domain** processing **methods** which will be **analogous to** temporal domain **IIR filtering**?”*

Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

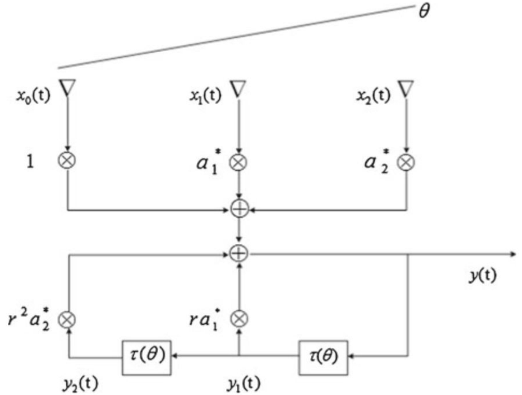
Related work

Trying to achieve recursive spatial response also inspired other works.

Related work

In [2], two methods have been proposed.

- **Estimating the delay between the signal's time of arrival (TOA) to consecutive sensors. Then, synthetically generate delayed instances of the output to emulate the recursive part of the filter.**
    - Involves temporal processing.
    - Sensitive to delay estimation errors.



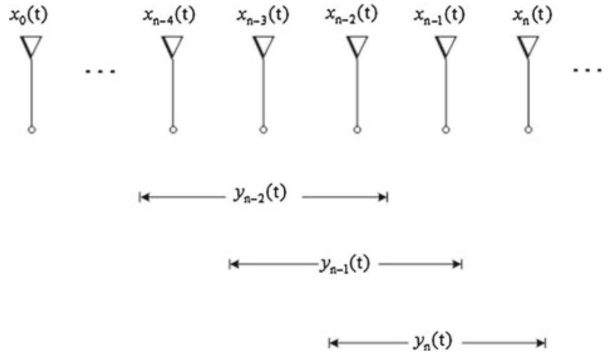
[2] Wen, Fuxi, Boon Poh Ng, and Vinod Veera Reddy. "Extending the concept of IIR filtering to array processing using approximate spatial IIR structure." *Multidimensional Systems and Signal Processing* 24.1 (2013): 157-179.

Related work

In [2], two methods have been proposed.

- **Treating the array as a set of consecutive overlapping sub arrays, where each of the sub arrays produces a time shifted instance of the output which are weighted and summed to generate the approximated recursive response.**

    - A truncated version of the IIR - not infinite.
    - Can be replaced by a mere FIR.



[2] Wen, Fuxi, Boon Poh Ng, and Vinod Veera Reddy. "Extending the concept of IIR filtering to array processing using approximate spatial IIR structure." *Multidimensional Systems and Signal Processing* 24.1 (2013): 157-179.

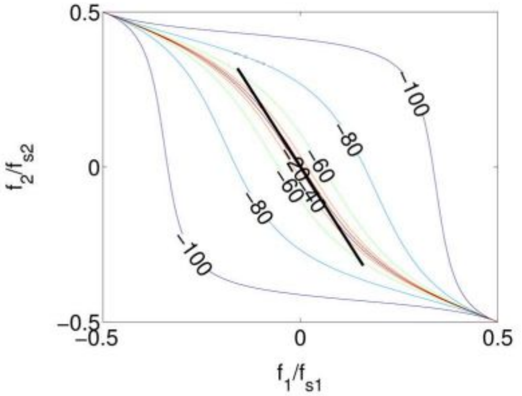
Related work

- **Ultra wideband beamforming [3] relies on the spatio-temporal straight (and tilted according to DOA) line representation of impinging plane wave.**

- **Then a 2D spatio-temporal filter is designed to enhance those straight tilted lines.**

    - Also involves temporal processing

    - Filter response “bends” close to the sampling frequencies (both temporal and spatial) - unwanted signals are enhanced.



[3] Hum, S. V., H. L. P. A. Madanayake, and L. T. Bruton. "UWB beamforming using 2-D beam digital filters." *IEEE Transactions on Antennas and Propagation* 57.3 (2009): 804-807.

Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

In this section:

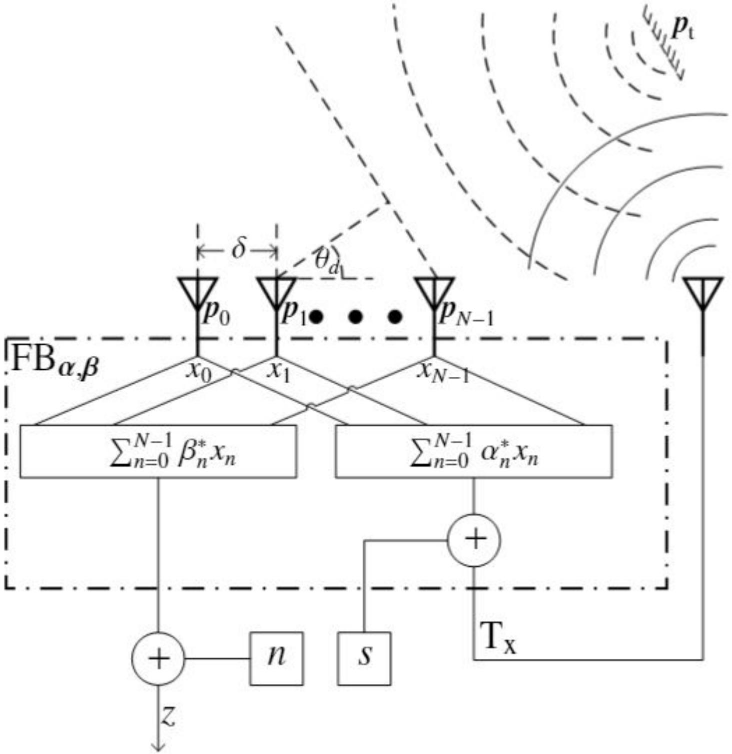
- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

- The proposed feedback beamformer
  - Architecture & analysis
  - Array configuration
  - Stability
  - Performance
  - Identifying the range estimation sensitivity.

- Overcoming the sensitivity
  - The dual frequency architecture
  - Analysis
  - Configuration
  - Simulation results
  - Example

Proposed solution – Setup

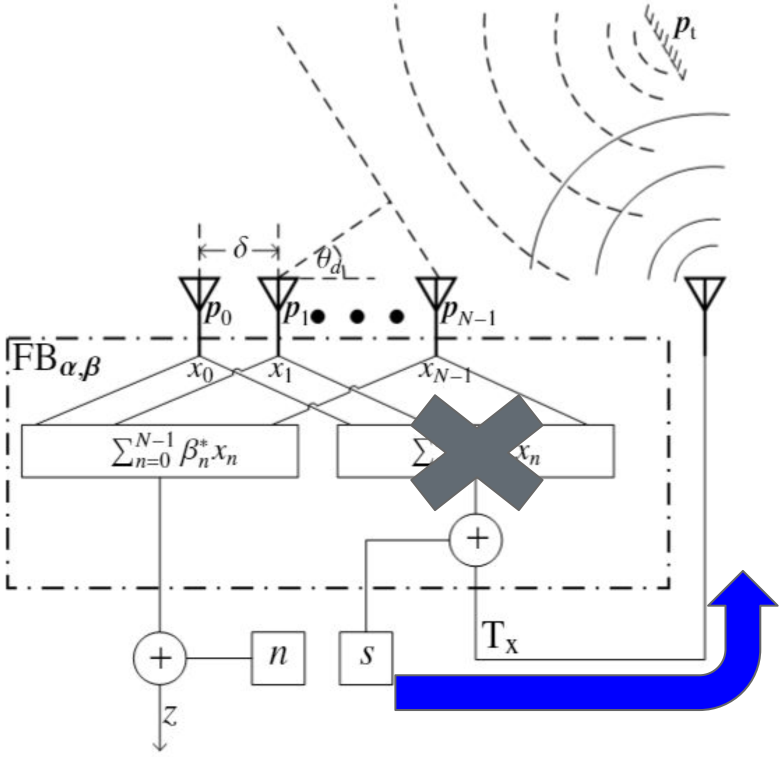
- ULA
- Adding a transmitter
- Target acts as a reflector
- Far field
- Simplifications:
    - Single stationary target (in p_t)
    - Planar (2D) problem
    - CW signal



[4] Woodbury, Max A. *Inverting modified matrices*. Statistical Research Group, 1950.

RADAR – visualization

1

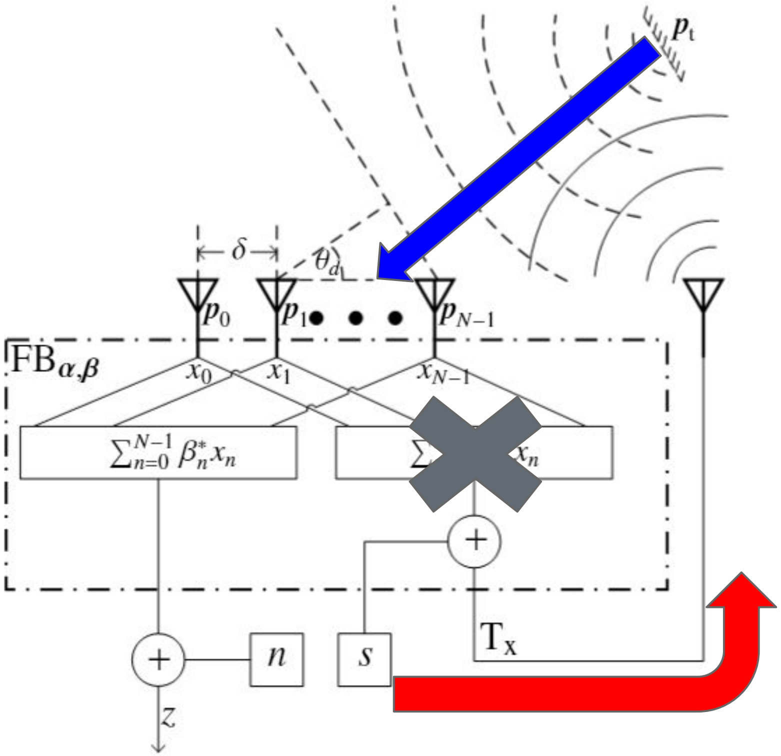


Typically signal is pulse based.

RADAR – visualization

1

2

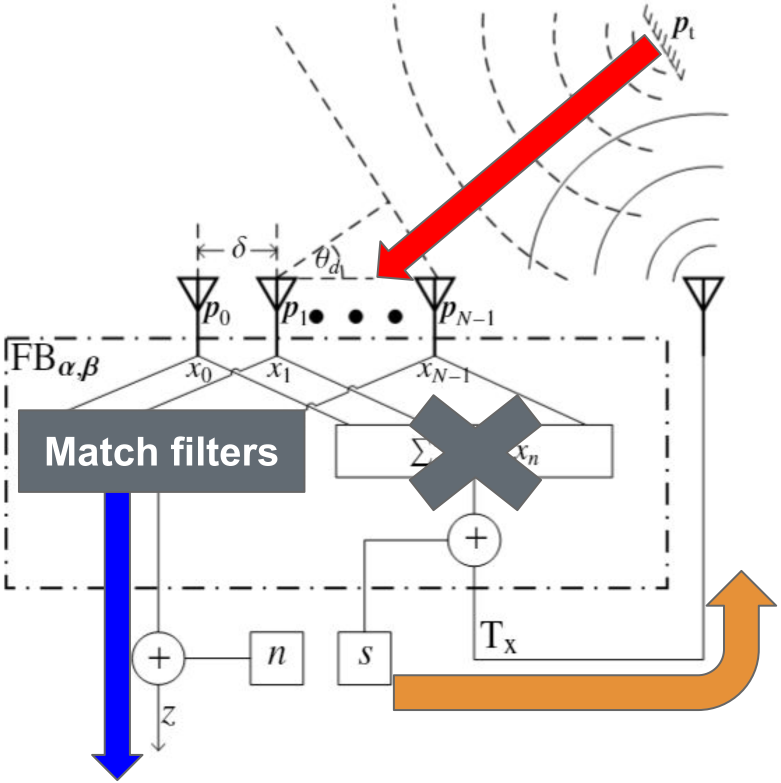


RADAR - visualization

1 *

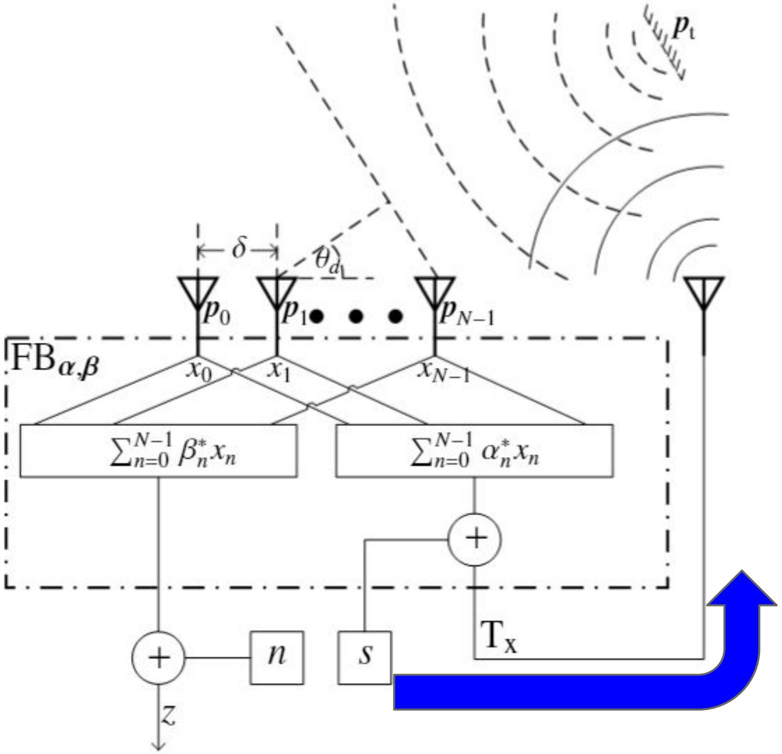
2

3



Spatial loop – visualization

1

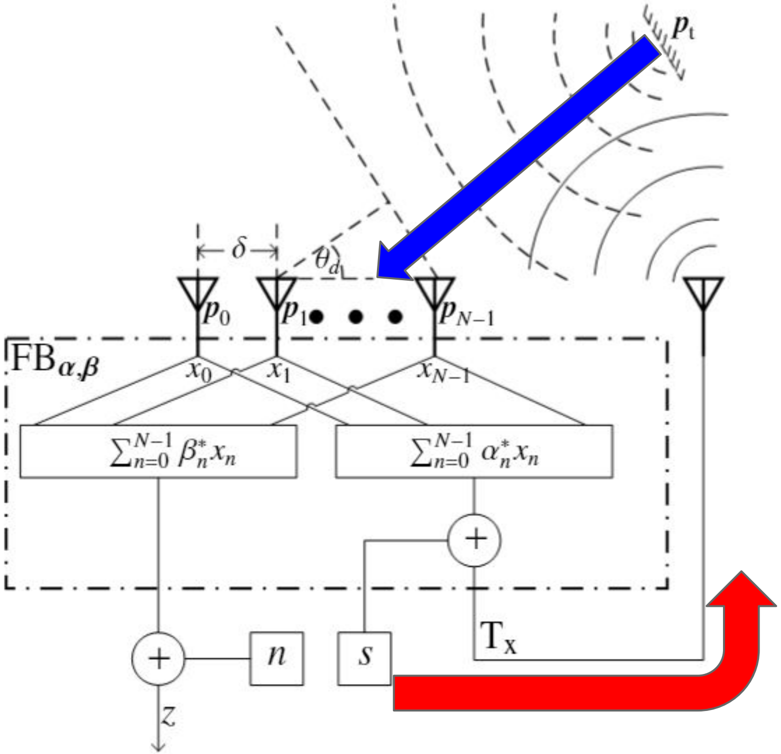


Signal is CW

Spatial loop – visualization

1

2

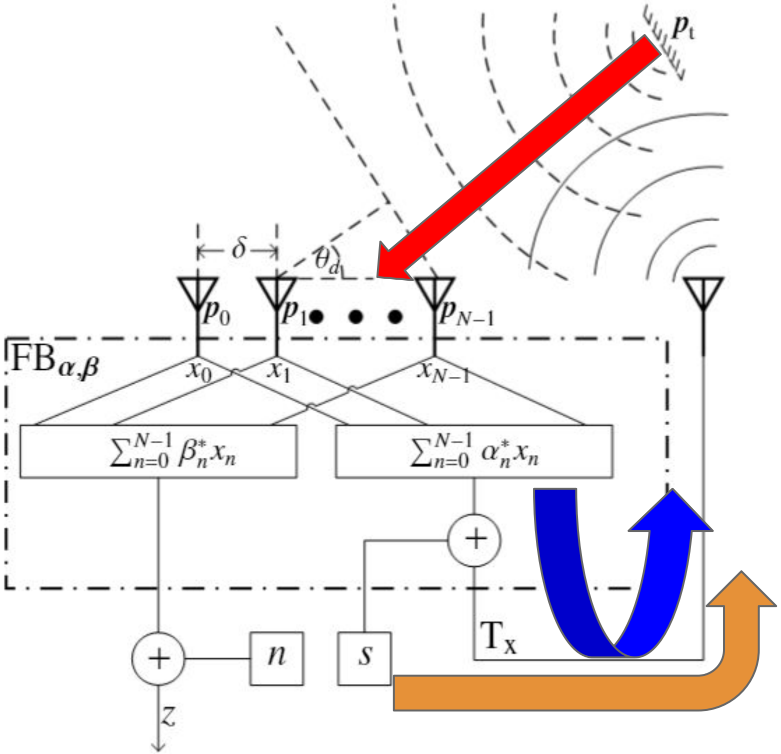


Spatial loop – visualization

1 *

2

3



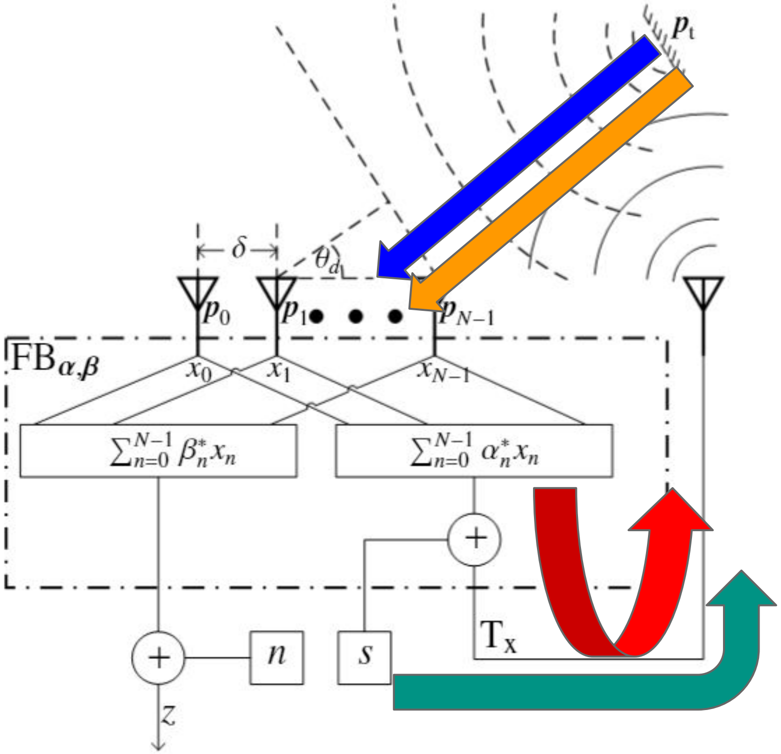
Spatial loop – visualization

1 *

2 *

3

4



Spatial loop – visualization

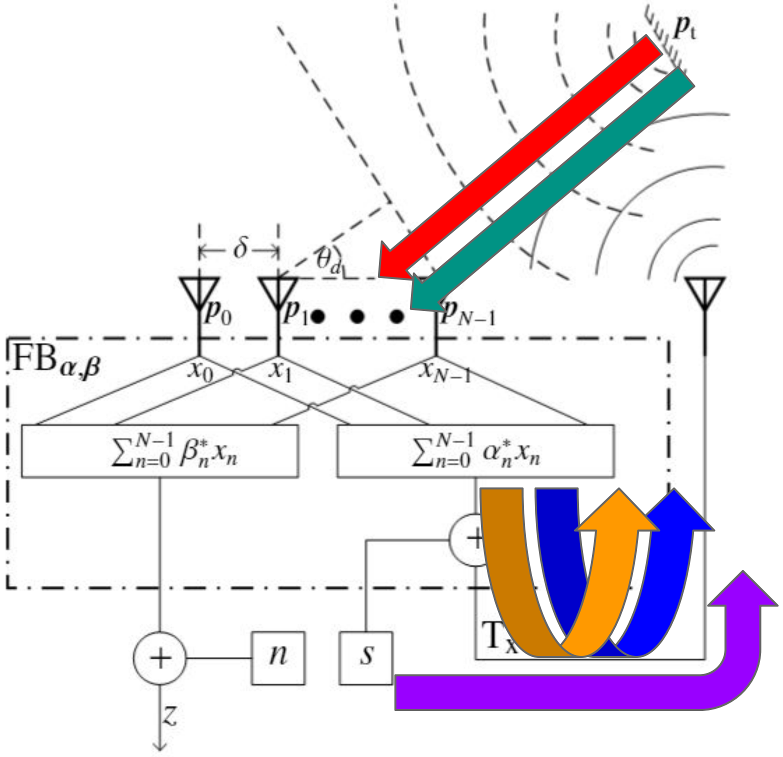
1 **

2 *

3*

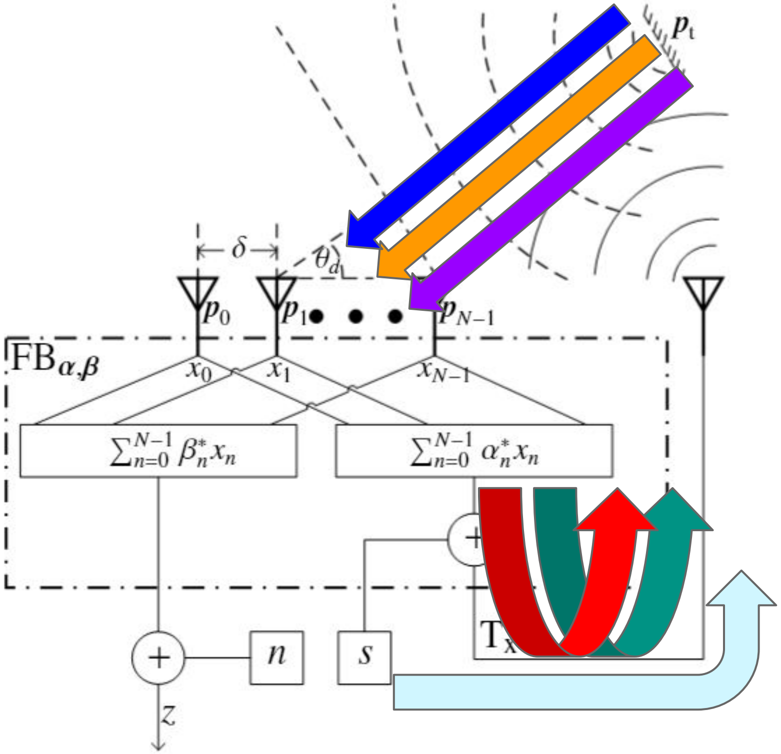
4

5



Spatial loop – visualization

1 **
2 **
3*
4*
5
6



Notations

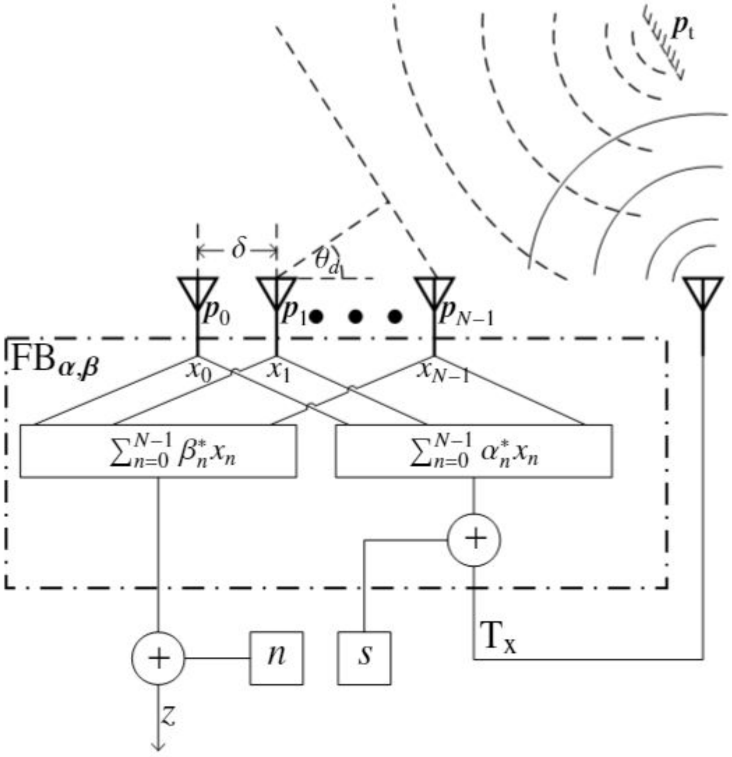
- d - steering vector

- g - Propagation related signal gain

  (assuming gain&delay only channel - considering narrowband CW signals)

- τ_{pd} - Propagation delay

- $\boldsymbol{\phi}$ - Round trip related signal phase $\phi \triangleq \omega\tau_{pd}$



System response

$$x_n(t) = g\left(s\left(t-\tau_{pd}-\tau_n\right) + \sum_{m=0}^{N-1}\alpha_m^* x_m\left(t-\tau_{pd}-\tau_n\right)\right)$$

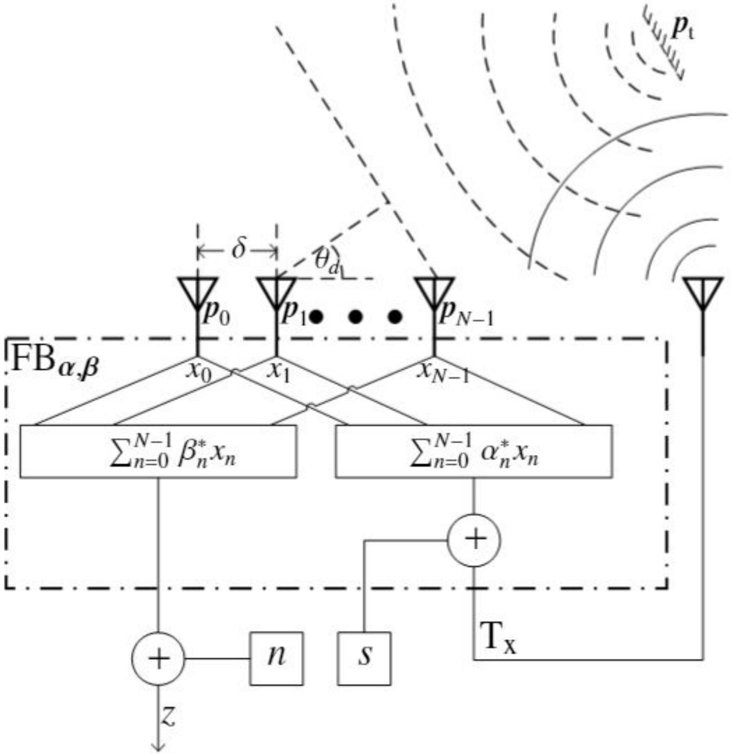
$$\boldsymbol{X} = g\left(\boldsymbol{I} - g\boldsymbol{d}\boldsymbol{\alpha}^H e^{-j\omega\tau_{pd}}\right)^{-1}\boldsymbol{d}S\exp\left(-j\omega\tau_{pd}\right).$$

[4]

$$= \frac{\boldsymbol{d} + \left(\boldsymbol{d}\boldsymbol{\alpha}^H\boldsymbol{d} - \boldsymbol{\alpha}^H\boldsymbol{d}\boldsymbol{d}\right)g\exp(-j\omega\tau)}{1-g\boldsymbol{\alpha}^H\boldsymbol{d}\exp(-j\omega\tau)}$$

$$= \frac{\boldsymbol{d}}{1-g\boldsymbol{\alpha}^H\boldsymbol{d}\exp(-j\omega\tau)}$$

IIR

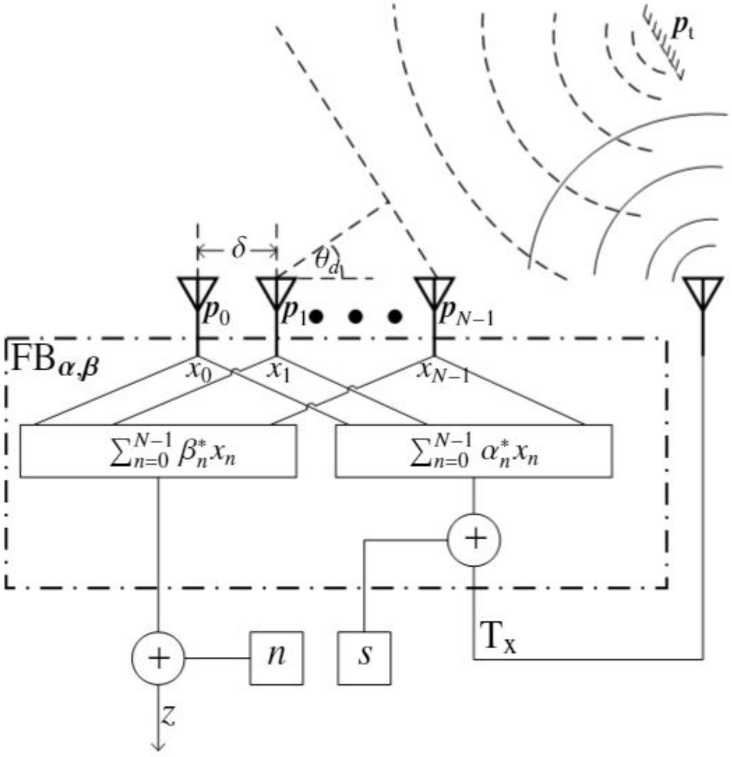


[4] Woodbury, Max A. *Inverting modified matrices*. Statistical Research Group, 1950.

Proposed solution – Setup

$$H_{\beta,\alpha} \triangleq \frac{Z}{S} = \frac{g\boldsymbol{\beta}^H \boldsymbol{d} \exp(-j\phi)}{1 - g\boldsymbol{\alpha}^H \boldsymbol{d} \exp(-j\phi)}$$

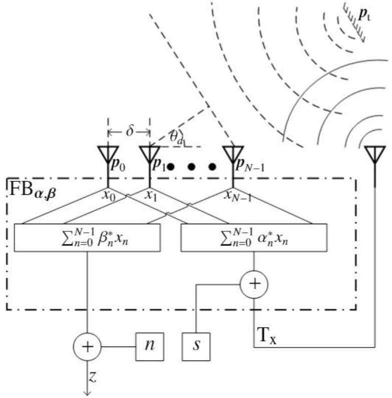
- Controllable denominator - (IIR)
- Frequency dependency suppressed
  - Gain
  - Steering vector
  - Propagation phase
- Range dependent ($\boldsymbol{\phi}$)



Fisher information matrix (FIM)

- To choose the coefficients, we use the Fisher information matrix (FIM)

- The computation of the FIM is done in the frequency domain, following the steps of [5] with the parameter vector η=[θ,ϕ]



$$J_{k,l}(\boldsymbol{\eta}) = \Re\left\{\frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2}\frac{1}{\Phi(\omega)}\mathfrak{F}^*\left\{\frac{\partial z(t)}{\partial \eta_k}\right\}\mathfrak{F}\left\{\frac{\partial z(t)}{\partial \eta_l}\right\}d\omega\right\} + \frac{T}{4\pi}\int_{-\omega_s/2}^{\omega_s/2}\frac{1}{\Phi^2(\omega)}\left(\frac{\partial \Phi(\omega)}{\partial \eta_k}\right)^*\frac{\partial \Phi(\omega)}{\partial \eta_l}d\omega$$

[5] Zeira, Ariela, and Arye Nehorai. "Frequency domain Cramer-Rao bound for Gaussian processes." *IEEE transactions on acoustics, speech, and signal processing* 38.6 (1990): 1063-1066.

Coefficient setting

$$J_{\theta_d\phi} = J^*_{\phi\theta} = \Re\left\{ \frac{1}{2\pi\sigma^2} \int_{-\omega_s/2}^{\omega_s/2} \frac{jg^2 \boldsymbol{\beta}^T \boldsymbol{d}^* \boldsymbol{\beta}^H \left(\boldsymbol{A}\boldsymbol{d} + g\boldsymbol{B}\boldsymbol{\alpha}^* \exp(-j\phi)\right)}{\left|1 - g\boldsymbol{\alpha}^H \boldsymbol{d} \exp(-j\phi)\right|^4} |\mathrm{S}|^2 \, d\omega \right\}$$

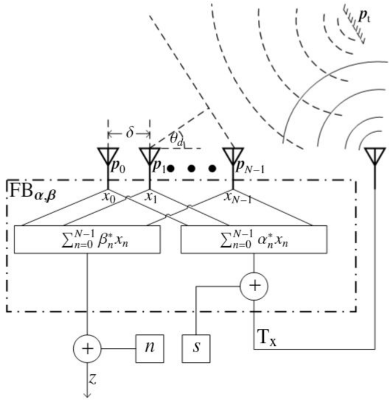
Odd argument over symmetric range = 0

$$J_{\theta_d\theta_d} = \frac{1}{2\pi\sigma^2} \int_{-\omega_s/2}^{\omega_s/2} \frac{\left|g\boldsymbol{\beta}^H \boldsymbol{A}\boldsymbol{d} - g^2 \boldsymbol{\beta}^H \boldsymbol{B}\boldsymbol{\alpha}^* \exp(-j\phi)\right|^2}{\underline{\left|1 - g\boldsymbol{\alpha}^H \boldsymbol{d} \exp(-j\phi)\right|^4}} |\mathrm{S}|^2 \, d\omega$$

$$J_{\phi\phi} = \frac{1}{2\pi\sigma^2} \int_{-\omega_s/2}^{\omega_s/2} \frac{\left|g\boldsymbol{\beta}^H \boldsymbol{d}\right|^2}{\underline{\left|1 - g\boldsymbol{\alpha}^H \boldsymbol{d} \exp(-j\phi)\right|^4}} |\mathrm{S}|^2 \, d\omega$$

Minimizing the response's denominator maximizes the residing spatial information in the system.

$$1 - g\boldsymbol{\alpha}^H \boldsymbol{d} \exp(-j\phi)$$

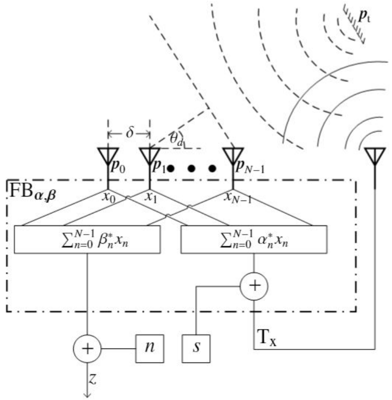


Coefficient setting

- With the FIM, we find that a generalization of the conventional beamformer (CB) maximizes (locally) the residing information.

$$\alpha^*_{\mathrm{CB,opt}} = \frac{\boldsymbol{d}^* \exp(j\phi)}{\hat{g}\,\|\boldsymbol{d}\|^2} \qquad \beta_{\mathrm{CB,opt}} = \alpha_{\mathrm{CB,opt}}.$$

- $\hat{g}$ is the signal's estimated propagation related gain



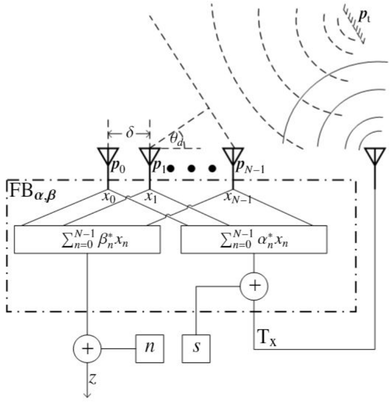
Error terms

Spatial performance

- $r = g/\hat{g}$ **Gain estimation mismatch**

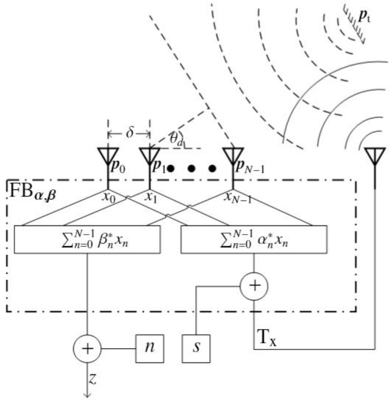
- **$\Delta\theta$ - DOA estimation phase mismatch**

- **$\Delta\phi$ - Range estimation phase mismatch**



Analysis scenarios

$$
\begin{array}{ll}
\text{Perfect alignment} & (\Delta\theta = 0 \ , \Delta\phi = 0)\,, \\
\text{Steering error} & (|\Delta\theta| > 0 \ , \Delta\phi = 0)\,, \\
\text{Range error} & (\Delta\theta = 0 \ , |\Delta\phi| > 0)\,, \\
\text{General} & (|\Delta\theta| > 0 \ , |\Delta\phi| > 0)\,.
\end{array}
$$

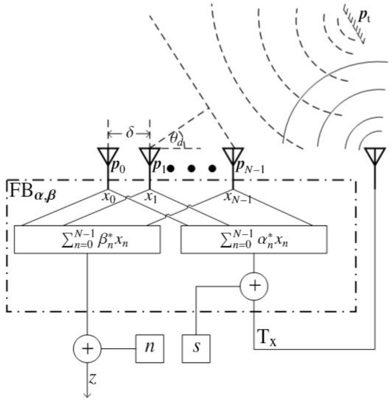


- In all scenarios, the gain error exists (r<1)

Stability

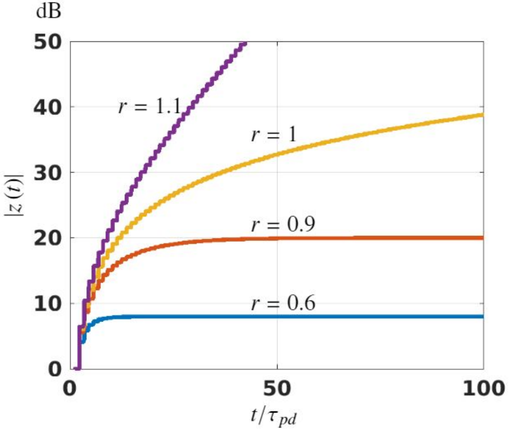
- Assuming “**ideal scenario**” & Applying the **CB generalization**

$$H_{\beta_{\mathrm{CB,opt}},\alpha_{\mathrm{CB,opt}}} = \frac{r}{1-r}$$



- Stability is for r<1.

- For r>1, the system is not stable.



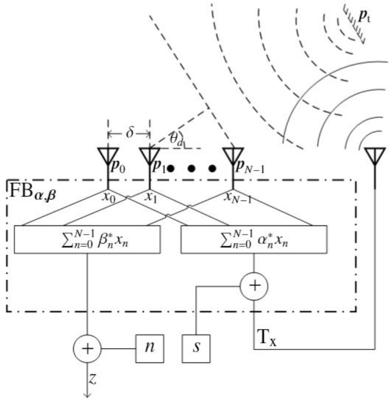
Steering error – i.e. $\Delta\phi=0$

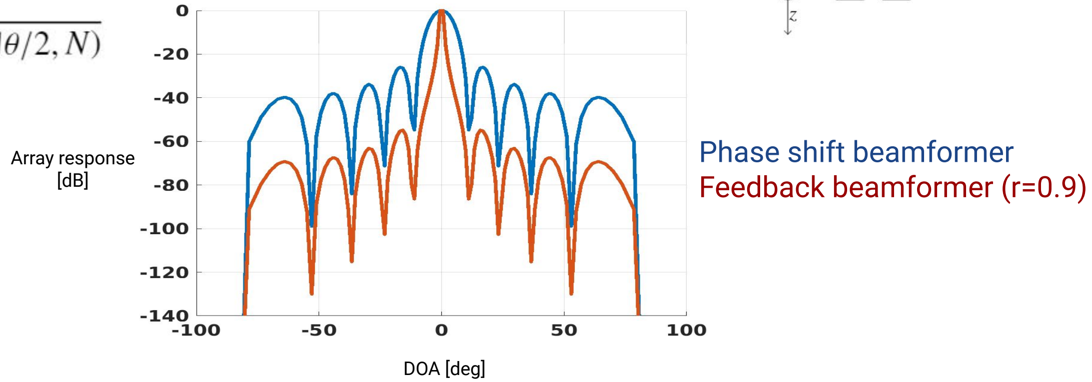
- **Normalized Dirichlet kernel**

$$\mathrm{D}(x, N) = \frac{1}{N}\frac{\sin(Nx)}{\sin(x)}$$

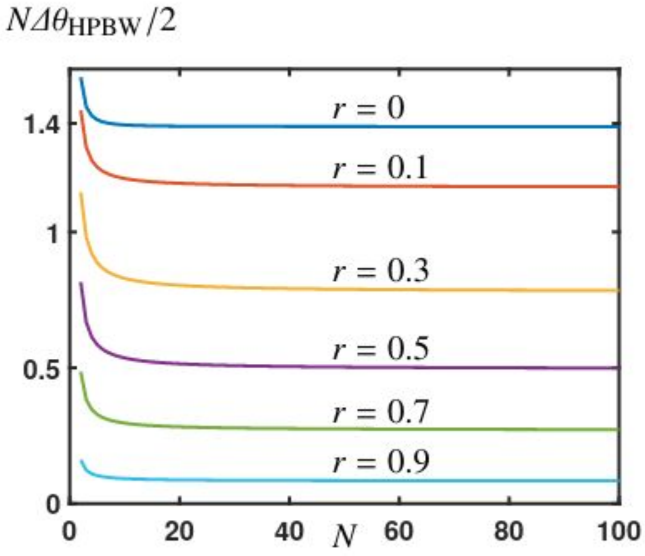
- **Normalized beampattern - (mainlobe gain = 1)**

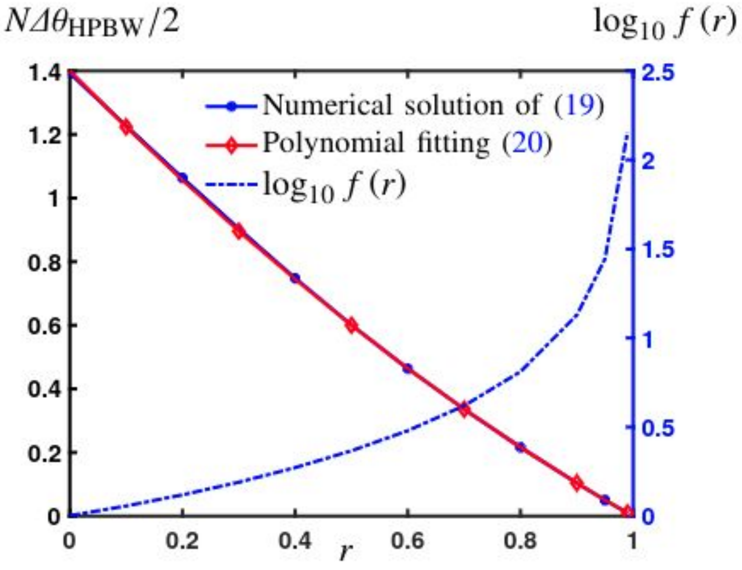
$$\mathcal{H} = \frac{(1-r)\,\mathrm{D}(\Delta\theta/2, N)}{\exp(j((N-1)\Delta\theta/2)) - r\mathrm{D}(\Delta\theta/2, N)}$$

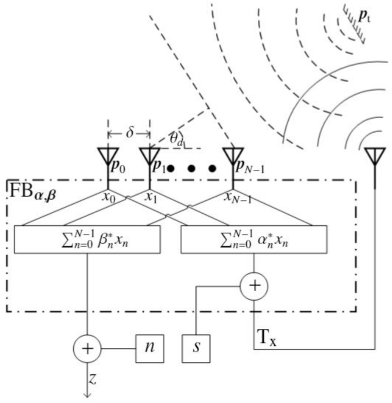




Performance – HPBW





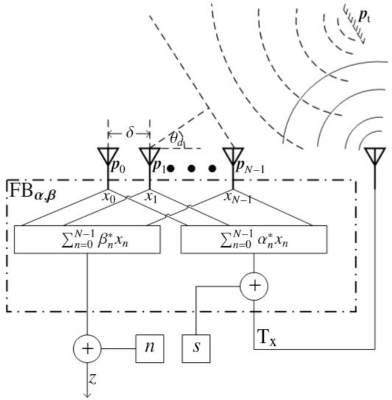


known [1] result

$$\Delta\theta_{\mathrm{HPBW}}/2 = 1.4/N$$

Performance – HPBW

$$\Delta\theta_{\mathrm{HPBW}}/2 \approx \frac{1.4}{f(r)N} \qquad f(r) \approx \frac{1.4}{(1-r)(-0.4r+1.4)}$$



- Aperture virtual increase
- Governed by r
- Infinite improvement for r->1

Proposed solution – Performance

Sidelobes gain

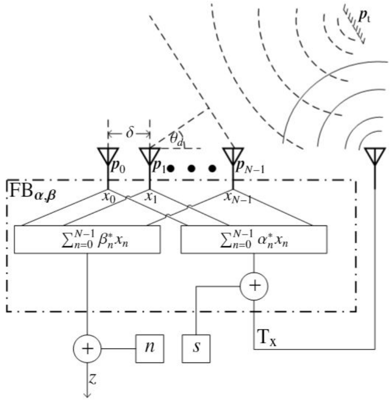
- The known [1] result of the conventional beamformer is

$$2/3\pi$$

- In feedback beamformer:

$$\frac{2(1-r)}{3\pi}$$

  - r -> 1 : Sidelobes vanish



Proposed solution – Performance

Directivity

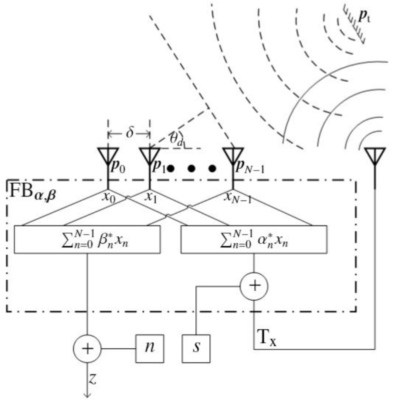
- The known result for the conventional beamformer is D = N

- Plugging the feedback beamformer's response into the directivity's definition gives

$$\mathcal{D}(N,r) = \frac{\mathcal{H}_{\Delta\theta=0,\Delta\phi=0,r}}{\frac{1}{2\pi}\int_0^{2\pi}\mathcal{H}_{\Delta\theta,\Delta\phi=0,r}\,d\Delta\theta} = \frac{2\pi}{\int_0^{2\pi}\mathcal{H}_{\Delta\theta,\Delta\phi=0,r}\,d\Delta\theta}$$

- In search of rational expressions for the directivity, we found that it follows:

$$\mathcal{D}(N,r) \approx \frac{N-r}{1-r}$$



As r tends to 1, the **directivity goes to infinity**

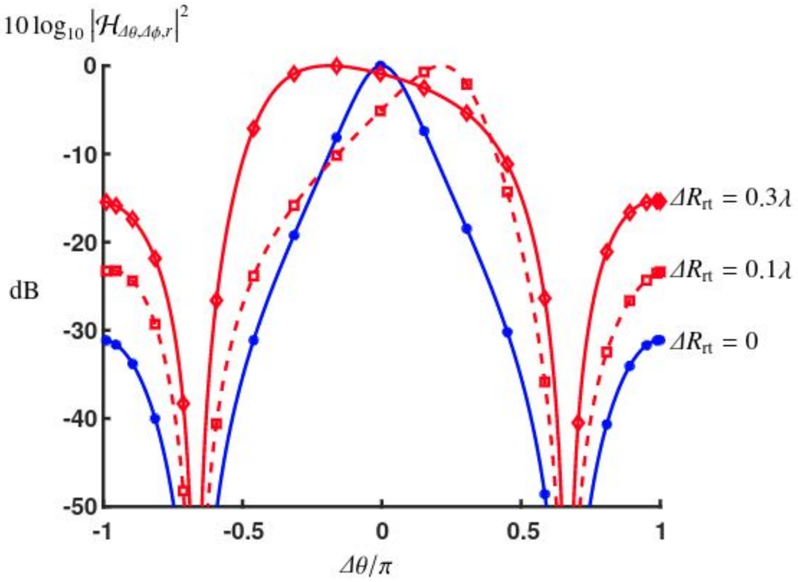
Performance summary

$$f(r) \approx \frac{1.4}{(1-r)(-0.4r+1.4)}$$

	ULA	FEEDBACK BEAMFORMING	IMPROVEMENT
HPBW	$1.4/N$	$1.4/(f(r)N)$	Narrower by a factor of $f(r)$
FIRST SIDELOBE GAIN	$2/3\pi$	$2(1-r)/3\pi$	smaller by a factor of $1-r$ for $N \gg 1$
DIRECTIVITY	N	$(N-r)/(1-r)$	$1/(1-r)$ times higher for $N \gg 1$

Proposed solution - Range estimation sensitivity

- Range estimation error $\Delta R_{\mathrm{rt}} = \dfrac{\Delta\phi\lambda}{2\pi}$
- Even mild errors of O(λ) dramatically distort the beampattern.
- The sensitivity is 2π periodic
- The sensitivity increases as r tends towards 1.



In this section:

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

- The proposed feedback beamformer
  - Architecture & analysis
  - Array configuration
  - Stability
  - Performance
  - Identifying the range estimation sensitivity.

- Overcoming the sensitivity
  - The dual frequency architecture
  - Analysis
  - Configuration
  - Simulation results
  - Example

So how do we overcome this sensitivity?

So how do we overcome this sensitivity?

What's the problem?
Get perfect range estimation.

So how do we overcome this sensitivity?

What's the problem?
Get perfect range estimation.

Lets also know the DOA...

So how do we overcome this sensitivity?

What's the problem?
Get perfect range estimation.

Lets also know the DOA...

Lets use low-frequency carrier

So how do we overcome this sensitivity?

What's the problem?
Get perfect range estimation.

Lets also know the DOA...

Lets use low-frequency carrier

Antenna’s length should be O(λ)
For tolerating errors of O(m), the antennas should also be O[m].

So how do we overcome this sensitivity?

What's the problem?
Get perfect range estimation.

Lets also know the DOA...

Lets use low-frequency carrier

Antenna’s length should be $O(\lambda)$
For tolerating errors of O(m), the antennas should also be O[m].

Maybe, use two close harmonics.

So how do we overcome this sensitivity?

What's the problem?
Get perfect range estimation.

Lets also know the DOA...

Lets use low-frequency carrier

Antenna’s length should be O(λ)
For tolerating errors of O(m), the antennas should also be O[m].

Maybe, use two close harmonics...

But how?

Intuition – two close harmonics

The Dual Frequency feedback BeamFormer - DFBF

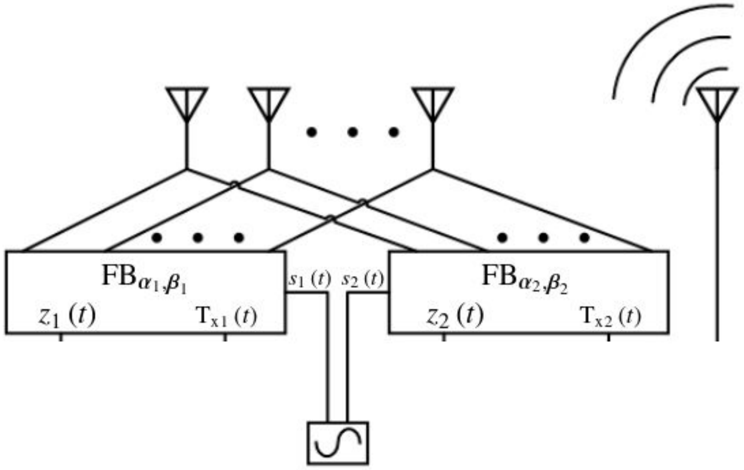
- Two independent single frequency feedback beamformers.

How to cancel cross interference?

How to combine the two output signals?

How to extract spatial data from the combined signal?

How to set the coefficients?



The Dual Frequency feedback BeamFormer - DFBF

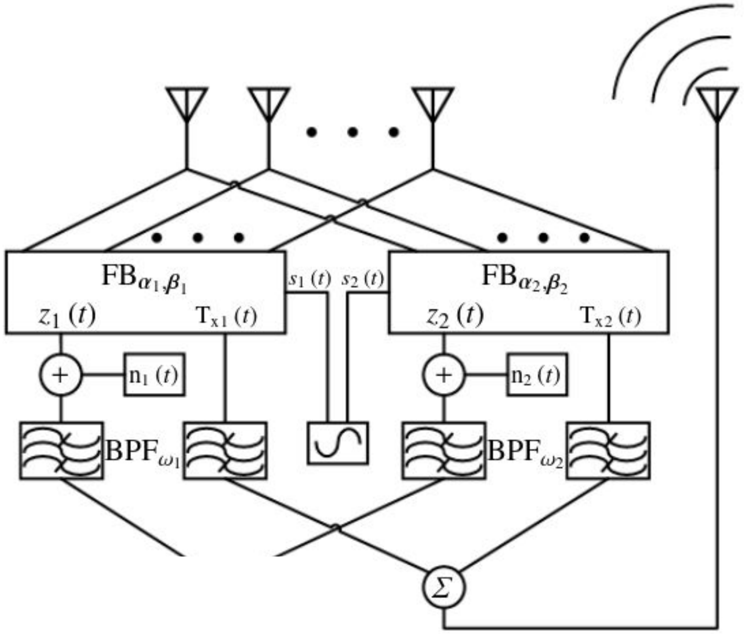
- Two independent single frequency feedback beamformers.

- Using band pass filters (BPF) to prevent cross interference.

How to combine the two output signals?

How to extract spatial data from the combined signal?

How to set the coefficients?

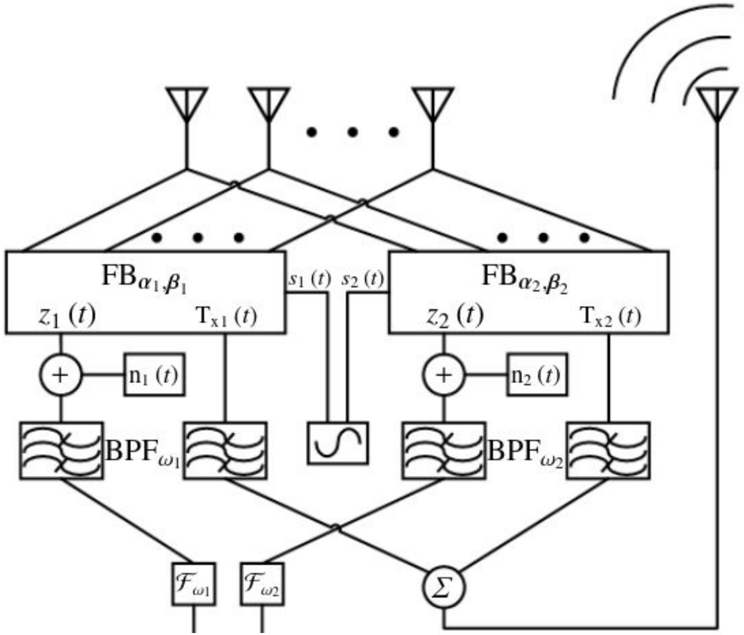


The Dual Frequency feedback BeamFormer - DFBF

- Two independent single frequency feedback beamformers.
- Using band pass filters (BPF) to prevent cross interference.
- Implementing fast Fourier transform (FFT) blocks As combining scalars is more convenient.


How to extract spatial data from the combined signal?

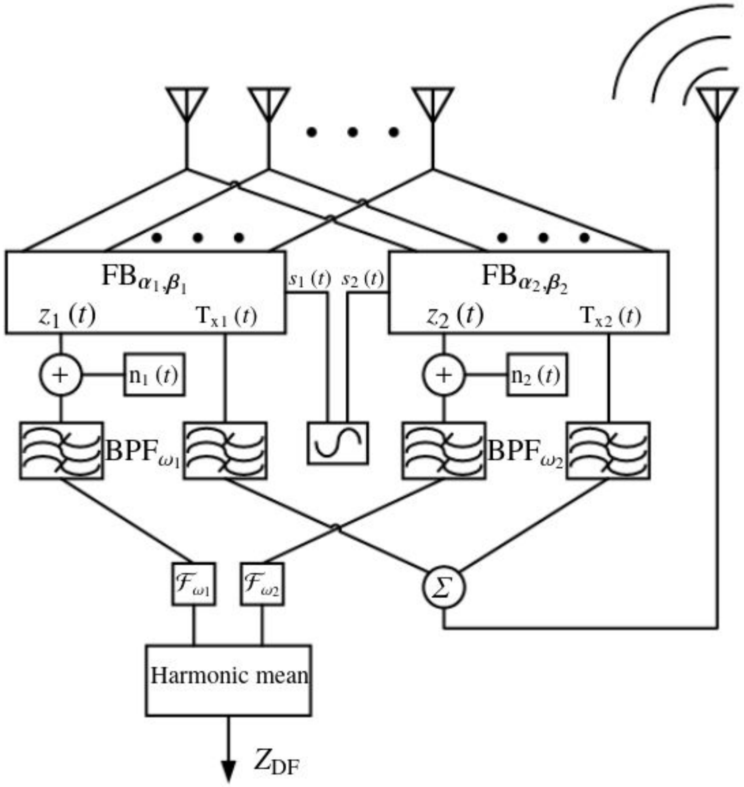

How to set the coefficients?



The Dual Frequency feedback BeamFormer - DFBF

- Two independent single frequency feedback beamformers.
- Using band pass filters (BPF) to prevent cross interference.
- Implementing fast Fourier transform (FFT) blocks
  As combining scalars is more convenient.
- The two independent FFT outputs serve as input to a harmonic mean block.

How to set the coefficients?



DFBF – Setup

How to set the coefficients?

$$Z_{\mathrm{DF}}^{-1} = \left| \frac{1 - g_1 \boldsymbol{\alpha}_1^H \boldsymbol{d}_1 e^{-j\phi_1}}{g_1 \boldsymbol{\beta}_1^H \boldsymbol{d}_1 e^{-j\phi_1}} + \frac{1 - g_2 \boldsymbol{\alpha}_2^H \boldsymbol{d}_2 e^{-j\phi_2}}{g_2 \boldsymbol{\beta}_2^T \boldsymbol{d}_2 e^{-j\phi_2}} \right|$$

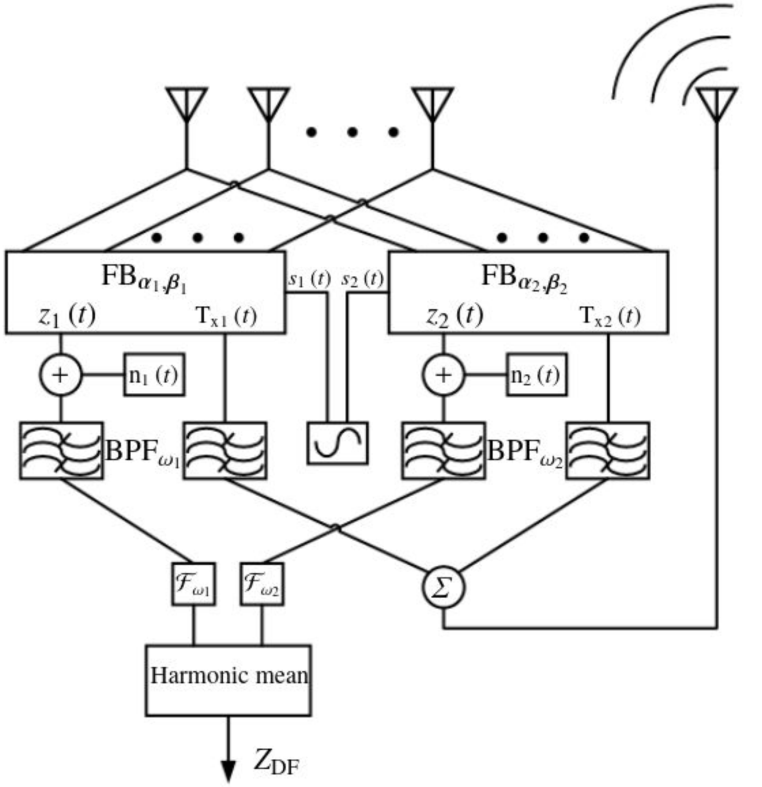
$$\boldsymbol{\alpha}_1 = \boldsymbol{\beta}_1, \ \boldsymbol{\alpha}_2 = -\boldsymbol{\beta}_2 = [1/\hat{g}_2, 0, \ldots, 0] \quad \Rightarrow \quad Z_{\mathrm{DF}} = \left| \frac{g_1 \boldsymbol{\beta}_1^H \boldsymbol{d}_1}{1 - \left(g_1 \boldsymbol{\beta}_1^H \boldsymbol{d}_1 / r_2 \right) \exp\left(-j\left(\phi_1 - \phi_2\right)\right)} \right|$$

DFBF – Setup

The final result:

$$Z_{DF} = \left| \frac{g_1 \boldsymbol{\beta}_1^H \boldsymbol{d}_1}{1 - \left(g_1 \boldsymbol{\beta}_1^H \boldsymbol{d}_1 / r_2\right) \exp\left(-j\left(\phi_1 - \phi_2\right)\right)} \right|$$

- Resembles the single frequency (SF) feedback beamformer.
- Range error sensitivity is linked to the diff between the two frequencies.



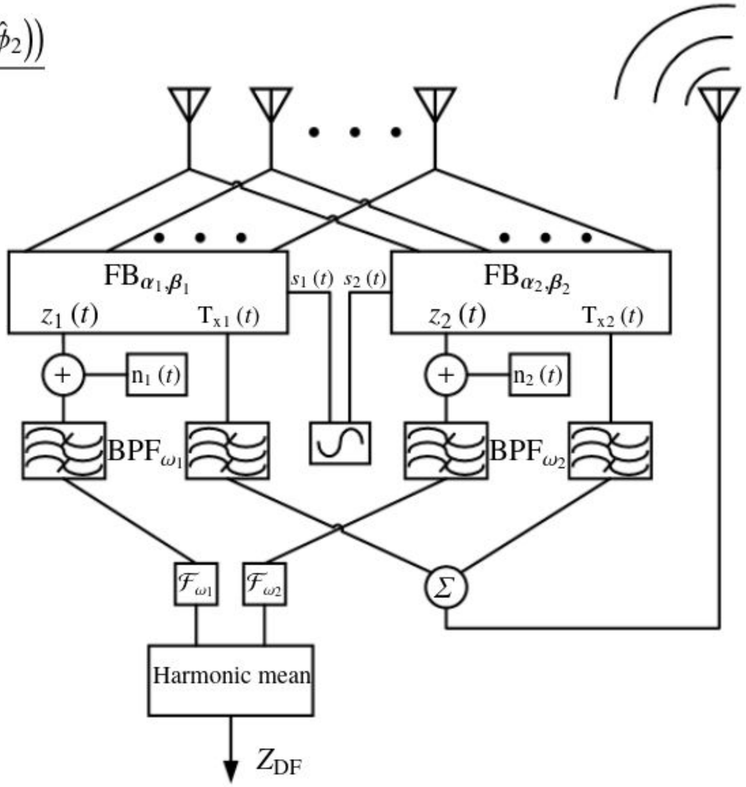
DFBF – Coefficients

Generalizing the CB approach, we set $\beta_1^* = \frac{-\hat{\mathbf{d}}_1^* \exp\left(j\left(\hat{\phi}_1 - \hat{\phi}_2\right)\right)}{\hat{g}_1 \left\|\hat{\mathbf{d}}_1\right\|^2}$

hence:

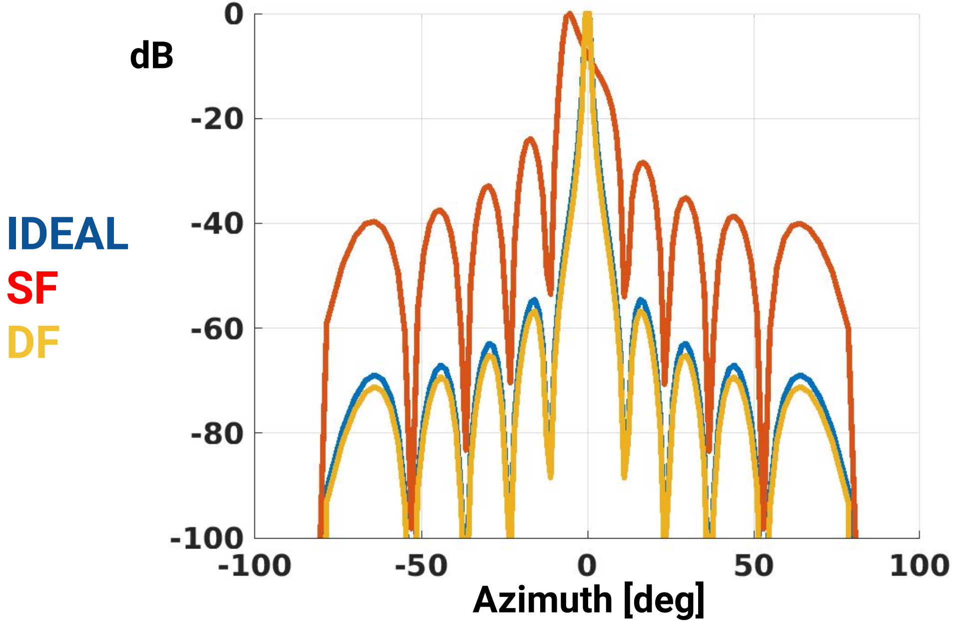
$$H_{\mathrm{DF,CB}}(\omega) = \left| \frac{r_1 \mathrm{D}(\Delta\theta/2, N)}{1 - \kappa \mathrm{D}(\Delta\theta/2, N) \exp\left(-j\left(\phi_2 - \phi_1 + (N-1)\Delta\theta/2\right)\right)} \right|$$

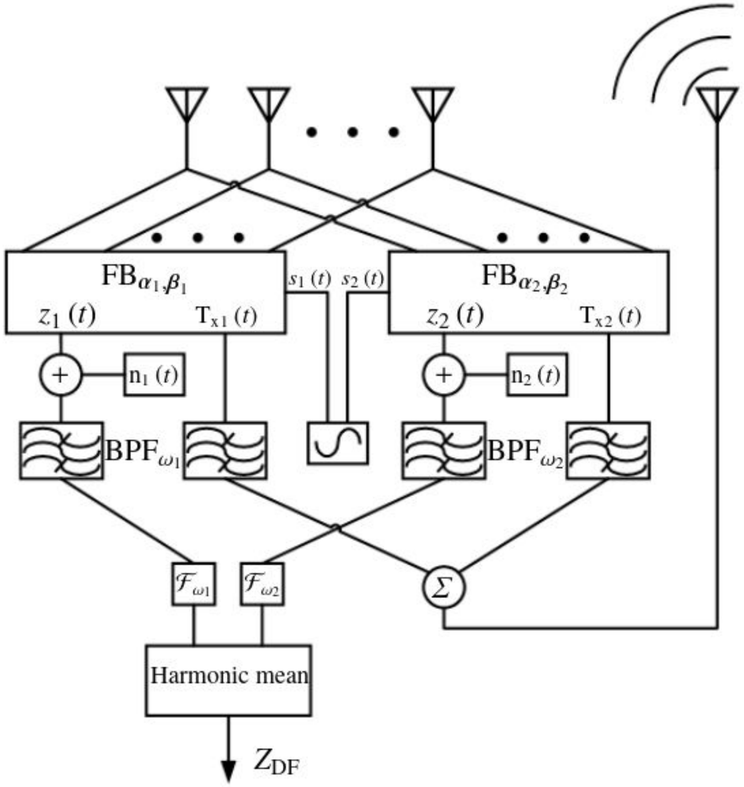
- $\kappa \triangleq r_1/r_2$ acts as the r in the SF version.
- As the carrier frequencies are close, r1 and r2 are similar - therefore κ tends to 1.
- By deliberately modifying r1, the designer may achieve any desired value of κ - thus controlling the beamwidth.



DFBF - Range estimation sensitivity assessment

- Comparing to SF - range error of ~333.3λ
- We set κ = 0.9





DFBF - Numerical example

- Carrier frequency - 10[GHz]
- Range estimation error - 10[meters]
- Maximum allowed phase shift - 0.01π

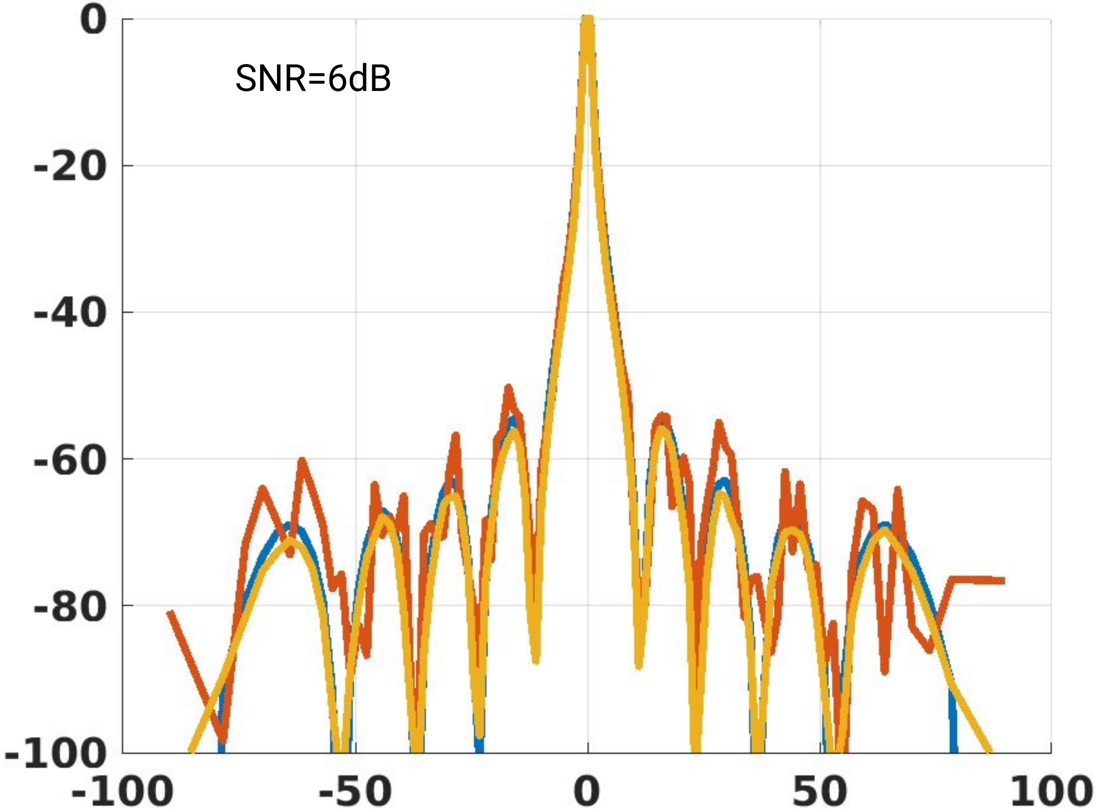
$$\left|(\omega_1 - \omega_2)\frac{\Delta R_{\mathrm{rt}}}{c}\right| < 0.01\pi.$$

$$|f_1 - f_2| < 0.005c/\Delta R_{\mathrm{rt}} = 150\,\mathrm{kHz}$$

DFBF – Low SNR performance

- Compare to SF
- No range estimation error
- $\kappa = 0.9$

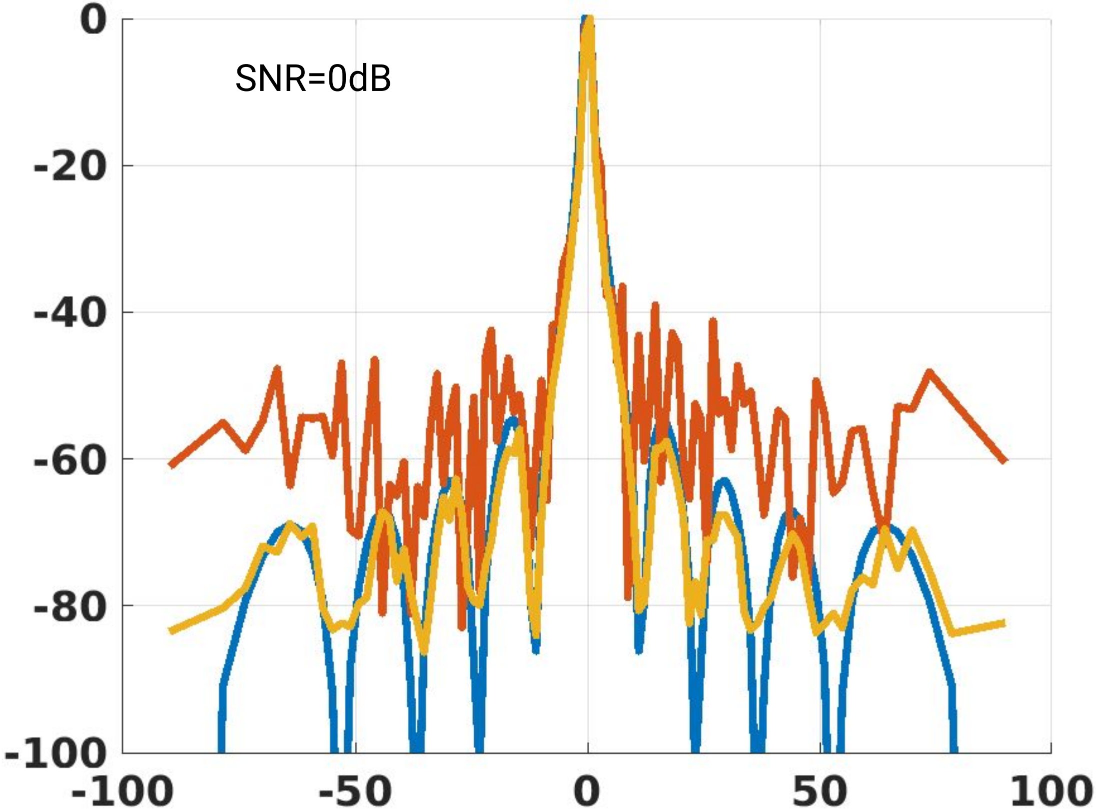
IDEAL
SF
DF



DFBF – Low SNR performance

- Compare to SF
- No range estimation error
- κ = 0.9

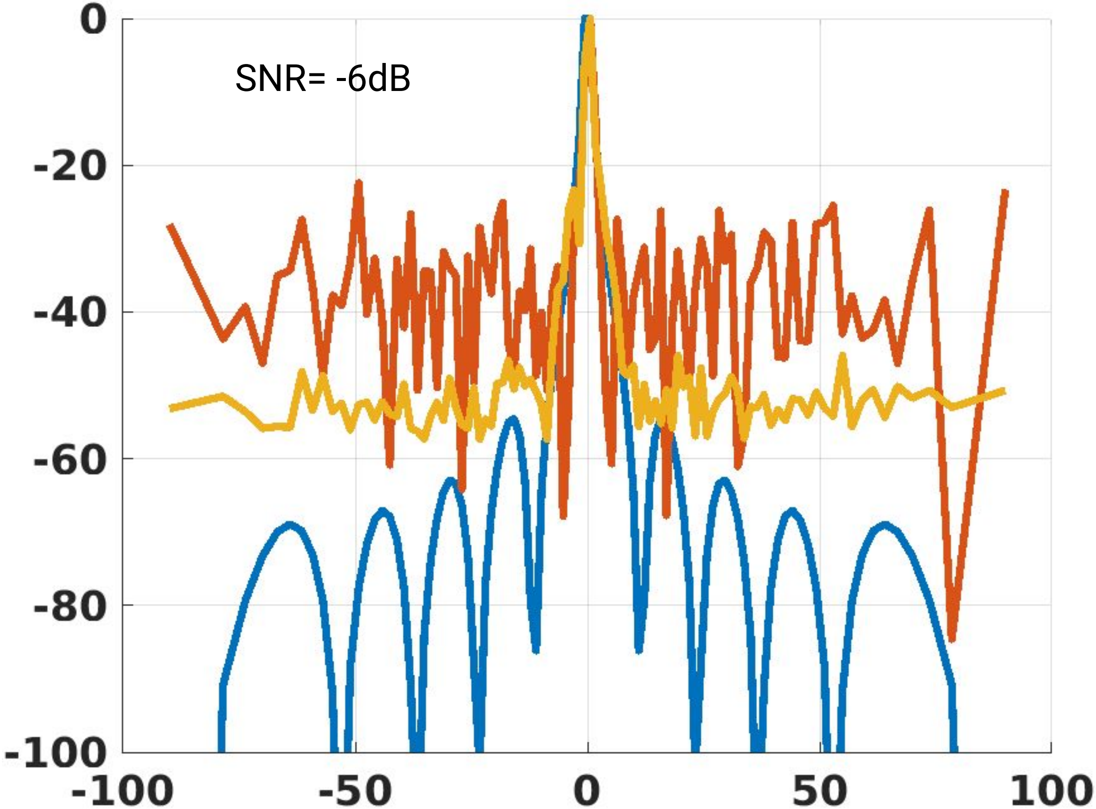
IDEAL
SF
DF



DFBF – Low SNR performance

- Compare to SF
- No range estimation error
- κ = 0.9

IDEAL
SF
DF



Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

Conclusions

- A spatial equivalent to temporal IIR filter has been proposed.
- Considering ideal scenarios, spatial performance is considerably improved.
- High sensitivity to range estimation errors was found.
- A dual-frequency architecture was presented, which mitigates that sensitivity.
- Simulations show that DF architecture also performs well even in low SNR cases.

Agenda

- Background
- Motivation
- Related Work
- Proposed solution
- Conclusions
- Future research

Future research leads

Dynamic targets

- Let the target of interest to move - i.e. $\boldsymbol{p}_t(t)$

- The feedback loop temporal analysis is modified

$$t_i(t) = \left\{ t - 2\tau \,\middle|\, \tau = \frac{R(t-\tau)}{c} \right\} \qquad x_n(t) = g\left(s(t_i(t)) + \sum_{m=0}^{N-1} \alpha_m^* x_m(t_i(t)) \right)$$

- Questions:
  - Doppler shifts -> feedback frequency correction?
  - Stability issues?
  - Pseudo static approximations?

Future research leads

Multiple targets

- Place P targets in the arena.
- The feedback loop temporal analysis is modified

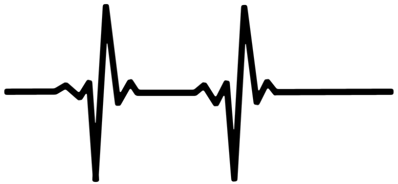
$$x_n(t) = \sum_{i=0}^{i=P-1}\left(g\left(s\left(t-\tau_{pd,i}-\tau_{n,i}\right)+\sum_{m=0}^{N-1}\alpha_m^* x_m\left(t-\tau_{pd,i}-\tau_{n,i}\right)\right)\right)$$

- Questions:
  - Cross interference?
  - Other waveforms?
  - Focused transmission? (requires transmission array)
  - Localization resolution?
  - Maximal detectable targets? Still governed by N?

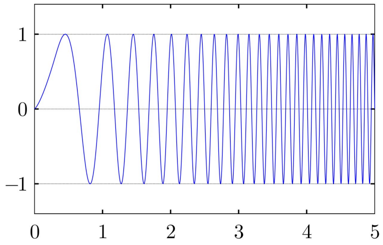
Future research leads

Waveforms modifications

- RADAR systems use pulse based signals
    - Lower duty-cycle -> Lower power consumption



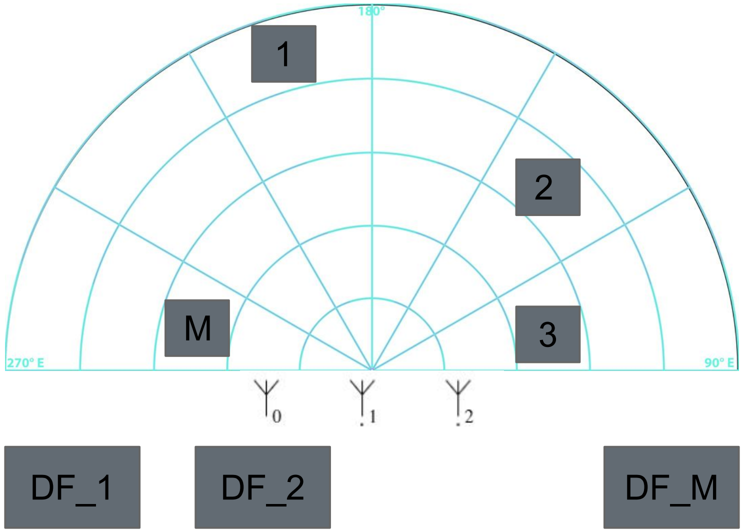
    - Wideband signals (in the chirp case) enhance spatial resolution in radial axis



Future research leads

Waveforms modifications

- Multiple simultaneous DF beamformers - where each covers a certain area in the arena.



Questions?

Thank you.