Robust Focusing for Wideband MVDR Beamforming

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***Abstract*—In this paper, we propose and study two robust methods for coherent focused wideband Minimum Variance Distortionless Response (MVDR) beamforming. The focusing procedure introduces a frequency dependent focusing error which causes performance degradation, especially at high Signal to Noise Ratio (SNR) values. The proposed robust methods aim at reducing the sensitivity of the coherent MVDR to focusing errors. The first method is based on modifying the beamformer optimization problem and generalizing it to bring into account the focusing transformations and the second is based on modifying the focusing scheme itself. A numerical study demonstrates a significant performance improvement of the proposed robust schemes when applied, using the Wavefield Interpolated Narrowband Generated Subspace (WINGS) focusing transformation.**

I. INTRODUCTION

This paper deals with coherent adaptive wideband beamformers which incorporate a frequency focusing procedure for signal subspace alignment (e.g. [1]). The main benefits of the coherent methods are low computational complexity, the ability to combat the signal cancellation problem and improved convergence properties. These advantages render the focusing wideband method useful, especially when data independent focusing methods may be used [2], [3]. However, further study shows that the sensitivity of the focused adaptive wideband beamformer to focusing errors can greatly deteriorate the performance of the adaptive beamformers especially at high Signal to Noise Ratio (SNR) scenarios. This important issue is addressed here by proposing and studying robust methods that combat this sensitivity.

The paper is organized as follows: in Section II, the problem is formulated. In Section III, we conduct a numerical and simulative study for the single source case using the Wavefield Interpolated Narrowband Generated Subspace (WINGS) [3] focusing method. WINGS is an interpolation based method, which does not require any apriori Direction Of Arrival (DOA) knowledge. In order to overcome the sensitivity to focusing errors we propose and investigate in Section IV two robust schemes for wideband focused beamforming. The first method is based on a modifying the Minimum Variance Distortionless Response (MVDR) beamformer by implementing a robust General-Rank (GR) beamforming scheme and the second is based on modifying the focusing transformation so that the focusing error is reduced in the direction of the desired source. In Section V, we conduct a performance analysis demonstrating the efficacy of the proposed methods.

II. PROBLEM FORMULATION

Consider an arbitrary array of N sensors sampling a wavefield generated by P wideband far-field sources, in the presence of additive noise. Let $\{\theta_i\}_{i=1}^{P}$ be the DOAs of the sources. Each T seconds of received data are divided into K snapshots and transformed into the frequency domain yielding

$$\mathbf{x}_k(w_j) = \mathbf{A}_{\boldsymbol{\theta}}(w_j)\mathbf{s}_k(w_j) + \mathbf{n}_k(w_j), \quad j = 1, 2, ...J, \; k = 1, 2, ...K \tag{1}$$

where $\mathbf{x}_k(w_j)$, $\mathbf{s}_k(w_j)$ and $\mathbf{n}_k(w_j)$ denote vectors whose elements are the discrete Fourier coefficients of the measurements, of the unknown sources signals and the noise, respectively at the *k*th snapshot and frequency w_j. $\mathbf{A}_{\boldsymbol{\theta}}(w_j) \equiv [\mathbf{a}_{\theta_1}(w_j), \mathbf{a}_{\theta_2}(w_j), ..., \mathbf{a}_{\theta_P}(w_j)]$ is the $N \times P$ direction matrix, where $\mathbf{a}_{\theta}(w)$, is the *array manifold* vector. We assume that the signal vectors $\mathbf{s}_k(w_j)$ and the noise vectors $\mathbf{n}_k(w_j)$ are independent samples of stationary, zero-mean circular complex Gaussian random process, with unknown covariance matrices $\mathbf{R}_s(w_j)$ and $\sigma^2(w_j)\mathbf{I}$, respectively. Let $\mathbf{T}(w_j)$ denote a transformation that maps the wideband array output from frequency w_j to frequency w_0

$$\mathbf{T}(w_j)\mathbf{A}_{\boldsymbol{\theta}}(w_j) \cong \mathbf{A}_{\boldsymbol{\theta}}(w_0), \tag{2}$$

where w_j are within the bandwidth of the signals and w_0 is the focused frequency.

Following [3], we may construct the focused time-domain vector

$$\mathbf{y}_k(n) = \sum_{j=1}^{J} \mathbf{T}(w_j)\mathbf{x}_k(w_j)e^{iw_j nT_s} \cong \mathbf{A}_{\boldsymbol{\theta}}(w_0)\mathbf{s}(n) + \widetilde{\mathbf{n}}(n), \tag{3}$$

where $\mathbf{s}(n)$ is the temporal vector of wideband unknown signals within the focused frequency band $[w_1 : w_J]$, T_s is the sampling frequency and $\widetilde{\mathbf{n}}(n)$ is the transformed noise. Since $\mathbf{y}_k(n)$ has a narrowband array manifold, we can use any narrowband adaptive beamformer matched to frequency w_0, such as the MVDR beamformer. The MVDR focused beamformer operates on the temporal focused data vector $\mathbf{y}_k(n)$ (3) whose sample covariance matrix is estimated by $\hat{\mathbf{R}}_x^f = \frac{1}{KJ}\sum_{k,n}\mathbf{y}_k(n)\mathbf{y}_k^H(n)$. The focused weight vector is simply computed in the time domain by $\hat{\mathbf{w}}_{\theta}^f = \frac{(\hat{\mathbf{R}}_x^f)^{-1}\mathbf{a}_{\theta}(w_0)}{\mathbf{a}_{\theta}^H(w_0)(\hat{\mathbf{R}}_x^f)^{-1}\mathbf{a}_{\theta}(w_0)}$, where w_0 is the focusing frequency and f stands for *focused*.

The focusing operation introduces a focusing error which influences the performance of the focused MVDR. This focusing error is expected to be relatively high in interpolation based focusing methods which focus all angular directions in comparison to data dependant focusing methods which require preliminary DOAs estimates. One of the focusing methods, based on interpolation, is the WINGS focusing method [3] which may be applied to any array with a known arbitrary geometry. WINGS focusing method minimizes ε_j, the L_2 norm of the focusing error over all possible directions

$$\varepsilon_j^2 \triangleq \frac{1}{N} \int_{\theta\,=\,-\pi}^{\pi} d\theta \left\| \mathbf{a}_\theta(w_0) - \mathbf{T}(w_j)\mathbf{a}_\theta(w_j) \right\|^2. \tag{4}$$

The WINGS focusing matrix minimizing (4) is given by [3]

$$\mathbf{T}(w_j) = \mathbf{G}(w_0)\mathbf{G}^\dagger(w_j), \tag{5}$$

where $\mathbf{G}(w)$ is a matrix which depend only on the array geometry and $\mathbf{G}^\dagger(w_j)$ denotes the pseudo-inverse of $\mathbf{G}(w_j)$.

Here we address the sensitivity problem of the focused beamformer to focusing errors by proposing and analyzing two robust focused beamforming schemes. The effectiveness of the proposed schemes are examined using the WINGS focusing transformation.

III. Analytic Array Gain of the focused MVDR

The analytic expression for the $\mathrm{SINR_{out}}$ of the MVDR focused beamformer is given by [4]

$$\mathrm{SINR_{out}} = \frac{\mathbf{b}_{\theta_1}(\mathrm{w_0}) \left(\sum_{\mathrm{j=1}}^{\mathrm{J}} \Psi(\mathrm{j}, \theta_1) \right) \mathbf{b}_{\theta_1}(\mathrm{w_0})^{\mathrm{H}}}{\mathbf{b}_{\theta_1}(\mathrm{w_0}) \left(\sum_{\mathrm{j=1}}^{\mathrm{J}} \left[\sum_{\mathrm{p=2}}^{\mathrm{P}} \Psi(\mathrm{j}, \theta_{\mathrm{p}}) + \mathbf{R}^{\mathrm{f}}_{\mathrm{n,j}} \right] \right) \mathbf{b}_{\theta_1}(\mathrm{w_0})^{\mathrm{H}}} \tag{6}$$

$$\mathbf{b}_\theta(w_0) = \mathbf{a}_\theta^H(w)(\mathbf{R}_x^f)^{-1}, \tag{7}$$

$$\Psi(j,\theta) = \sigma_{s_1}^2(w_j)\mathbf{a}_{\theta_1}^f(w_j)(\mathbf{a}_\theta^f(w_j))^H. \tag{8}$$

$\mathbf{R}_{n,j}^f = \mathbf{T}(w_j)\mathbf{R}_n(w_j)\mathbf{T}^H(w_j)$ is the focused noise covariance matrix, and

$$\mathbf{R}_x^f = E\left\{\mathbf{y}_k(n)\mathbf{y}_k^H(n)\right\}, \tag{9}$$

is the covariance matrix of the focused data vector $\mathbf{y}_k(n)$ (3). The analytic Array Gain (AG) is given by

$$\mathrm{AG} = \mathrm{SINR_{out}}/\mathrm{SINR_{in}}, \tag{10}$$

where $\mathrm{SINR_{in}}$ is the Signal to Interference and Noise ratio at the input of the beamformer.

We now use (10) to investigate the sensitivity to focusing transformation errors of the focused wideband MVDR beamformer. We start with an illustration of this sensitivity for the single source case.

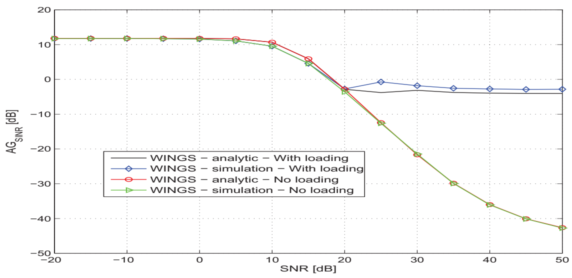

Fig. 1. AG versus SNR, for the case of a single source and perfect knowledge of it's DOA. Presented are both the cases that loading term was either added or not in order to limit the output noise gain.

A. *Single source example*

Let us examine the AG for the single source case in the presence of additive white noise. We evaluate the analytic AG given by (10) and compare it to the simulative AG based on Monte-Carlo simulations of the MVDR focused beamformer. We take a circular complex Gaussian wideband acoustic source propagating towards a linear array of N=20 sensors in velocity of 1500 m/sec. The source DOA $\boldsymbol{\theta} = 105°$ is assumed to be known perfectly, where $90°$ is the broadside direction. The bandwidth of the source is 600Hz taken around f_c = 1500Hz and the spectrum is taken to be flat in the relevant bandwidth. The focusing frequency is $f_0 = 1500$Hz. The observation time T is taken as 10 seconds and divided into $K = 46$ snapshots. Each snapshot of data is transformed to the frequency domain using an FFT of 1024 bins yielding $J = 129$ frequency bins in the relevant bandwidth. The spacing between two adjacent sensors is $d = \frac{\lambda_{\min}}{2}$, where $\lambda_{\min}$ corresponds to the highest frequency of the bandwidth. Fig. 1 shows the asymptotic and the simulative AG versus SNR for the coherent focused WINGS MVDR beamformer. Also shown is the performance when a loading term has been added to the covariance matrix before inversion. This operation limits the norm of the beamformer coefficients vector yielding a robust beamformer. We can see that the loading term improves the performance especially in high SNR values. Yet, in both cases we can see a decrease in the AG as the SNR increases. In [4] we show analytically that it occurs due to the focusing error in the desired source direction and that the AG is approximately inversely proportional to the square of the SNR for large values of the SNR. In the next section we propose two methods to improve the performance sensitivity to focusing errors in high SNR values.

IV. Robust MVDR focused beamformers for coherent wideband array processing

In this section we propose and examine two methods designed to combat the problem of sensitivity of the focused MVDR at high SNR values .

A. *General-Rank focused MVDR (GR-MVDR)*

Let us examine more closely the structure of the signal component in the focused covariance matrix $\mathbf{R}_x^f$ (9). Inserting

$\mathbf{R}_x(w_j) = \sigma_s^2(w_j)\mathbf{a}_{\theta_d}(w_j)\mathbf{a}_{\theta_d}^H(w_j) + \mathbf{R}_n(w_j)$ into (9) we get

$$\mathbf{R}_x^f = \mathbf{R}_s^f + \sum_{j=1}^{J} \mathbf{T}(w_j)\mathbf{R}_n(w_j)\mathbf{T}^H(w_j) \quad (11)$$

where $\mathbf{R}_s^f$ is the signal component of the focused covariance matrix

$$\mathbf{R}_s^f = \sum_{j=1}^{\mathbf{J}} \sigma_s^2(w_j)\mathbf{T}(w_j)\mathbf{a}_{\theta_d}(w_j)\mathbf{a}_{\theta_d}^H(w_j)\mathbf{T}^H(w_j). \quad (12)$$

From the above structure we see that the rank of the signal component covariance is higher than one. Therefore, we can use the general rank MVDR beamformer e.g. [5]. For the case where the source spectrum $\sigma_s^2(w_j)$ is known, we may find the Minimum Variance solution for the weight vector by maintaining a distortionless array response to the signal covariance

$$\min_{\mathbf{w}} \mathbf{w}^H(w_0)\mathbf{R}_x^f\mathbf{w}(w_0) \quad \textit{subject to } \mathbf{w}^H(w_0)\mathbf{R}_s^f\mathbf{w}(w_0) = \mathbf{1}. \quad (13)$$

Following [5], the solution of (13) is given by

$$\mathbf{w}_{\mathrm{GR-MVDR}}^f = \mathcal{P}\left\{(\mathbf{R}_x^f)^{-1}\mathbf{R}_s^f\right\}, \quad (14)$$

where $\mathcal{P}\{\cdot\}$ denotes the *principal eigenvector* of a matrix. In [5] a robust version handling the uncertainties in the knowledge of $\mathbf{R}_x^f$ is also derived, based on the concept of the narrowband diagonal loading. We now extend the robust narrowband version of [5] to the focused wideband case. We are interested in limiting the white noise gain of the beamformer. In case of the focused beamformer, the output noise power is given by

$$\sigma_{n_{out}}^2 = \sigma_n^2(\mathbf{w}_\theta^f)^H \left(\frac{1}{J}\sum_{l=1}^{J}\mathbf{T}(w_l)\mathbf{T}^H(w_l)\right)\mathbf{w}_\theta^f, \quad (15)$$

where we assume that the noise spectrum is frequency independent, i.e. $\sigma_n^2(w) = \sigma_n^2, \forall w$. Thus, limiting the white noise gain yields the following additional quadratic constraint

$$(\mathbf{w}_\theta^f)^H\mathbf{Q}\mathbf{w}_\theta^{\mathbf{f}} \leq T_0, \quad (16)$$

where $\mathbf{Q} \triangleq \frac{1}{J}\sum_{l=1}^{J}\mathbf{T}(w_l)\mathbf{T}^H(w_l)$ and T_0 is a design parameter. Solving (13) with the additional constraint (16), by using the lagrange multipliers method, we get

$$(\mathbf{R}_x^f + \beta\mathbf{Q})^{-1}\mathbf{R}_s\mathbf{w}_\theta^f = \frac{1}{\lambda}\mathbf{w}_\theta^f. \quad (17)$$

The solution to (17) is given by [5]

$$\mathbf{w}_{\mathrm{GR-MVDR-QL}}^f = \mathcal{P}\left\{(\mathbf{R}_x^f + \beta\mathbf{Q})^{-1}\mathbf{R}_s^f\right\}, \quad (18)$$

where β is the loading factor. Note that the GR focused MVDR requires a-priori knowledge of the spectral shape of the source $\sigma_s^2(w_j)$. Following [5] we use a robust version combating a small signal spectrum mismatch

$$\mathbf{w}_{\mathrm{ROBUST-GR-MVDR}}^f = \mathcal{P}\left\{(\mathbf{R}_x^f + \beta\mathbf{Q})^{-1}(\mathbf{R}_s^f - \varepsilon\mathbf{I})\right\}, \quad (19)$$

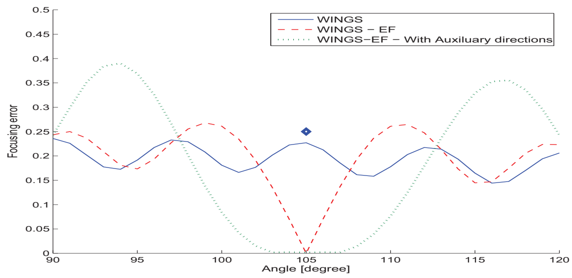

Fig. 2. Focusing error versus angle for the various robust methods. Also presented the non-robust WINGS for comparison. The diamond marks the true source direction.

where ε is the norm of the error in $\mathbf{R}_s^f$.

B. Enhanced Focusing (EF)

WINGS has a relatively large focusing error due to the panoramic focusing requirement. Adding an additional error component in the desired source direction to the LS minimization term (5) of the WINGS enables us to reduce the error in the source direction. In this case, the minimization term of the WINGS (4) becomes:

$$\varepsilon_j^2 = \frac{1}{N}\left\|[\widetilde{\mathbf{G}}(w_0) - \mathbf{T}(w_j)\widetilde{\mathbf{G}}(w_j)]\right\|_F^2, \quad (20)$$

where

$$\widetilde{\mathbf{G}}(w) = [\mathbf{a}_{\theta_d}(w)\ ,\ \mathbf{G}(w)] \quad (21)$$

and θ_d is the desired source direction. This solution achieves better performance as shown in the next section. Yet, it requires an accurate estimation of the desired source direction which is a drawback. In order to increase the robustness of this solution to DOAs uncertainties, we add 4 auxiliary directions at $-2, -1, 1, 2$ degrees relative to the assumed desired source direction. In this case, (21) becomes

$$\widetilde{\mathbf{G}}(w) = [\mathbf{a}_{\theta_d-2}(w), \mathbf{a}_{\theta_d-1}(w), \mathbf{a}_{\theta_d}(w), ..., \mathbf{a}_{\theta_d+2}(w),\ \mathbf{G}(w)]. \quad (22)$$

Fig. 2 illustrates the benefit of adding the auxiliary directions. We can see that WINGS-EF with the auxiliary directions as in (22) is robust to direction errors of approximately 2 degrees.

V. Performance analysis of the robust focused MVDR beamformer

In this section, the performance of the proposed robust focused MVDR schemes, is numerically studied for the single source case. The simulation parameters are identical to those in Section III-A. The signal spectrum which is required for the GR-MVDR is assumed to be flat in accordance with the simulation. There is an error at the desired source direction of 1.5 degrees.

A. Sensitivity to source DOA

Fig. 3 shows the analytic and simulative AG versus SNR of the focused WINGS-MVDR robust and non robust methods with a loading term. One can see that both robust schemes

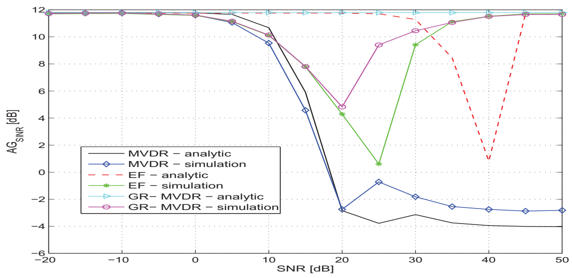

Fig. 3. AG versus SNR of the various solutions for robust focused WINGS MVDR. Single source case with an error of 1.5 degrees between the assumed and actual DOA.

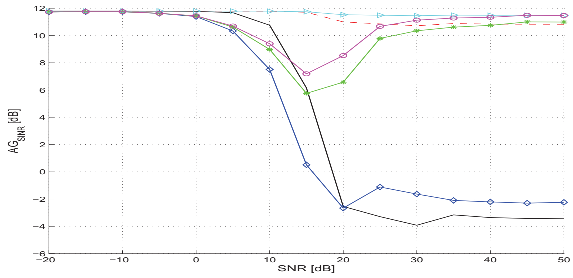

Fig. 5. Array gain for AR source spectrum with a known DOA and a maximal deviation of 1dB between the actual and assumed spectrum. The legend is like in fig. 4.

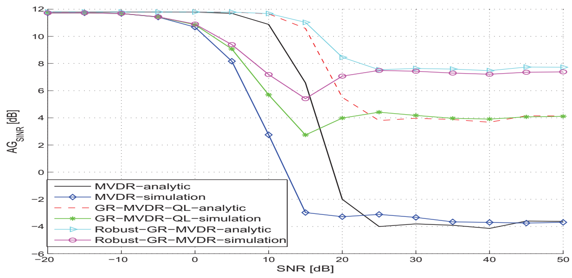

Fig. 4. Array gain of focused MVDR, focused GR-MVDR-QL(18) and focused ROBUST-GR-MVDR(19) methods for AR source spectrum with a known DOA. A maximal deviation of 3.5dB between the actual and assumed spectrum.

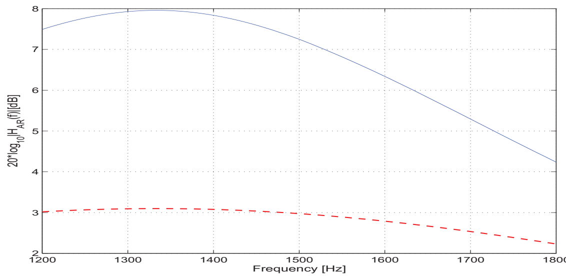

Fig. 6. Spectrum of the Auto-Regressive signal. 1dB maximal deviation (dashed) and 3.5dB maximal deviation (solid).

improve the performance of the focused WINGS MVDR, bringing the AG closer towards the ideal values. Both methods exhibit robustness to the DOA error.

B. Sensitivity to source spectrum

In this section we examine the sensitivity of the focused GR-MVDR to errors in the spectral shape of the source. Note that we assume a flat signal spectrum in accordance with the above example in which, white Gaussian sources were simulated. Figs.4 and 5 demonstrate the performance of the focused MVDR, focused GR-MVDR-QL (18) and focused ROBUST-GR-MVDR (19) methods when the source is shaped by an Auto-Regressive filter of a single pole whose spectrum is plotted at Fig. 6. In Figs. 4 and 5 we examine, respectively, 3.5dB and 1dB maximal spectral deviation between the actual and the assumed spectrum. From these examples we see that the robust extension of the focused GR-MVDR (19) method can handle a spectral deviation smaller than 1dB. In practice, for a larger deviation, spectrum estimation of the desired source should be used.

VI. Conclusions

Two robust methods for wideband focused beamforming have been proposed and investigated in this paper. The proposed methods aim at reducing the sensitivity of the beamformer's performance to focusing error, especially at high SNR scenarios. This sensitivity is more significant in interpolation based focusing methods which do not require preliminary estimates of the DOAs. This independence of the focusing procedure on the preliminary DOAs estimates is a desirable property, therefore, designing robust MVDR beamformer for interpolation based focusing schemes approaches is of great importance. The first method is based on modifying the MVDR beamformer using the General-Rank (GR) approach and the second is based on modifying the focusing scheme. We examine the proposed methods by applying them to the WINGS focusing method. The results indicate that both EF and GR-MVDR focusing methods can improve the performance of the WINGS significantly, especially at high SNR values. The GR-MVDR requires a spectral source estimation and the EF method requires an estimation of the desired signal DOA. However, the GR-MVDR method is computational advantageous over the EF method, since spectral estimation is considerably less complex than data dependent calculation of the focusing transformations.

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