Quality Analysis and Enhancement of Reverberated Speech using Microphone Arrays

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Motivation

- Reverberation
  - Late & uncorrelated reflections
  - Quality & intelligibility degradation, temporal smearing

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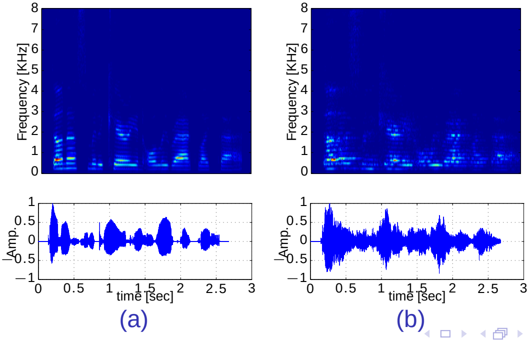

General Overview

- Unsolved problem:
  - Reverberation in speech signals
- The solution we propose:
  - Use microphone arrays
- The topic we will discuss:
  - Enhancement & reverberation suppression
  - Quality analysis of speech signals

General Outline

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I Robust Regularized Superdirective Beamforming

II The Tunable Superdirective Beamformer

III Microphone Array Reverberation Assessment

IV Conclusions

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Part I

Robust Regularized Superdirective Beamforming

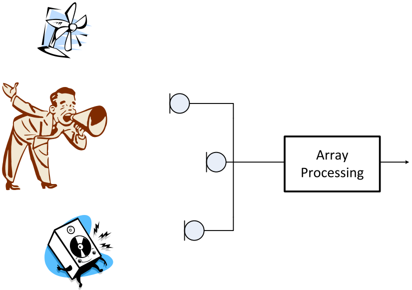
Based on:

- R. Berkun, I. Cohen, and J. Benesty "Combined Beamformers for Robust Broadband Regularized Superdirective Beamforming," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 23, no. 5, pp. 877–886, May 2015.
- R. Berkun, I. Cohen, and J. Benesty "User Determined Superdirective Beamforming," *Proc. 28th IEEE Convention of Electrical and Electronics Engineers in Israel, IEEEI-2014, Eilat, Israel, 3–5 December 2014*.

Outline

Background

- Multi-channel vs. single microphone
  - Preserve spatial information
  - Higher processing flexibility
- Microphone array benefits:
  - Source of interest extraction
  - Handling multiple sources
  - Superior noise suppression
- Impact factors:
  - Number of sensors
  - Array geometry
  - Processing algorithm



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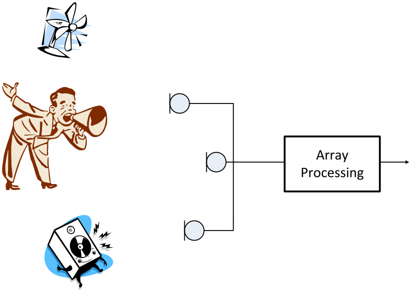

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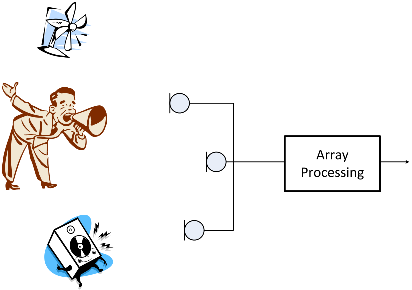

Background – Contd.

- Microphone array beamforming:
  - Class of Multichannel signal processing algorithms
  - Extraction & enhancement of desired signals
  - Noise & reverberation suppression
- Delay & Sum (DS) beamformer
  - Low gain for diffuse noise
  - Inferior performance for reverberation

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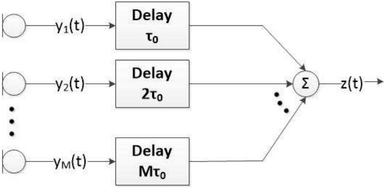

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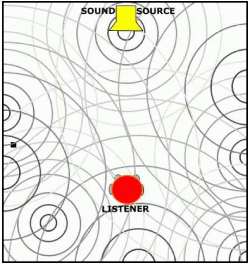
- Reverberation
  - Late & uncorrelated reflections
- Superdirective beamforming:
  - Supergain for diffuse noise
  - White noise amplification
- The optimal beamformer:
  - High gain for diffuse noise
  - Control of the white noise amplification



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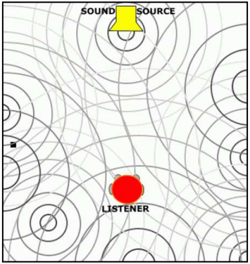

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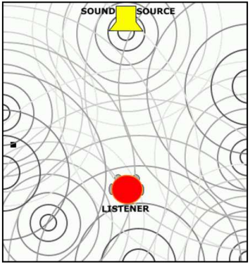
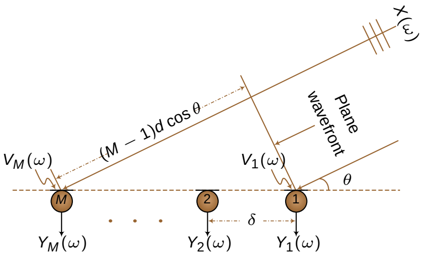


Signal Model

- Uniform Linear Array (ULA) with M omnidirectional microphones
- Fixed beamformers with small δ
- Azimuth angle θ
- Mainlobe at endfire direction ($\theta = 0°$)



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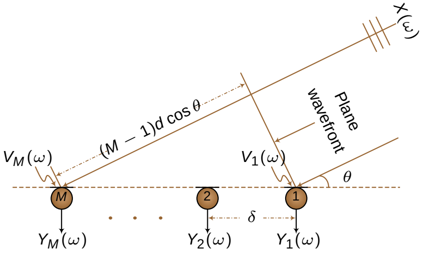
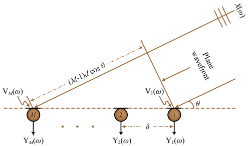


Signal Model – Contd.

- The steering vector (of length M):
  - $\mathbf{d}(\omega,\theta) = \begin{bmatrix} 1 & e^{-\jmath\omega\tau_0\cos\theta} & \cdots & e^{-\jmath(M-1)\omega\tau_0\cos\theta} \end{bmatrix}^T$

- The observation signal vector:

$$\begin{aligned}\mathbf{y}(\omega) &= \begin{bmatrix} Y_1(\omega) & Y_2(\omega) & \cdots & Y_M(\omega) \end{bmatrix}^T \\ &= \mathbf{x}(\omega) + \mathbf{v}(\omega) \\ &= \mathbf{d}(\omega)X(\omega) + \mathbf{v}(\omega)\end{aligned}$$



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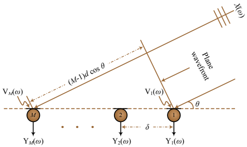
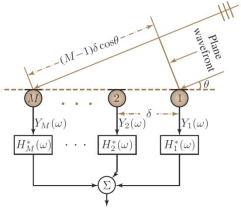


Problem Formulation

- The objective:
  - Apply $\mathbf{h}(\omega)$ to the observation vector $\mathbf{y}(\omega)$
  - The beamformer output:

$$\begin{aligned} Z(\omega) &= \mathbf{h}^H(\omega)\mathbf{y}(\omega) \\ &= \mathbf{h}(\omega)\mathbf{d}(\omega)X(\omega) + \mathbf{h}^H(\omega)\mathbf{v}(\omega) \end{aligned}$$



  - The distortionless constraint

$$\mathbf{h}^H(\omega)\mathbf{d}(\omega) = 1$$

Performance Measures

- SNR Gain:

$$\mathcal{G}[\mathbf{h}(\omega)] = \frac{\mathrm{oSNR}[\mathbf{h}(\omega)]}{\mathrm{iSNR}(\omega)} = \frac{\left|\mathbf{h}^H(\omega)\mathbf{d}(\omega)\right|^2}{\mathbf{h}^H(\omega)\mathbf{\Gamma}_{\mathbf{v}}(\omega)\mathbf{h}(\omega)}$$

  - $\mathbf{\Gamma}_{\mathbf{v}}(\omega) = \frac{\mathbf{\Phi}_{\mathbf{v}}(\omega)}{\phi_{V_1}(\omega)}$

White Noise Gain

- White Noise Gain (WNG):

$$\mathcal{W}[\mathbf{h}(\omega)] = \mathcal{G}\left[\mathbf{h}(\omega) \,\middle|\, \mathbf{\Gamma}_{\mathbf{v}}(\omega) = \mathbf{I}_M\right] \leq M, \ \forall \mathbf{h}(\omega)$$

  - For WNG: $\mathbf{\Gamma}_{\mathbf{v}}(\omega) = \mathbf{I}_M$
- WNG - robustness measure against:
  - Array weight imperfections
  - Sensor position mismatches
  - Uncorrelated noise
  - Microphone thermal noise
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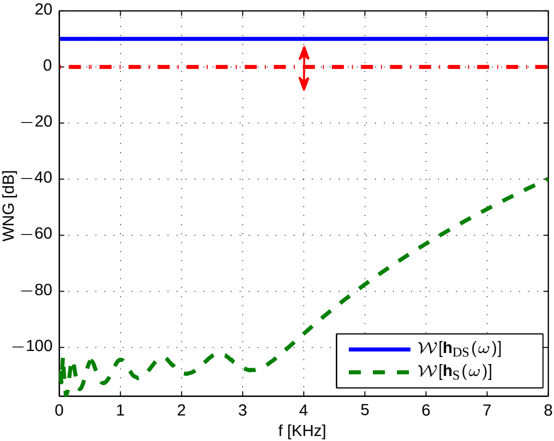
White Noise Gain – Contd.



White noise gain of the DS (solid line) and superdirective (dashed line) beamformers, with $M = 10$ microphones, and $\delta = 1$ cm.

Directivity Factor

- Gain for diffuse noise
  - $\mathbf{\Gamma}_{\mathbf{v}}(\omega) = \mathbf{\Gamma}_{\mathrm{d}}(\omega)$
- Directivity Factor (DF):

$$\mathcal{D}[\mathbf{h}(\omega)] = \mathcal{G}\left[\mathbf{h}(\omega) \,\middle|\, \mathbf{\Gamma}_{\mathbf{v}}(\omega) = \mathbf{\Gamma}_{\mathrm{d}}\right] \leq M^2, \ \forall \mathbf{h}(\omega)$$

- $\mathcal{D}[\mathbf{h}_{\mathrm{S}}(\omega)] = \mathcal{D}_{\max}(\omega) \xrightarrow[\delta \to 0]{} M^2$

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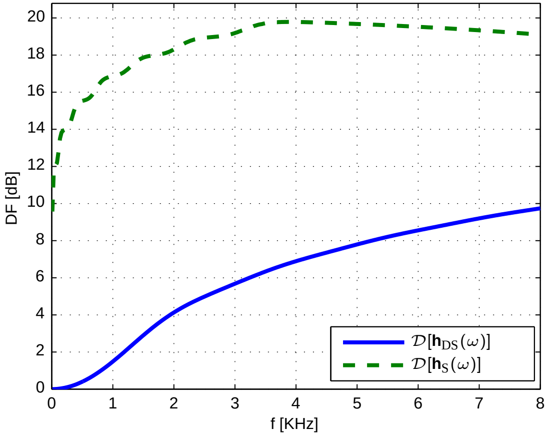

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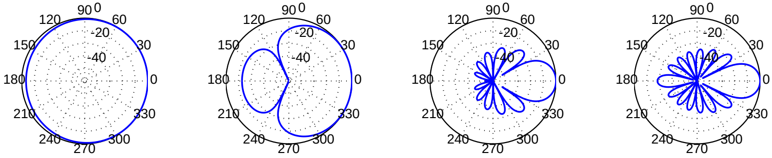
Beampattern

- Angular directivity-factor response
  - Beamformer response as function of $d(\omega, \theta)$
- Beampattern:

$$\begin{aligned}\mathcal{B}[\mathbf{h}(\omega), \theta] &= \mathbf{d}^H(\omega, \theta)\mathbf{h}(\omega) \\ &= \sum_{m=1}^{M} H_m(\omega) e^{\jmath(m-1)\omega\tau_0 \cos\theta}\end{aligned}$$

Beampattern – Contd.



(a) $f = 1KHz$ (b) $f = 3KHz$ (c) $f = 1KHz$ (d) $f = 3KHz$

Beampattern of the DS (a,b) and superdirective (c,d) beamformers, with $M = 8$ microphones, and $\delta = 1$ cm.

The Delay & Sum (DS) Beamformer

- Maximizes the WNG
  - subject to distortionless constraint

$$\mathbf{h}_{\mathrm{DS}}(\omega) = \frac{\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\mathbf{d}(\omega)} = \frac{\mathbf{d}(\omega)}{M}$$

- $\mathcal{W}[\mathbf{h}_{\mathrm{DS}}(\omega)] = M = \mathcal{W}_{\max}$
- $\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)] \geq 1$

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The Superdirective Beamformer

- Maximizes the DF
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$$\mathbf{h}_{\mathrm{S}}(\omega) = \frac{\mathbf{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\mathbf{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega)}$$

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Prior Work

- Superdirective beamformer with white noise gain constraint:
  - Introduced by [Cox et al., 1987]
  - Optimal constrained solution
  - Multistep reconstruction algorithm
- Optimization approaches:
  - Linear programming [Vorobyov et al., 2003]
  - Different mismatch formulations [Doclo and Moonen, 2007]
  - Minimax & non-linear optimization [Doclo and Moonen, 2003]
- The proposed approach:
  - Closed-form solution
  - Effective tradeoff
    - High gain for diffuse noise (directivity factor)
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The Regularized Superdirective Beamformer

- Maximizes the DF [Cox et al., 1987, Benesty et al., 2008]
  - Constraint on the WNG
  - subject to distortionless constraint

$$\mathbf{h}_{\mathrm{S},\epsilon}(\omega) = \frac{\left[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega) + \epsilon \mathbf{I}_M\right]^{-1} \mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\left[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega) + \epsilon \mathbf{I}_M\right]^{-1} \mathbf{d}(\omega)}$$

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$$\mathbf{h}_{\mathrm{S},\epsilon}(\omega) \Big| \, \boldsymbol{\Gamma}_{\mathrm{d}}(\omega) \leftarrow \boldsymbol{\Gamma}_{\epsilon}(\omega)$$

The Proposed Beamformer

- Linear combination of the DS (maximum WNG) and the regularized superdirective (high DF) beamformer:

$$\mathbf{h}_{\alpha,\epsilon}(\omega) = \frac{[\boldsymbol{\Gamma}_{\epsilon}^{-1}(\omega) + \alpha(\omega)\mathbf{I}_M]\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)[\boldsymbol{\Gamma}_{\epsilon}^{-1}(\omega) + \alpha(\omega)\mathbf{I}_M]\mathbf{d}(\omega)} = \frac{\mathbf{h}_{\mathrm{S},\epsilon}(\omega)}{1 + \alpha_{\epsilon}(\omega)} + \frac{\mathbf{h}_{\mathrm{DS}}(\omega)}{1 + \alpha_{\epsilon}^{-1}(\omega)}$$

  - $\alpha_{\epsilon}(\omega) = \alpha(\omega)\frac{\mathcal{W}_{\max}}{\mathcal{D}_{\max,\epsilon}}$
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  - ϵ controls the regularization

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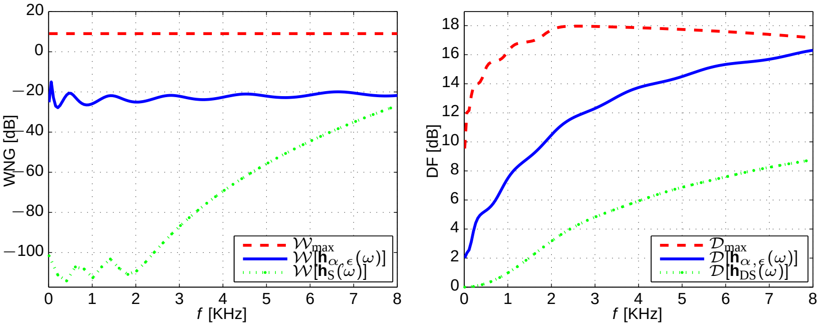

(a) WNG (b) DF

Array gains of $\mathbf{h}_{\alpha,\epsilon}(\omega)$ beamformer (solid line), with $\alpha = 1$, $\epsilon = 1 \cdot 10^{-5}$, $M = 8$, and $\delta = 1$ cm.

Fixed-WNG Beamformer

$$\mathcal{W}[\mathbf{h}_{\alpha,\epsilon}(\omega)] = f\{\mathcal{W}[\mathbf{h}_{\mathrm{DS}}(\omega)], \mathcal{W}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)]\} \quad (1)$$

- Design $\mathbf{h}_{\alpha,\epsilon}(\omega)$ such that
  - $\mathcal{W}_0$ is a constant user-determined WNG response
  - $\mathcal{W}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)] < \mathcal{W}_0 < M, \ \forall\omega$
- Substitute $\mathcal{W}_0$ in (1)

$$\Rightarrow \quad \alpha_\epsilon(\omega) = \cdots$$

$$\Rightarrow \quad \mathbf{h}_{\alpha_+,\epsilon}(\omega) = \frac{\mathbf{h}_{\mathrm{S},\epsilon}(\omega)}{1+\alpha_{\epsilon+}(\omega)} + \frac{\mathbf{h}_{\mathrm{DS}}(\omega)}{1+\alpha_{\epsilon+}^{-1}(\omega)}$$

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Fixed-DF Beamformer

$$\mathcal{D}[\mathbf{h}_{\alpha,\epsilon}(\omega)] = f\{\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)], \mathcal{D}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)]\} \quad (2)$$

- Design $\mathbf{h}_{\alpha,\epsilon}(\omega)$ such that
  - $\mathcal{D}_0$ is a fixed user-determined DF response
  - $\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)] < \mathcal{D}_0 < \mathcal{D}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)], \forall\omega$
- Substitute $\mathcal{D}_0$ in (2)

$$\Rightarrow \quad \alpha_\epsilon(\omega) = \cdots$$

$$\Rightarrow \quad \mathbf{h}_{\tilde{\alpha}_+,\epsilon}(\omega) = \frac{\mathbf{h}_{\mathrm{S},\epsilon}(\omega)}{1+\tilde{\alpha}_{\epsilon+}(\omega)} + \frac{\mathbf{h}_{\mathrm{DS}}(\omega)}{1+\tilde{\alpha}_{\epsilon+}^{-1}(\omega)}$$

Fixed-DF Beamformer

$$\mathcal{D}[\mathbf{h}_{\alpha,\epsilon}(\omega)] = f\{\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)], \mathcal{D}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)]\} \stackrel{!}{=} \mathcal{D}_0 \tag{2}$$

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- Substitute $\mathcal{D}_0$ in (2)

$$\Rightarrow \quad \alpha_\epsilon(\omega) = \cdots$$

$$\Rightarrow \quad \mathbf{h}_{\tilde{\alpha}_+,\epsilon}(\omega) = \frac{\mathbf{h}_{\mathrm{S},\epsilon}(\omega)}{1+\tilde{\alpha}_{\epsilon+}(\omega)} + \frac{\mathbf{h}_{\mathrm{DS}}(\omega)}{1+\tilde{\alpha}^{-1}_{\epsilon+}(\omega)}$$

Fixed-DF Beamformer

$$\begin{aligned} \mathcal{D}[\mathbf{h}_{\alpha,\epsilon}(\omega)] &= f\{\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)], \mathcal{D}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)]\} \\ &\stackrel{!}{=} \mathcal{D}_0 \end{aligned} \quad (2)$$

- Design $\mathbf{h}_{\alpha,\epsilon}(\omega)$ such that
  - $\mathcal{D}_0$ is a fixed user-determined DF response
  - $\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)] < \mathcal{D}_0 < \mathcal{D}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)], \ \forall\omega$
- Substitute $\mathcal{D}_0$ in (2)

$$\Rightarrow \quad \alpha_\epsilon(\omega) = \cdots$$

$$\Rightarrow \quad \mathbf{h}_{\tilde{\alpha}_+,\epsilon}(\omega) = \frac{\mathbf{h}_{\mathrm{S},\epsilon}(\omega)}{1 + \tilde{\alpha}_{\epsilon+}(\omega)} + \frac{\mathbf{h}_{\mathrm{DS}}(\omega)}{1 + \tilde{\alpha}_{\epsilon+}^{-1}(\omega)}$$

Fixed-DF Beamformer - Contd.

- Another interpretation of the DF:

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{\left|\mathcal{B}[\mathbf{h}(\omega), 0°]\right|^2}{\frac{1}{2}\int_0^{\pi} \left|\mathcal{B}[\mathbf{h}(\omega), \theta]\right|^2 \sin\theta d\theta}$$

- For the fixed-DF beamformer:
  - $\mathcal{D}_0^{-1} = \frac{1}{2}\int_0^{\pi} \left|\mathcal{B}[\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega), \theta]\right|^2 \sin\theta d\theta$

$\Rightarrow$ Frequency-invariant beampattern beamformer (FIBP) [Traverso et al., 2014]

Fixed-DF Beamformer - Contd.

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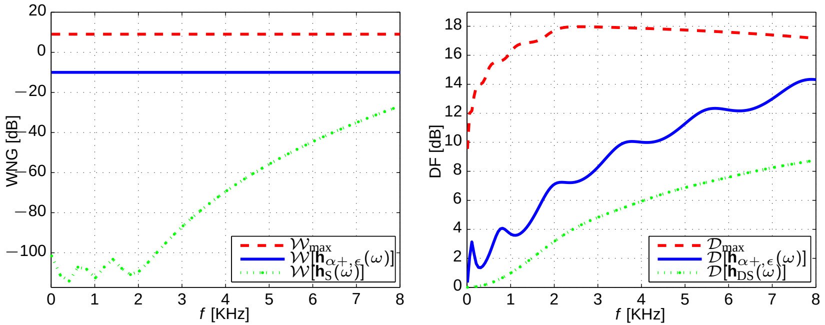
$\Rightarrow$ Frequency-invariant beampattern beamformer (FIBP) [Traverso et al., 2014]

$$\mathbb{E}_\theta\{\left|\mathcal{B}[\mathbf{h}(\omega), \theta]\right|^2\} = const, \ \forall\omega$$

Fixed-WNG Simulation

- $\mathcal{W}[\mathbf{h}_{\alpha+,\epsilon}(\omega)] = \mathcal{W}_0(\omega) = -10$ dB

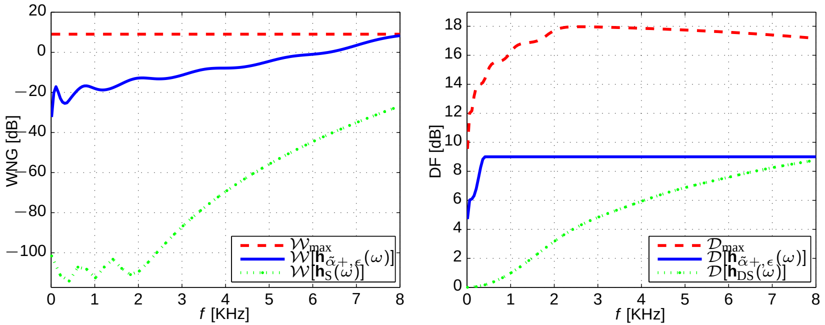


(a) WNG (b) DF

Array gains of $\mathbf{h}_{\alpha+,\epsilon}(\omega)$ (solid line). $\epsilon = 1 \cdot 10^{-4}$, $M = 8$ and $\delta = 1$ cm.

Fixed-DF Simulation

- $\mathcal{D}[\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega)] = \mathcal{D}_0(\omega) = 9$ dB



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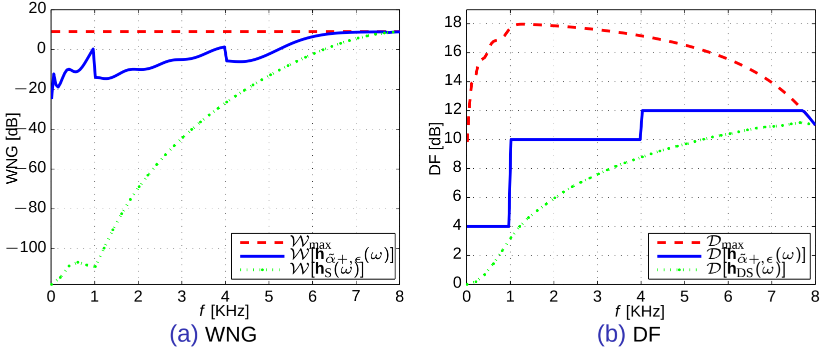
Array gains of $\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega)$ (solid line). $\epsilon = 1 \cdot 10^{-4}$, $M = 8$ and $\delta = 1$ cm.

User-Determined DF Simulation

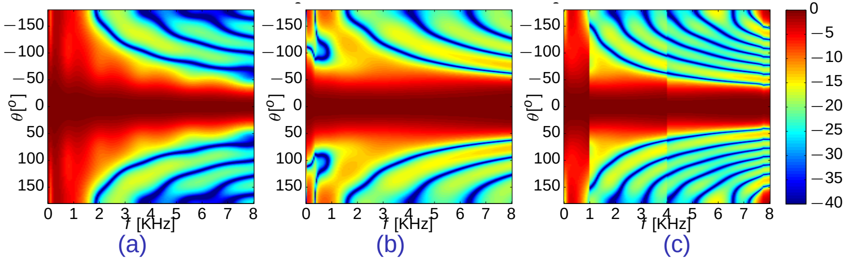
- Multiband $\mathcal{D}[\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega)] = \mathcal{D}_0(\omega)$



Array gains of $\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega)$ (solid line) with multiband $\mathcal{D}_0(\omega)$.
$\epsilon = 1 \cdot 10^{-4}$, $M = 8$, and $\delta = 1$ cm.

Beampattern Simulation

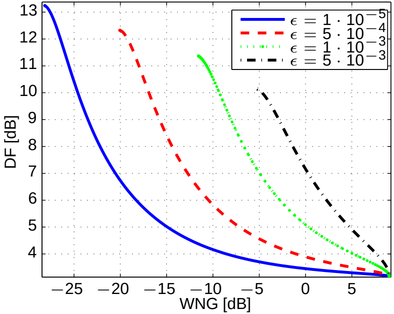
- Fixed-WNG & fixed-DF beampattern $\mathcal{B}[\mathbf{h}(\omega), \theta]$



The squared beampattern [dB] with $\epsilon = 1 \cdot 10^{-4}$, $M = 8$. (a) $|\mathcal{B}[\mathbf{h}_{\alpha+,\epsilon}(\omega), \theta]|^2$, with $\delta = 1$ cm. (b) $|\mathcal{B}[\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega), \theta]|^2$ ($\delta = 1$ cm). (c) $|\mathcal{B}[\mathbf{h}_{\tilde{\alpha}+,\epsilon}(\omega), \theta]|^2$ of the multiband user determined DF beamformer ($\delta = 2$ cm).

WNG–DF Tradeoff

- The DF curve vs. WNG of $\mathbf{h}_{\alpha,\epsilon}(\omega)$ for increasing $\alpha(\omega) = \alpha$



The DF curve versus WNG, of $\mathbf{h}_{\alpha,\epsilon}(\omega)$. $M = 8$, $\delta = 1$ cm, $f = 2$ KHz, and α is monotonically increased from 0 to ∞.

Part II

The Tunable Superdirective Beamformer

Based on:

- R. Berkun, I. Cohen, and J. Benesty “A Tunable Beamformer for Robust Superdirective Beamforming,” submitted to: *Signal Processing, IEEE Transactions on*.

Outline

Motivation

- Constrained optimization
  - No closed-form expressions
- Setting the regularization factor:
  - Iterative optimization process [Cox et al., 1987]
  - Ad hoc methods [Van Trees, 2002]
  - Optimize the Signal-to-Interference-plus-Noise-Ratio (SINR) [Mestre and Lagunas, 2006]
- The proposed approach:
  - Maximize DF *and* WNG

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Performance Measures

Directivity Factor

- The DF:

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{|\mathcal{B}[\mathbf{h}(\omega), 0°]|^2}{\frac{1}{2}\int_0^{\pi} |\mathcal{B}[\mathbf{h}(\omega), \theta]|^2 \sin\theta d\theta} = \frac{\left|\mathbf{h}^H(\omega)\mathbf{d}(\omega)\right|^2}{\mathbf{h}^H(\omega)\mathbf{\Gamma}_{0,\pi}(\omega)\mathbf{h}(\omega)}$$

- $\mathbf{\Gamma}_{\mathrm{d}} = \mathbf{\Gamma}_{0,\pi}(\omega) = \frac{1}{2}\int_0^{\pi} \mathbf{d}(\omega,\theta)\mathbf{d}^H(\omega,\theta)\sin\theta d\theta$

The Optimization problem

- Minimize some white noise $+$ some diffuse noise energy:

$$\min_{\mathbf{h}(\omega)} \mathbf{h}^H(\omega)\left[\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)\right]\mathbf{h}(\omega)$$
$$\text{subject to} \quad \mathbf{h}^H(\omega)\mathbf{d}(\omega) = 1$$

- $\epsilon_\psi = \frac{1-\cos\psi}{2}, \qquad 0 \le \psi \le \pi$
- $\mathbf{\Gamma}_{\psi,\pi}(\omega) = \frac{1}{2}\int_\psi^\pi \mathbf{d}(\omega,\theta)\mathbf{d}^H(\omega,\theta)\sin\theta d\theta$
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- ψ controls the compromise between DF & WNG

The Tunable Beamformer

- The Tunable beamformer:

$$\mathbf{h}_{\mathrm{T},\psi}(\omega) = \frac{[\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)]^{-1}\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)[\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)]^{-1}\mathbf{d}(\omega)}$$

- ψ determines the regularization factor
- $0 \leq \epsilon_\psi \leq 1$

Generalization

- Additional linear constraints
  - A null in the direction π: $\mathbf{h}^H(\omega)\mathbf{d}(\omega,\pi) = 0$
- The optimization problem:

$$\min_{\mathbf{h}(\omega)} \mathbf{h}^H(\omega)[\epsilon_\psi \mathbf{I}_M + \Gamma_{\psi,\pi}(\omega)]\mathbf{h}(\omega)$$

$$\text{subject to} \quad \mathbf{h}^H(\omega)\mathbf{C}(\omega) = \mathbf{i}^T$$

$$\mathbf{C}(\omega) = [\ \mathbf{d}(\omega) \quad \mathbf{d}(\omega,\pi)\ ],$$

$$\mathbf{i} = [\ 1 \quad 0\ ]^T$$

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$$\mathbf{C}(\omega) = [\ \mathbf{d}(\omega) \quad \mathbf{d}(\omega,\pi)\ ],$$

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Generalization - Contd.

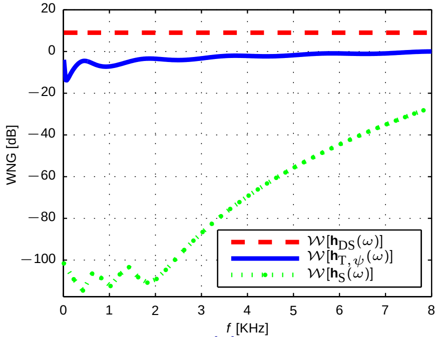
- The generalized beamformer:

$$\mathbf{h}_{\mathrm{C},\psi}(\omega) = \boldsymbol{\Upsilon}_{\psi}^{-1}(\omega)\mathbf{C}(\omega)[\mathbf{C}^{H}(\omega)\boldsymbol{\Upsilon}_{\psi}^{-1}(\omega)\mathbf{C}(\omega)]^{-1}\mathbf{i}$$

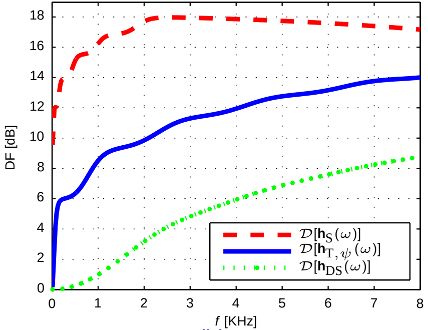
$$\boldsymbol{\Upsilon}_{\psi}(\omega) = \epsilon_{\psi}\mathbf{I}_{M} + \boldsymbol{\Gamma}_{\psi,\pi}(\omega)$$

WNG & DF Simulation

- The Tunable beamformer WNG & DF simulation



(a) WNG

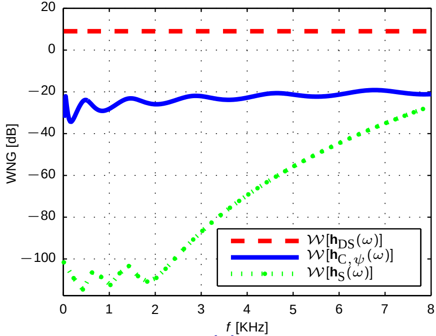


(b) DF

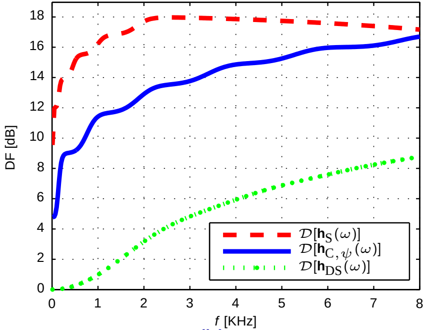
Array gains of $\mathbf{h}_{\mathrm{T},\psi}(\omega)$ (solid line) with $\psi = 10^\circ$. $M = 8$ and $\delta = 1$ cm.

WNG & DF Simulation - Contd.

- The generalized beamformer WNG & DF simulation



(a) WNG

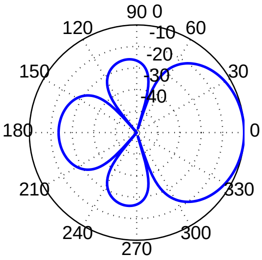


(b) DF

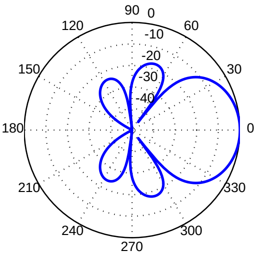
Array gains of $\mathbf{h}_{C,\psi}(\omega)$ (solid line) with $\psi = 0.5^\circ$. $M = 8$ and $\delta = 1$ cm.

Beampattern Simulation

- The Tunable beamformer and the generalized beamformer beampattern $\mathcal{B}[\mathbf{h}(\omega), \theta]$



(a) $\mathbf{h}_{\mathrm{T},\psi}(\omega)$

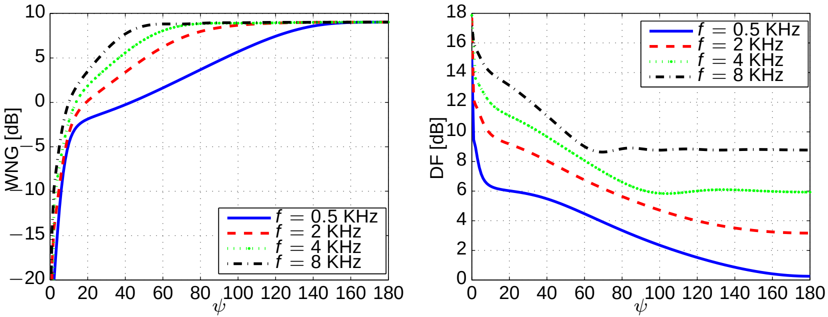


(b) $\mathbf{h}_{\mathrm{C},\psi}(\omega)$

The normalized beampattern $\mathcal{B}[\mathbf{h}(\omega), \theta]$ [dB] versus θ (at $f = 2$ KHz), with $M = 8$ and $\delta = 1$ cm. (a) $\psi = 10^\circ$. (b) $\psi = 0.5^\circ$.

Array Gains vs. ψ Simulation

- The Tunable beamformer WNG and DF vs. ψ



(a) WNG (b) DF

(a)-(b) Gain curves of $\mathbf{h}_{\mathrm{T},\psi}(\omega)$ versus ψ, with $M = 8$ microphones and $\delta = 1$ cm, at selected frequencies.

Part III

Microphone Array Reverberation Assessment

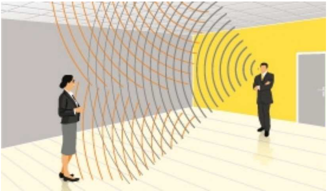
Based on:

- R. Berkun, I. Cohen, “Microphone Array Power Ratio for Quality Assessment of Reverberated Speech,” *Silencing the Echoes - Processing of Reverberant Speech, EURASIP Journal on Advances in Signal Processing*, vol. 2015, no. 1, pp. 1–11, June 2015.

Outline

Motivation

- Reverberation
  - Late & uncorrelated reflections
- Speech communication systems
  - Distributed microphones environment
  - Quality monitoring
    - Select the best-quality channel
    - Pick-up least reverberant signal
- Quality measurement of reverberated speech



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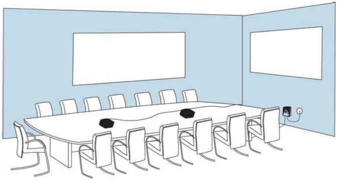

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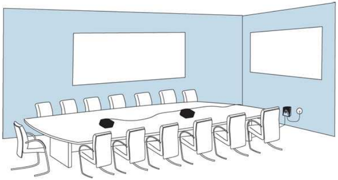

Related Work

- Reverberation measurement methods:
  - Quantifying system characteristics
    - Direct-to-reverberation energy ratio [Kuttruff, 2009]

$$\text{DRR} = \frac{E_d}{E_r} = \frac{\int_0^{T_d} h^2(\tau)d\tau}{\int_{T_d}^{\infty} h^2(\tau)d\tau}$$

  - Comparison to clean reference [Naylor and Habets, 2010]
  - Direct signal-based evaluation
    - Signal energy
    - Signal-to-noise ratio (SNR) [Wolf and Nadeu, 2014]

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Related Work – Contd.

- Signal-based quality measures:
  - Signal-to-diffuse ratio estimation
    - Spatial complex coherence between microphones [Jeub et al., 2011]
    - Direct & diffuse part segregation using beamforming [Thiergart et al., 2014, Hioka et al., 2012]
  - Modulation spectral analysis: Speech to reverberation modulation energy ratio (SRMR) [Falk et al., 2010]
- Low correlation of signal-based measures with subjective listening tests [Goetze et al., 2010]
  - High correlation of the *Clarity* measure (C50) [Kuttruff, 2009]

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Related Work – Contd.

- The optimal solution:
  - Simple & reliable measure
  - Estimate quality & reverberation level
  - Signal-based
  - Correlative to objective parameters

Problem Formulation

- The measured signal:

$$z(t) = \int_{-\infty}^{t} s(\tau)h(t-\tau)d\tau + v(t),$$

  - $s(t)$ anechoic speech signal
  - $h(t)$ causal room impulse response (RIR)
  - $v(t) = 0$ ambient noise

- Reverberated RIR model:

$$h(t) = \begin{cases} h_d(t), & \text{for } 0 \leq t < T_r \\ h_r(t), & \text{for } t \geq T_r \\ 0, & \text{otherwise,} \end{cases}$$

- The objective:
  - Estimate the reverberation (quality & DRR)

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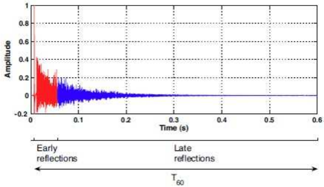

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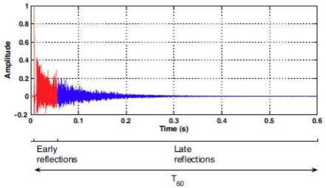

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Signal Model

- Statistical Room Acoustics (SRA) model [Polack, 1988]:

$$h_d(t) = \begin{cases} b_d(t)e^{-\delta t}, & \text{for } 0 \leq t < T_r \\ 0 & \text{otherwise,} \end{cases}$$

  - $b_d(t) \sim \mathcal{N}(0, \sigma_d^2)$
  - $\delta = \frac{3 \ln 10}{T_{60}}$

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⇒ The measured signal energy:

$$\mathbb{E}_z\{z^2(t)\} = \mathbb{E}_z\{z_d^2(t)\} + \mathbb{E}_z\{z_r^2(t)\}$$

  - $\lambda_s(t) = \mathbb{E}_s\{s^2(t)\}$
  - $\mathbb{E}_z\{z_d^2(t)\} = f(\lambda_s(t), \sigma_d^2, T_r)$
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  - $\mathbb{E}_z\{z_d^2(t)\} = f(\lambda_s(t), \sigma_d^2, T_r)$
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  - $b_r(t) \sim \mathcal{N}(0, \sigma_r^2)$

⇒ The measured signal energy:

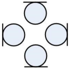
$$\mathbb{E}_z\{z^2(t)\} = \mathbb{E}_z\{z_d^2(t)\} + \mathbb{E}_z\{z_r^2(t)\}$$

  - $\lambda_s(t) = \mathbb{E}_s\{s^2(t)\}$
  - $\mathbb{E}_z\{z_d^2(t)\} = f(\lambda_s(t), \sigma_d^2, T_r)$
  - $\mathbb{E}_z\{z_r^2(t)\} = f(\lambda_s(t), \sigma_r^2, T_r)$

Directional Array Response

- Expansion of the SRA model

- Unidirectional microphone array
  - Directional elements
  - Beamforming



- $g^{\text{dir/opp}}(\theta)$ - The microphone directional gain at angle θ

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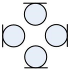

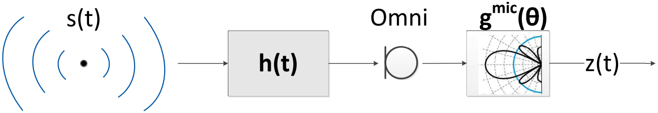

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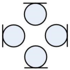

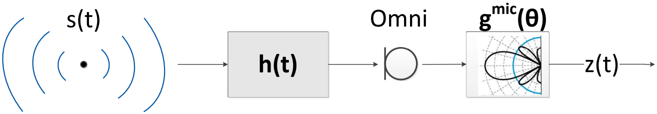

- $g^{\text{dir/opp}}(\theta)$ - The microphone directional gain at angle θ

Directional Array Response - Contd.

⇒ The direct microphone signal energy:

$$\mathbb{E}_z\{[z^{\text{dir}}(t)]^2\} = [g^{\text{dir}}(\theta)]^2 \cdot \mathbb{E}_z\{z_d^2(t)\} + \frac{1}{\Omega}\int_\Omega [g^{\text{dir}}(\theta')]^2 d\theta' \cdot \mathbb{E}_z\{z_r^2(t)\}$$

⇒ The opposite microphone signal energy:

$$\mathbb{E}_z\{[z^{\text{opp}}(t)]^2\} = \frac{1}{\Omega}\int_\Omega [g^{\text{opp}}(\theta')]^2 d\theta' \cdot \mathbb{E}_z\{z_r^2(t)\}$$

Directional Array Response - Contd.

⇒ The direct microphone signal energy:

$$\mathbb{E}_z\{[z^{\text{dir}}(t)]^2\} = [g^{\text{dir}}(\theta)]^2 \cdot \mathbb{E}_z\{z_d^2(t)\} + \frac{1}{\Omega}\int_{\Omega}[g^{\text{dir}}(\theta')]^2 d\theta' \cdot \mathbb{E}_z\{z_r^2(t)\}$$

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The Proposed Measure

- Assuming the microphones are calibrated:
  - $\bar{g}^2 = \frac{1}{\Omega}\int_\Omega [g^{\text{dir}}(\theta')]^2 d\theta' = \frac{1}{\Omega}\int_\Omega [g^{\text{opp}}(\theta')]^2 d\theta'$
- The Power Ratio between the direct & opposite microphones:

$$\frac{\mathbb{E}_z\{[z^{\text{dir}}(t)]^2\}}{\mathbb{E}_z\{[z^{\text{opp}}(t)]^2\}} = \frac{[g^{\text{dir}}(\theta)]^2}{\bar{g}^2} \cdot [\frac{\sigma_d^2}{\sigma_r^2}(e^{2\delta T_r} - 1)] + 1$$

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The Proposed Measure - Contd.

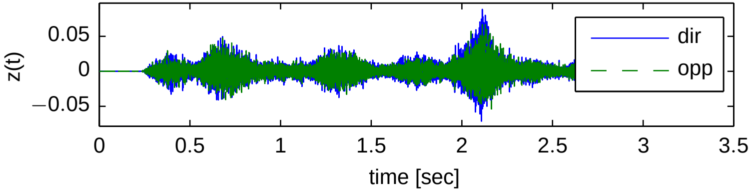
- Replace $\mathbb{E}_z\{\cdot\} \leftrightarrow$ temporal smoothing

$\Rightarrow$ The Directional Power Ratio quality measure:

$$\mathrm{PR}(t) = \frac{P^{\mathrm{dir}}(t)}{P^{\mathrm{opp}}(t)} = \frac{\int_{t-T}^{t} [z^{\mathrm{dir}}(\tau)]^2 d\tau}{\int_{t-T}^{t} [z^{\mathrm{opp}}(\tau)]^2 d\tau} = \frac{[g^{\mathrm{dir}}(\theta)]^2}{\bar{g}^2} \cdot \mathrm{DRR}(t) + 1$$

The Proposed Measure - Contd.

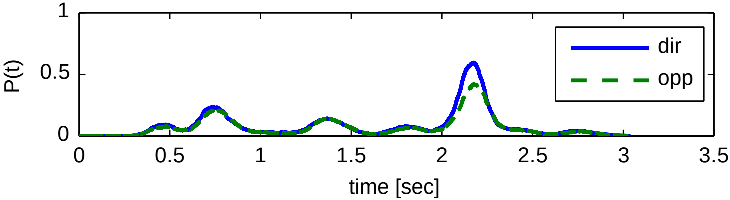
Measuring the power ratio of a speech signal. source-microphone distance $= 2$ m, $T_{60} = 1$ sec.



(a) The measured signal $z(t)$

The Proposed Measure - Contd.

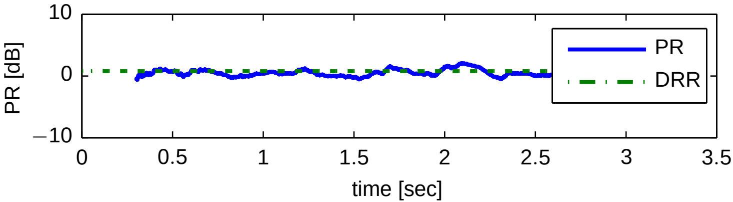
Measuring the power ratio of a speech signal. source-microphone distance $= 2$ m, $T_{60} = 1$ sec.



(b) The directional measured power $P(t)$.

The Proposed Measure - Contd.

Measuring the power ratio of a speech signal. source-microphone distance $= 2$ m, $T_{60} = 1$ sec.



(c) The measured power ratio PR(t) (solid line) and $\frac{[g^{\text{dir}}(\theta)]^2}{\bar{g}^2} \cdot \text{DRR} + 1$ (dashed-dotted line).

The Proposed Measure - Contd.

- Blind DRR estimator - $\widehat{\mathrm{DRR}}(t)$:

$$\text{PR-DRR}(t) = \frac{\bar{g}^2}{[g^{\text{dir}}(\theta)]^2} \cdot \left(\frac{P^{\text{dir}}(t)}{P^{\text{opp}}(t)} - 1 \right) = \frac{\bar{g}^2}{[g^{\text{dir}}(\theta)]^2} \cdot \frac{P^{\text{dir}}(t) - P^{\text{opp}}(t)}{P^{\text{opp}}(t)}$$

Simulations
Experiment Conditions

- Performed experiments:
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  - Increasing T_{60} with fixed source-microphone distance
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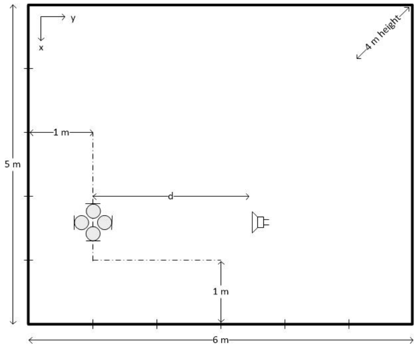

Quality Measure Performance

- Reference quality measures
  - SRMR [Falk et al., 2010]
  - Envelope Variance (EV) [Wolf and Nadeu, 2014]
- Correlation coefficients with:
  - *Clarity* (C50) [Kuttruff, 2009]
  - ITU-T P.563 [Recommendation, 2004]
  - ITU-T P.862 (PESQ) [Recommendation, 2001]

Input type		White noise	Speech signals		
		Correlation ref.	Correlation ref.		
Test type	Algorithm	C50	C50	PESQ	P. 563
$T_{60} = 0.3$ sec,	PR	0.999	**0.999**	0.911	0.712
Vs.	SRMR	-0.27	**0.845**	0.973	0.934
increasing distance	EV	-0.66	**0.931**	0.994	0.875
distance = 0.5 m,	PR	0.944	**0.951**	0.899	0.562
Vs.	SRMR	0.392	**0.640**	0.991	0.873
increasing T_{60}	EV	0.235	**0.614**	0.984	0.912

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Blind DRR Estimator Performance

- Reference DRR measure
  - CDR-based DRR [Jeub et al., 2011]
- Correlation coefficient with:
  - DRR

Input type		White noise	Speech signals
Test type	Algorithm	Correlation ref. DRR	Correlation ref. DRR
$T_{60} = 1$ sec, Vs. increasing distance	PR-DRR	0.999	0.999
	CDR	0.964	0.972
distance $= 2$ m, Vs. increasing T_{60}	PR-DRR	0.999	0.999
	CDR	0.852	0.913

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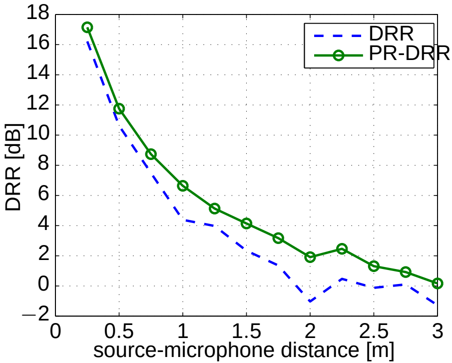
distance $= 2$ m, Vs. increasing T_{60}	PR-DRR	0.999	0.999
	CDR	0.852	0.913

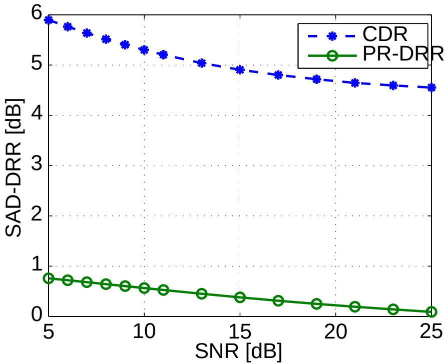
Blind DRR Estimator Performance - Contd.

The proposed DRR estimate vs. distance. PR-DRR [dB] (solid-circled line), and the true DRR [dB] (dashed-line), vs. source-microphone distance, with fixed $T_{60} = 0.3$ sec.



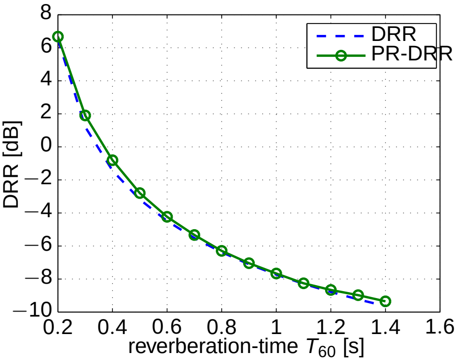
Blind DRR Estimator Performance - Contd.

Performance of the DRR estimate vs. SNR. Absolute difference of the proposed DRR estimate PR-DRR [dB] (solid-circled line), and of Jeub et al. CDR-based DRR estimate [dB] (dashed-asterisk line), vs. SNR [dB]. $T_{60} = 0.3$ sec and source-microphone distance $= 0.5$ m.



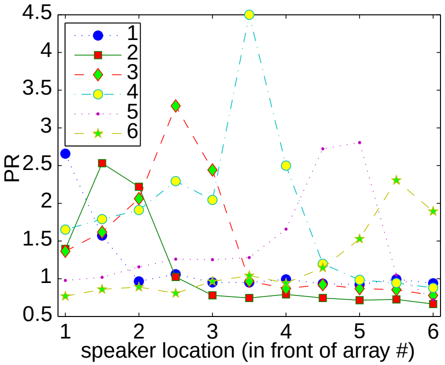
Blind DRR Estimator Performance - Contd.

The proposed DRR estimate vs. T_{60} - Off main-lobe. PR-DRR [dB] (solid-circled line), and the true DRR [dB] (dashed line), vs. T_{60}. (source–receiver angle $\in [-30^\circ .. +30^\circ]$, source-microphone distance $= 2$ m)



Blind DRR Estimator Performance - Contd.

Recorded speech PR measure vs. source location. The measured PR of all microphone arrays (1 – 6) vs. the source position (hall of size 15 × 10 × 6 m, with 3 m spacing between adjacent arrays)



Conclusions

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Robust Regularized Superdirective Beamforming

- New approach to robust regularized beamforming
  - Combination of conventional and regularized beamformers
  - Achieve robust distortionless high-gain solution
  - Filter parameters determine the frequency response
  - Control of the WNG–DF tradeoff
- Derived closed-form beamformers with user-determined WNG or DF
  - A unique form of the FIBP

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Conclusion

The Tunable Superdirective Beamformer

- Novel method to control the white noise & diffuse noise energy minimization
- New perspective for beamforming and angular noise field analysis
- Tunable regularization parameter
  - Closed-form expression for the regularization parameter $0 \leq \epsilon_\psi \leq 1$
- Generalized multiple linear constraints beamformer

Conclusion

Microphone Array Reverberation Assessment

- New approach to reverberation ratio estimation
  - Expansion of a well-founded statistical model, using directional microphone arrays
  - Signal-based acoustic quality measure
  - Blind DRR estimator
- Robust performance
  - Correlative to state-of-the-art quality & DRR measures
  - Robust against additive noise

Future research

- Regularized Superdirective Beamforming:
  - Different noise fields
  - Tailored frequency-dependent regularization
  - Broadside beamforming, differential microphones, etc.
- The Tunable Beamformer:
  - Linking the tunable regularization to the WNG & DF
  - Noise field angular analysis
- Reverberation assessment:
  - Microphone beampattern analysis
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Thank You

Thank You!

Questions?

- Contact: berkun@tx.technion.ac.il
- Prof. Israel Cohen: http://webee.technion.ac.il/people/IsraelCohen/
- Prof. Jacob Benesty: http://externe.emt.inrs.ca/users/benesty/

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The Delay & Sum (DS) Beamformer

- Maximizes the WNG
  - subject to distortionless constraint

$$\mathbf{h}_{\mathrm{DS}}(\omega) = \frac{\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\mathbf{d}(\omega)} = \frac{\mathbf{d}(\omega)}{M}$$

- $\mathcal{W}[\mathbf{h}_{\mathrm{DS}}(\omega)] = M = \mathcal{W}_{\max}$
- $\mathcal{D}[\mathbf{h}_{\mathrm{DS}}(\omega)] = \frac{M^2}{\mathbf{d}^H(\omega)\mathbf{\Gamma}_{\mathrm{d}}(\omega)\mathbf{d}(\omega)} \geq 1$

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The Superdirective Beamformer

- Maximizes the DF
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$$\mathbf{h}_{\mathrm{S}}(\omega) = \frac{\boldsymbol{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\boldsymbol{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega)}$$

- $\mathcal{W}[\mathbf{h}_{\mathrm{S}}(\omega)] = \mathcal{W}_{\max}\cos^2\varphi(\omega)$
  - $\cos\varphi(\omega) = \cos\left[\mathbf{d}(\omega), \boldsymbol{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega)\right] = \frac{\mathbf{d}^H(\omega)\boldsymbol{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega)}{\sqrt{\mathbf{d}^H(\omega)\mathbf{d}(\omega)}\sqrt{\mathbf{d}^H(\omega)\boldsymbol{\Gamma}_{\mathrm{d}}^{-2}(\omega)\mathbf{d}(\omega)}}$
- $\mathcal{D}[\mathbf{h}_{\mathrm{S}}(\omega)] = \mathbf{d}^H(\omega)\boldsymbol{\Gamma}_{\mathrm{d}}^{-1}(\omega)\mathbf{d}(\omega) = \mathcal{D}_{\max}(\omega)$

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The Regularized Superdirective Beamformer

- Maximizes the DF [Cox et al., 1987, Benesty et al., 2008]
  - Constraint on the WNG
  - subject to distortionless constraint

$$\mathbf{h}_{\mathrm{S},\epsilon}(\omega) = \frac{\left[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega) + \epsilon \mathbf{I}_M\right]^{-1} \mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\left[\boldsymbol{\Gamma}_{\mathrm{d}}(\omega) + \epsilon \mathbf{I}_M\right]^{-1} \mathbf{d}(\omega)}$$

  - Define: $\boldsymbol{\Gamma}_{\epsilon}(\omega) = \boldsymbol{\Gamma}_{\mathrm{d}}(\omega) + \epsilon \mathbf{I}_M$
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- $\mathcal{D}[\mathbf{h}_{\mathrm{S},\epsilon}(\omega)] = \frac{\left[\mathbf{d}^H(\omega)\boldsymbol{\Gamma}_{\epsilon}^{-1}(\omega)\mathbf{d}(\omega)\right]^2}{\mathbf{d}^H(\omega)\boldsymbol{\Gamma}_{\epsilon}^{-1}(\omega)\boldsymbol{\Gamma}_{\mathrm{d}}(\omega)\boldsymbol{\Gamma}_{\epsilon}^{-1}(\omega)\mathbf{d}(\omega)}$

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