Multichannel seismic modeling and inversion based on Markov-Bernoulli random field

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SUMMARY

We introduce a multichannel blind deconvolution algorithm for seismic signals based on Markov-Bernoulli random field modeling. The proposed model accounts for layer discontinuities resulting from splitting, merging, starting or terminating layers within the region of interest. We define a set of reflectivity states and legal transitions between reflector configurations of adjacent traces, and subsequently extract sequences of reflectors that are connected across the traces by legal transitions. The improved performance of the proposed algorithm and its robustness to noise, compared to a competitive algorithm, are demonstrated using simulated and real seismic data examples.

INTRODUCTION

Multichannel seismic modeling and inversion is desired for estimating the reflectivity sequences and wavelet from measured noisy traces. Mendel et al. (1981) assume an autoregressive moving-average (ARMA) model and use a maximum likelihood estimator for the reflectivity. Kaaresen and Taxt (1998) assume that the wavelet is of short duration. Santamaria et al. (1999) use a Gaussian mixture model for the reflectivity sequence. While statistical methods generally require a large data set for derivation of a good estimate (Mendel, 1991; Lazear, 1993), sparsity of the reflectivity sequences can be exploited to cope with the ill-posed nature of the basic blind deconvolution problem (Kaaresen and Taxt, 1998; Jeffs, 1998), and to improve the performance of non-blind deconvolution methods (Meilhac et al., 2001). Channel sparsity enables efficient channel estimation, which is suitable for relatively short traces (Kaaresen and Taxt, 1998; Kaaresen, 1998).

Multichannel blind deconvolution (see Tong and Perreau (1998) and references therein, Xu et al. (1995); Luo and Li (1998)) is often more advantageous and more robust than single-channel blind deconvolution. Certain relations between spatially near channels are used to regularize the problem. Lateral continuity of the reflectors across channels has been used in Kaaresen and Taxt (1998) to further improve the channel estimates. Idier and Goussard (1993b) model the 2-D structure of the underground reflectivity as a Markov-Bernoulli random field, and impose lateral continuity to generate deconvolution results that are superior to those obtainable by single-channel deconvolution methods. However, since the parametric models used in these works result in a non-convex optimization problem, a global optimal solution is very difficult to achieve. Usually some sort of constrained search is performed within a group of possible solutions for the locations of reflectors (such as the Single Most Likely Replacement approach in Kormylo and Mendel (1982)), and a typical tradeoff remains between the extent of the search, the computational complexity, and the optimality of the final solution. Lavielle (1991) has modeled the two dimensional reflectivity as a Markov random field, and used Simulated Annealing and a maximum a posteriori probability (MAP) criterion for its estimation.

Recently, we have proposed a deconvolution method that attempts to maximize a MAP criterion using dynamic programming (Heimer et al., 2007; Heimer and Cohen, 2008). A search is performed among continuous paths of reflectors instead of single reflectors, and the best continuous reflector paths are chosen by dynamic programming. We showed that our approach recovers the reflectivity better than the Iterative Windowed Maximization (IWM) algorithm of Kaaresen and Taxt (1998), and particularly its advantage is more significant when the SNR is low. However, our reflectivity model did not take into account layer discontinuities. As a result, the application of our algorithm was limited to areas of mostly continuous layers.

In this paper, which summarizes the results in Heimer and Cohen (2009), we introduce an improved method for estimating the two dimensional reflectivity pattern by using a Markov-Bernoulli random field modeling. Unlike the model proposed in Heimer et al. (2007), the Markov-Bernoulli random field accounts for layer discontinuities resulting from splitting, merging, starting or terminating layers within the region of interest. The algorithm performs a search only among a subgroup of two dimensional reflectivity patterns that fit into the model. The Viterbi algorithm (Forney, 1973) is employed for efficiently finding the most likely reflectivity pattern in each iteration. We define a set of reflectivity states and legal transitions between reflector configurations of adjacent traces, and subsequently extract sequences of reflectors, connected across the traces by legal transitions. The performance of the proposed algorithm is investigated for reflectivity patterns that contain discontinuities. Improved performance and robustness to noise are demonstrated for simulated and real seismic data.

SIGNAL MODEL

We assume M received signals (traces) $z^{(m)}[n]$ of length $N+K-1$, each generated by a single input signal $h[n]$ of length K passing through a channel $x^{(m)}[n]$ of length N, which represents the reflectivity sequence of the mth trace. The output signal of channel m can be written as

$$z^{(m)}[n] = \sum_{k=0}^{K-1} h[k]\, x^{(m)}[n-k] + e^{(m)}[n] \tag{1}$$

for $m = 1,\ldots,M$ and $n = 1,\ldots,N+K-1$, where $e^{(m)}[n]$ denotes white Gaussian noise, which is statistically independent of the reflectivity sequence $x^{(m)}[n]$ and $h[n]$. We denote by $\mathbf{z}_m \in \boldsymbol{R}^{N+K-1}$ a single trace m and by $\mathbf{z} \in \boldsymbol{R}^{(N+K-1)\times M}$ the concatenation of $\mathbf{z}_m$, $m = 1,\ldots,M$.

Let $\mathbf{x} \in \boldsymbol{R}^{N\times M}$ represent the two dimensional reflectivity, and let $\mathbf{q} \in \{0,1\}^{N\times M}$ denote a binary matrix representing the existence of reflectors in $\mathbf{x}$, i.e. $q(n,m) = 1$ if there is a reflector in row n and column m of $\mathbf{x}$, otherwise $q(n,m) = 0$. Let $\acute{\mathbf{t}}, \bar{\mathbf{t}}, \grave{\mathbf{t}} \in \{0,1\}^{N\times M}$ be binary matrices representing transition variables of reflectors in $\mathbf{x}$ for ascending, horizontal, and descending layers respectively, i.e. $\acute{t}(n,m) = 1$ if $q(n,m) = 1$, $q(n-1,m+1) = 1$ and these two reflectors belong to the same layer boundary, otherwise $\acute{t}(n,m) = 0$. In a similar way $\bar{t}(n,m) = 1$ if $q(n,m) = 1$, $q(n,m+1) = 1$ and these two reflectors belong to the same layer boundary, otherwise $\bar{t}(n,m) = 0$. Also $\grave{t}(n,m) = 1$ if $q(n,m) = 1$, $q(n+1,m+1) = 1$ and these two reflectors belong to the same layer boundary, otherwise $\grave{t}(n,m) = 0$. Using these definitions, the two dimensional reflectivity model is defined as follows:

1) For each column m, the sequences $\{\acute{t}(n,m)\}_n$, $\{\bar{t}(n,m)\}_n$, $\{\grave{t}(n,m)\}_n$ and $\{q(n,m)\}_n$ are white (binary) processes.

2) $p\left(\acute{\mathbf{t}}, \bar{\mathbf{t}}, \grave{\mathbf{t}}\right) = p\left(\acute{\mathbf{t}}\right) p\left(\bar{\mathbf{t}}\right) p\left(\grave{\mathbf{t}}\right)$.

3) $p\{\acute{t}(n,m-1) = a, \bar{t}(n,m-1) = b, \grave{t}(n,m-1) = c, q(n,m)\} = p\{q(n,m), \acute{t}(n,m) = a, \bar{t}(n,m) = b, \grave{t}(n,m) = c\}$.

4) $p\{\acute{t}(n,m-1) = 0, \bar{t}(n,m-1) = 0, \grave{t}(n,m-1) = 0 \,|\, q(n,m) = 0\} = 1$.

5) $p\{\acute{t}(n,m) = 1\} = \acute{\mu}$, $p\{\bar{t}(n,m) = 1\} = \bar{\mu}$, $p\{\grave{t}(n,m) = 1\} = \grave{\mu}$.

6) $p\{q(n,m) = 1\} = \lambda$.

7) $p\{q(n,m) = 1 \,|\, \acute{t}(n+1,m-1) = 0, \bar{t}(n,m-1) = 0, \grave{t}(n-1,m-1) = 0\} = \varepsilon$.

Properties 5, 6 and 7 define the parameters of the reflectivity model $\acute{\mu}, \bar{\mu}, \acute{\mu}, \lambda, \varepsilon$, which are related by $\lambda = 1 - (1-\acute{\mu})(1-\bar{\mu})(1-\grave{\mu})(1-\varepsilon)$.

The amplitude field of the reflectivity pattern is characterized by two parameters r and σ_a^2. A reflector in $x(n_1, m-1)$ is said to be a predecessor of a reflector in $x(n_2,m)$ if $q(n_1,m-1) = 1$, $q(n_2,m) = 1$, $|n_1 - n_2| \leq 1$, and the transition variable connecting these two reflectors is equal to 1. Then the amplitude field is defined as follows:

8) If a reflector in $x(n_2,m)$ has no predecessors, or has more than one predecessors, then $x(n_2,m) \sim N\left(0, \sigma_a^2\right)$.

9) If a reflector in $x(n_2,m)$ has a predecessor $x(n_1,m-1)$, then $x(n_2,m) \sim N\left(r x(n_1,m-1), \left(1-r^2\right)\sigma_a^2\right)$.

It is shown in Idier and Goussard (1993b,a) that these definitions describe a Markov-Bernoulli random field of reflectors with Gaussian amplitudes, which is homogeneous and symmetric. In each column, the binary variable representing the existence of a reflector is Bernoulli distributed with a parameter λ. The parameter σ_a^2 determines the variance of the amplitudes and the parameter r determines the correlation between reflector amplitudes along a layer boundary.

ESTIMATION PROCEDURE

Let $\mathbf{x}_m$ and $\mathbf{q}_m$ denote, respectively, the m^{th} columns of $\mathbf{x}$ and $\mathbf{q}$, and let $\mathbf{a}_m$ denote a vector of reflector amplitudes in the reflectivity sequence $\mathbf{x}_m$. Note that the length of $\mathbf{a}_m$ may vary for different traces, according to the number of reflectors in a specific trace m. Maximum a posteriori estimation of the locations and amplitudes of reflectors is given by

$$\{\widehat{\mathbf{q}}, \widehat{\mathbf{a}}\} = \arg\max \;\; p(\mathbf{q}, \mathbf{a}|\mathbf{z}) \,. \tag{2}$$

Let $\widehat{\mathbf{q}}^t$ be an estimate of $\mathbf{q}$ in iteration t, and let $\widehat{\mathbf{q}}_m^{t,n}$ be the mth column of the matrix $\widehat{\mathbf{q}}^t$, with the insertion of a new reflector in the nth sample. Let $\widehat{\mathbf{a}}\left(\widehat{\mathbf{q}}_m^{t,n}\right)$ denote an estimate of the reflectors amplitudes in $\widehat{\mathbf{q}}_m^{t,n}$ given the observations $\mathbf{z}_m$. A maximum a posteriori estimator for the reflectors amplitudes is defined by

$$\widehat{\mathbf{a}}(\mathbf{q}_m) = \arg\max_{\mathbf{a}} p(\mathbf{a}_m|\mathbf{q}_m, \mathbf{z}_m) \,. \tag{3}$$

Using a matrix form of (1), i.e., $\mathbf{z}_m = \mathbf{H}_{\mathbf{q}_m}\mathbf{a}_m + \mathbf{e}_m$, where $\mathbf{H}_{\mathbf{q}_m}$ is composed of replicas of h translated to the locations of each reflector, we obtain $\mathbf{z}_m|\mathbf{q}_m, \mathbf{a}_m \sim N\left(\mathbf{H}_{\mathbf{q}_m}\mathbf{a}_m, \sigma_e^2\mathbf{I}\right)$ and $\mathbf{a}_m|\mathbf{q}_m \sim N\left(\mathbf{0}, \sigma_a^2\mathbf{I}\right)$. Hence (3) yields the following known estimator:

$$\widehat{\mathbf{a}}(\mathbf{q}_m) = \left(\mathbf{H}_{\mathbf{q}_m}^T\mathbf{H}_{\mathbf{q}_m} + \frac{\sigma_e^2}{\sigma_a^2}\mathbf{I}\right)^{-1} \mathbf{H}_{\mathbf{q}_m}^T \mathbf{z}_m \,. \tag{4}$$

For finding sequences of reflectors across traces, we define the following set of reflectivity states and legal transitions between reflectors across traces. For each m, we define N states which represent reflector configurations with a single additional reflector in the nth sample with respect to the current estimate (if there is already a reflector at that sample in the current estimate, then there is no change). An additional state is preserved for identifying reflector configurations that remain the same as in the previous iteration. A legal transition exists from each of the first N states in a trace to the three closest states in the next trace, i.e. the reflectors in consecutive traces are in the same sample or are separated by one sample. In addition, a legal transition exists from each of the first N states in a trace to the last state in the next trace, and from the last state in a trace to each of the first N states in the next trace. A legal path is a sequence of M states, one from each trace, that are connected by legal transitions. We define the maximum a posteriori estimated path as

$$\begin{aligned}
&\left\{\widehat{\mathbf{q}}_1^{t+1}, \widehat{\mathbf{a}}\left(\widehat{\mathbf{q}}_1^{t+1}\right), \ldots, \widehat{\mathbf{q}}_M^{t+1}, \widehat{\mathbf{a}}\left(\widehat{\mathbf{q}}_M^{t+1}\right)\right\} \\
&\quad = \underset{\{\text{Legal Paths}\}}{\arg\max} \; p\left\{\mathbf{q}_1^{t,n_1}, \widehat{\mathbf{a}}\left(\mathbf{q}_1^{t,n_1}\right), \ldots, \mathbf{q}_M^{t,n_M}, \widehat{\mathbf{a}}\left(\mathbf{q}_M^{t,n_M}\right) |\mathbf{z}\right\} \\
&\quad = \underset{\{\text{Legal Paths}\}}{\arg\max} \; p\left\{\mathbf{z}_1|\mathbf{q}_1^{t,n_1}, \widehat{\mathbf{a}}\left(\mathbf{q}_1^{t,n_1}\right)\right\} p\left\{\widehat{\mathbf{a}}\left(\mathbf{q}_1^{t,n_1}\right)|\mathbf{q}_1^{t,n_1}\right\} \\
&\qquad p\left\{\mathbf{q}_1^{t,n_1}\right\} \prod_{m=2}^{M} p\left\{\mathbf{z}_m|\mathbf{q}_m^{t,n_m}, \widehat{\mathbf{a}}\left(\mathbf{q}_m^{t,n_m}\right)\right\} p\left\{\mathbf{q}_m^{t,n_m}|\mathbf{q}_{m-1}^{t,n_{m-1}}\right\} \\
&\qquad \prod_{m=2}^{M} p\left\{\widehat{\mathbf{a}}\left(\mathbf{q}_m^{t,n_m}\right)|\mathbf{q}_m^{t,n_m}, \mathbf{q}_{m-1}^{t,n_{m-1}}, \widehat{\mathbf{a}}\left(\mathbf{q}_{m-1}^{t,n_{m-1}}\right)\right\}
\end{aligned} \tag{5}$$

A path can be efficiently obtained by using the Viterbi algorithm (Forney, 1973) as follows. Let $s_{n,m}$ denote a state in the

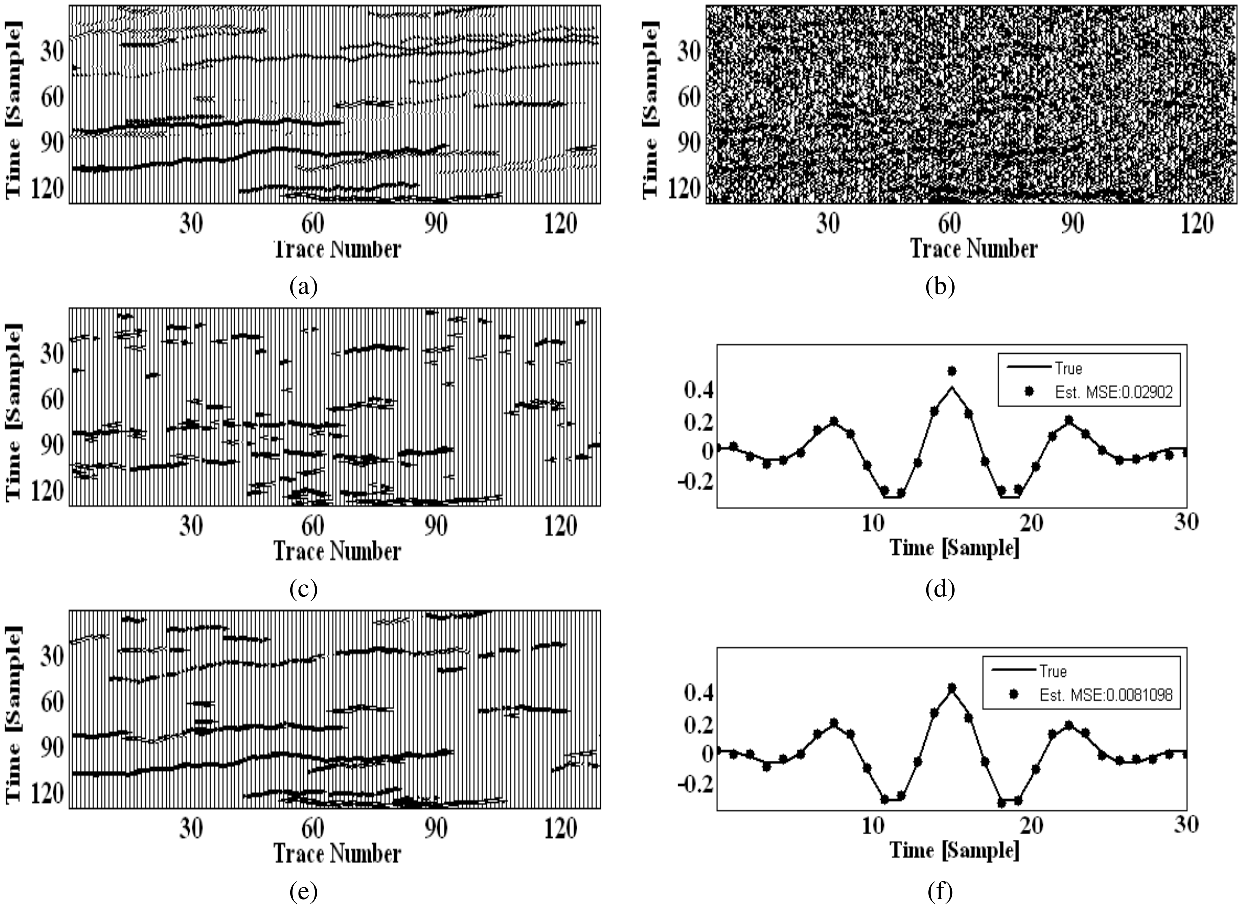

Figure 1: Performances of the proposed Markov-Bernoulli deconvolution (MBD) algorithm and the IWM algorithm in a simulated blind scenario under low SNR conditions (-5dB). (a) True reflectivity pattern; (b) received traces; (c) and (d), reflectivity and wavelet estimates obtained by using the IWM algorithm; (e) and (f), reflectivity and wavelet estimates obtained by using the proposed algorithm.

mth trace characterized by an additional reflector in the nth sample compared to a previous iteration, i.e., reflector state $\mathbf{q}_m^{t,n}$. Let $P_{n_m,m}$ denote a path of states across the traces $[1..m]$ that ends in the state $s_{n_m,m}$. Let $D(P) \triangleq p(P|\mathbf{z})$ be the probability of the states in the path P given the observations $\mathbf{z}$, and let $P^o_{n_m,m} \triangleq \underset{P_{n_m,m}}{\arg\max}\ D(P_{n_m,m})$ be the path that ends with the state $s_{n_m,m}$, whose probability given the observations is maximal. Let $B_{n_m,m}$ represent the set of states from which there is a legal transition to the state $s_{n_m,m}$. Recal that the processes are Markov processes of the first order, we have

$$n^o_{m-1} \triangleq \underset{n_{m-1}\in B_{n_m,m}}{\arg\max} \frac{D\left(P^o_{n_{m-1},m-1}\right) p\left(\mathbf{z}_m|s_{n_m,m}\right) p\left(s_{n_m,m}|s_{n_{m-1},m-1}\right)}{p(\mathbf{z})} \tag{6}$$

$$P^o_{n_m,m} = \left(P^o_{n^o_{m-1},m-1} \vdots s_{n_m,m}\right) \tag{7}$$

where $\vdots$ describes the concatenation of the state on its right to the path on its left. Equations (6) and (7) provide an efficient way of calculating the sequence of states in the states set whose probability given the observations is maximal. We initialize $P^o_{n_1,1} \triangleq s_{n_1,1}$ for $n_1 = 1,\ldots,N$, and then use (6) and (7) to recursively calculate $P^o_{n_M,M}$ for $n_M = 1,\ldots,N$. Finally, we choose

$$n^o_M \triangleq \underset{n_M}{\arg\max}\ D\left(P^o_{n_M,M}\right)$$

and then we obtain the final desired path:

$$P^o \triangleq P^o_{n^o_M,M}.$$

EXPERIMENTAL RESULTS

Simulated Data

Hundred and thirty traces of 130 samples length are generated by convolving a wavelet with a pattern of reflectors, according to the proposed Markov-Bernoulli model. The traces are corrupted by white Gaussian noise (SNR $= -5$ dB). The SNR is defined by SNR $\triangleq \left(\lambda\sigma_a^2/\sigma_e^2\right)\sum_k h^2[k]$, where σ_e denotes the standard deviation of the noise. A comparison is made between the IWM algorithm of Kaaresen and Taxt (1998) and the proposed algorithm. The results are presented in Figure 1.

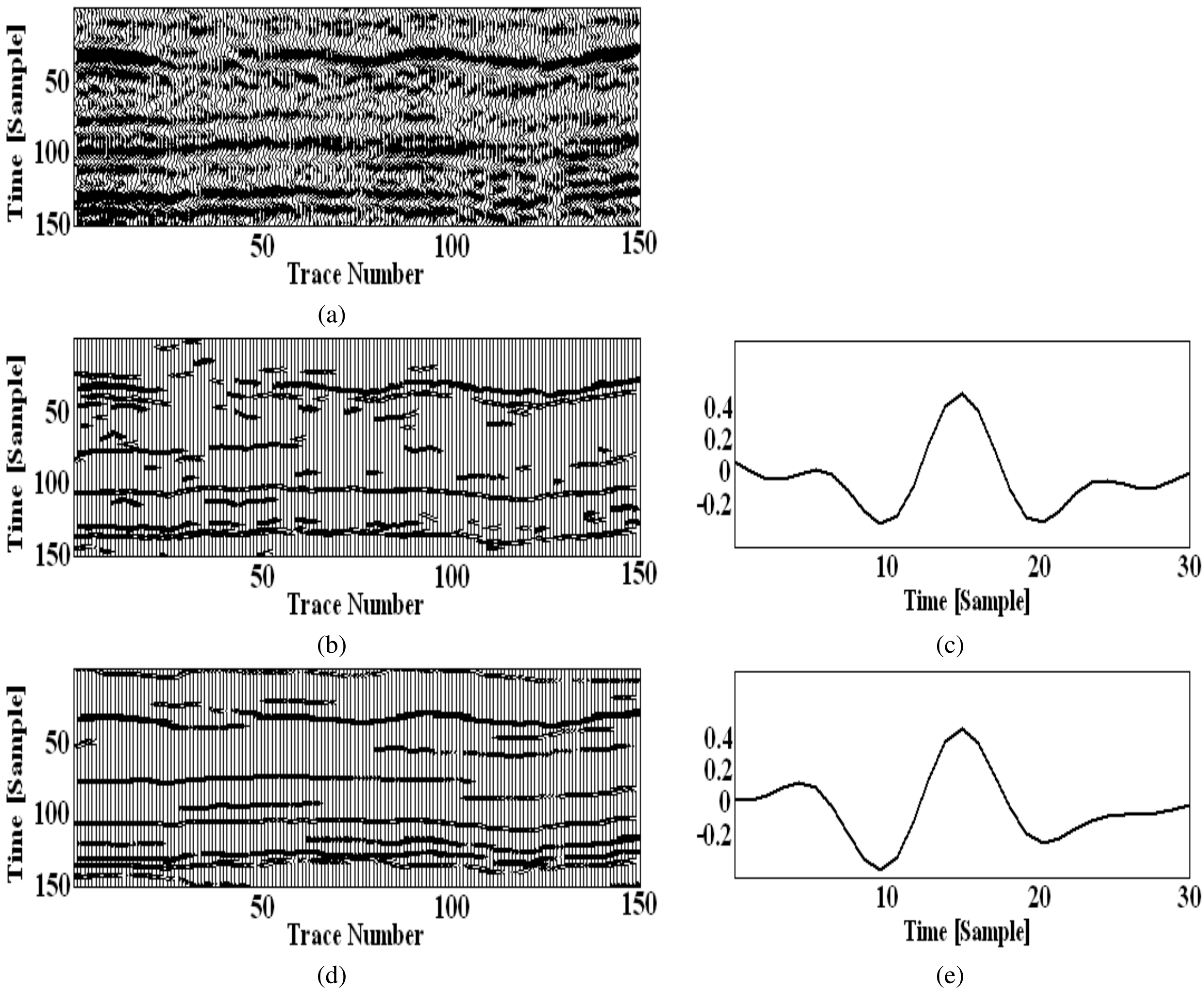

Figure 2: Performances of the MBD and IWM algorithms for real seismic data. (a) Received traces; (b) and (c), reflectivity and wavelet estimates obtained by using the IWM algorithm; (d) and (e), reflectivity and wavelet estimates obtained by using the MBD algorithm.

It is clear from the results that the proposed algorithm recovers the true reflectors more accurately than the IWM algorithm. Furthermore, the wavelet estimate obtained by the proposed algorithm is characterized by lower MSE (MSE $= \left\|\mathbf{h} - \widehat{\mathbf{h}}\right\|_2$). A comparison of Figures 1(a) and (c) shows that the IWM algorithm generates false reflectors in some areas, and misdetects true reflectors in other areas. Hence, the improvement by using the proposed algorithm is not related to the specific choice of sparsity parameter in the IWM algorithm, since a larger sparsity parameter would cause additional true reflectors to disappear, and a smaller sparsity parameter would lead to additional false reflectors.

Real Data

Figure 2(a) shows real seismic data (courtesy of GeoEnergy Inc., Texas) containing 150 traces of 150 samples long. The reflectivity and wavelet estimates obtained by using the IWM and proposed algorithms are presented in Figure 2. Since the true layer structure is unknown, one can only appreciate the continuous nature of the channel estimates obtained by using the proposed algorithm.

CONCLUSION

We have presented an algorithm for multichannel seismic deconvolution that is based on Markov-Bernoulli random field modeling of the lateral dependency between reflectors in consecutive traces. The computational complexity of the proposed algorithm is generally higher than that of the IWM algorithm. However, some reduction in complexity can be achieved by selecting in each iteration more than one path. Furthermore, when estimating the amplitudes for a new configuration of reflectors in a certain trace, not all the amplitudes need to be estimated, since reflectors that are distant from a newly added reflector or a removed reflector remain with nearly the same estimated amplitude values.

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