Design Methods of Sparse and Robust Differential Microphone Arrays (DMAs)

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Ph.D. research under the advisement of Prof. Israel Cohen

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Outline

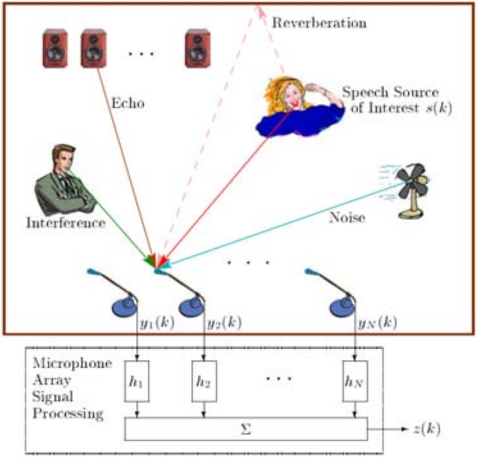
Microphone arrays

Applications:

- Teleconferencing
- Hands-free communications
- Mobile phones
- Hearing aids

Relevant problems:

- Noise reduction
- Echo reduction
- Dereverberation
- Source localization
- Source separation
- Cocktail party



From J. Benesty et al, 2008

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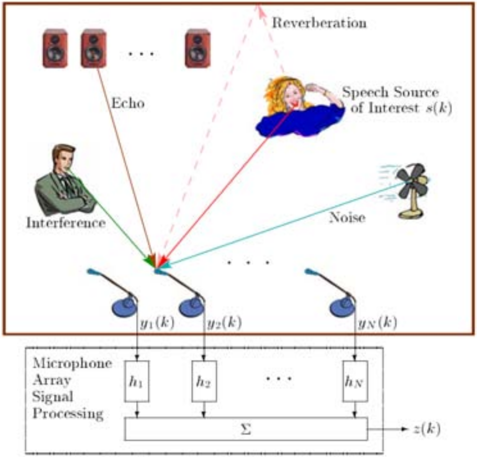
} Beamforming



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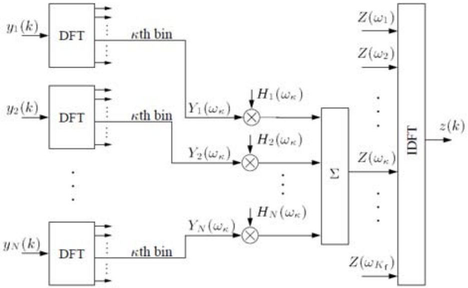
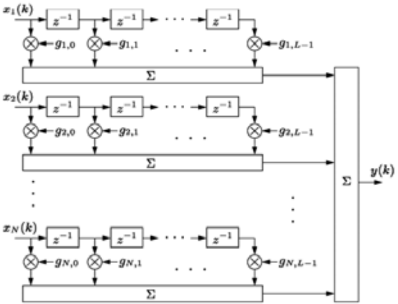
Block diagram of a Broadband beamformer

Frequency-domain



Time-domain



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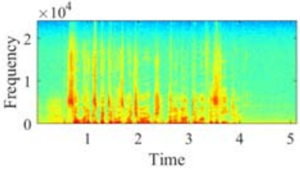
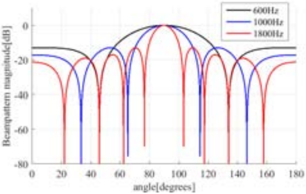
NBBF
Apps

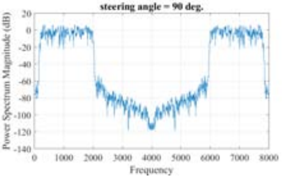
The broadband nature of speech signals

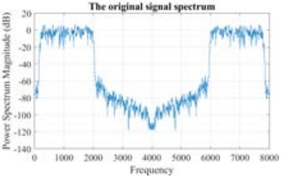


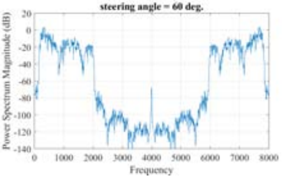
- Typically, speech signals are broadband
- Frequency-dependent directivity pattern
- $BW_{3dB} \propto \cos^{-1}(c/M\delta f)$



Gaussian source from broadside, linear array, $M = 10$ mic. , $\delta = 0.08$[m], DAS beamformer







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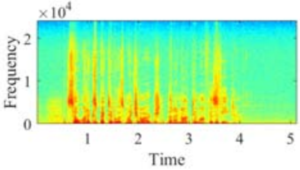

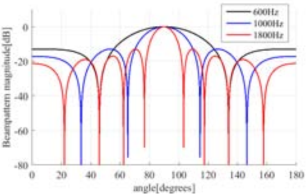

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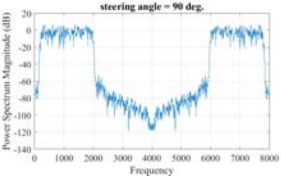

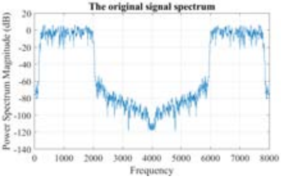

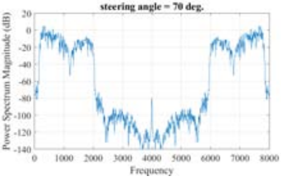

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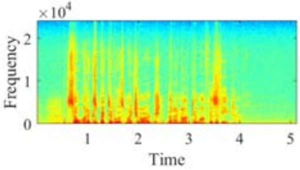

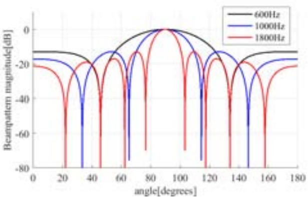

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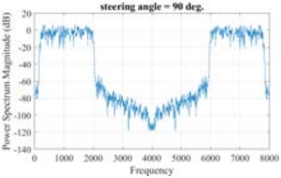
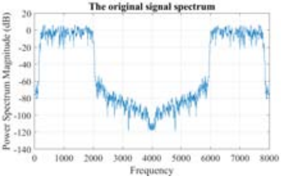
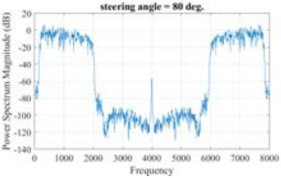




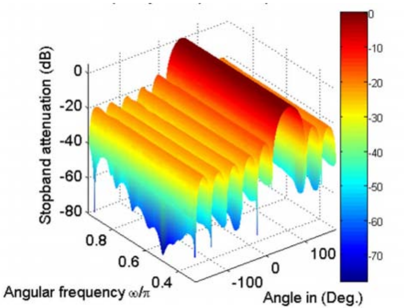


Classical design approaches of frequency-invariant (FI) broadband beamforming

Direct optimization [Crocco et al., 11′][Kellerman et al., 09′]:

$$J(\mathbf{h}) = \sum_{f_j} \sum_{\theta_l} \left| B\left(f_j, \theta_l, \mathbf{h}\right) - B_D\left(\theta_l\right) \right|^2$$

$$\begin{aligned} \underset{\mathbf{h}}{\text{minimize}} \quad & J(\mathbf{h}) \\ \text{subject to} \quad & \|\mathbf{h}\|^2 < \alpha \\ & \text{DF} > \beta \end{aligned}$$

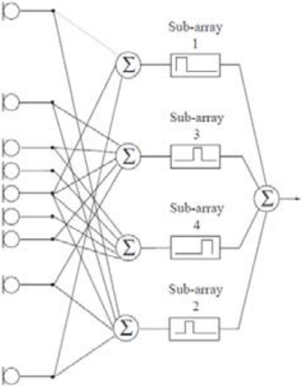


DF - Directivity Factor

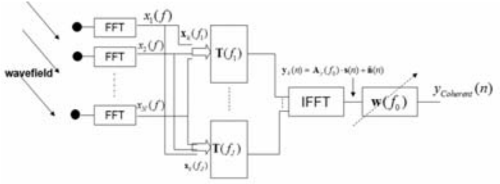
The vector **h** can be either in the frequency domain or the time domain.

Classical approaches to design of frequency-invariant (FI) broadband beamforming (2)

Nested arrays and unequally spaced arrays [Ward et al., 95′, Flanagan et al. 91′]



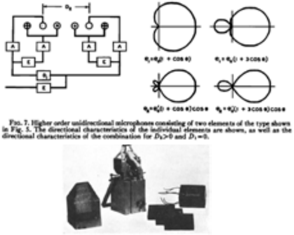
Coherent subspace methods [Hung & Kaveh , 88′, Buchris et al., 12′]



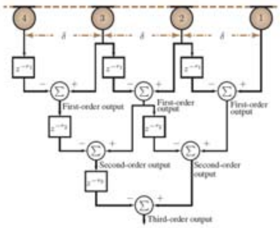
$$\mathbf{x}_k\left(f_j\right) = \mathbf{D}_\theta\left(f_j\right)\mathbf{s}_k\left(f_j\right) + \mathbf{n}_k\left(f_j\right)$$

$$\mathbf{T}\left(f_j\right)\mathbf{D}_\theta\left(f_j\right) \cong \mathbf{D}_\theta\left(f_0\right)$$

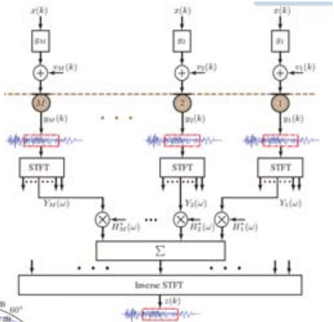
Alternative approach: Differential Microphone Arrays (DMAs)



[Olson, 33′, 46′]

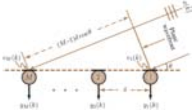


[Elko, 95′,00′]

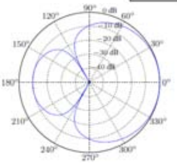


[Benesty, Cohen, 2012′, 2015′]

Model assumption:



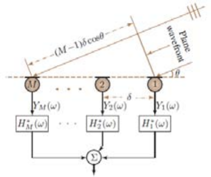
- $\delta \ll \lambda$
- Steering at the endfire ($\theta = 0°$)



FI Beampattern of Nth order DMAs:

$$\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos^n \theta$$

Performance measures

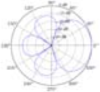


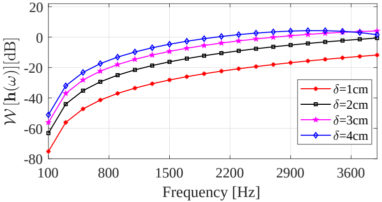
The Beampattern $\mathcal{B}\left[\mathbf{h}(\omega), \theta\right] = \mathbf{d}^H(\omega, \cos\theta)\mathbf{h}(\omega)$

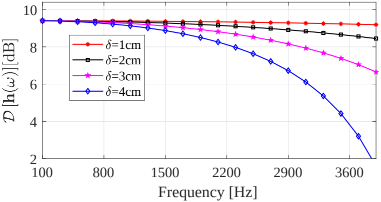
$$\mathbf{h}(\omega) = \left[H_1(\omega), H_2(\omega), ..., H_M(\omega)\right]^T$$

white noise gain (WNG) **directivity factor (DF)**

$$\mathcal{W}\left[\mathbf{h}(\omega)\right] = \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \theta_s)\right|^2}{\mathbf{h}^H(\omega)\,\mathbf{h}(\omega)}, \quad \mathcal{D}\left[\mathbf{h}(\omega)\right] = \frac{2\pi\left|\mathcal{B}\left[\mathbf{h}(\omega), \theta_s\right]\right|^2}{\int_0^{2\pi}\left|\mathcal{B}\left[\mathbf{h}(\omega), \theta\right]\right|^2 d\theta}$$

Example: Second-order hypercardioid: 





Large δ $\rightarrow$ improved WNG
Small δ $\rightarrow$ better frequency-invariance BP

Differential Microphone Arrays: pros and cons

$$\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos^n \theta$$

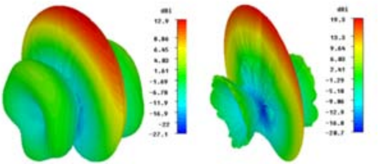
Pros:

- Very compact in size
- They can form frequency-invariant beampattern
- Superdirective: $DF \to M^2$

Cons:

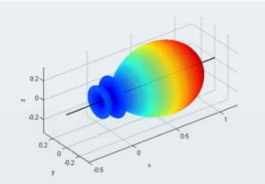
- White noise amplification, especially at low frequencies
- Desired signal direction is from the endfire (for linear geometry)

Beampattern of conventional beamformer

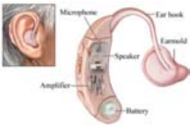


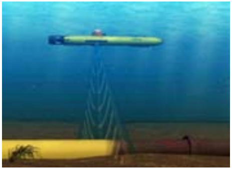
$M = 6$ $\quad$ $M = 20$

Beampattern of DMAs



Differential Microphone Arrays: potential applications





Journal Publications

- Y. Buchris, I. Cohen, and J. Benesty, "On the design of time-domain differential microphone arrays," *Applied Acoustics*, vol. 148, pp. 212-222, May 2019.
- Y. Buchris, I. Cohen, and J. Benesty, "Frequency-domain design of asymmetric circular differential microphone arrays," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, no. 4, pp. 760-773, April. 2018.
- Y. Buchris, A. Amar, I. Cohen, and J. Benesty, "Incoherent Synthesis of Sparse Arrays for Frequency-Invariant Beamforming," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482-495, March 2019.
- Y. Buchris, I. Cohen, J. Benesty, and A. Amar, "Joint Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern," submitted to *IEEE/ACM Transactions on Audio, Speech, and Language Processing*.

Thesis Overview and Main Contributions

This research is focused in several practical design aspects of DMAs and superdirective beamformers. There are four main contributions:

- A new formulation in the time domain for DMAs design, which may be useful in cases where minimal delay is required, and consumes less computational resources when short filters are sufficient
- Derivation of a general asymmetric model for circular DMAs which provides more flexible designs with higher array gains, compared to symmetric circular DMAs
- An incoherent sparse design for DMAs which obtains superior performance, with a reduced number of sensors and reduced computational complexity
- A greedy based sparse design of DMAs for other geometries like concentric arrays, where the beampattern can be steered azimuthally to any direction

Design and analysis of linear time-domain DMAs - motivation

- Previous work on time-domain beamforming, mainly considered the narrowband or the frequency-dependent broadband model [Van Trees 02′, Godara 95′]
- A small number of recent papers have considered the case of time-domain FI design [De sena, 12′][Nongpiur, 13′][Mabande, 11′][Godara, 07′][Yan, 06′,07′][Ward, 96′]
- Their basic approach is to optimize the filters by incorporating several constraints in the frequency domain
- Typically, constraints like FI beampattern, minimal WNG, SL level, etc., are considered
- The performance measures were also defined in the frequency domain

Design and analysis of linear time-domain DMAs - motivation(2)

- In this work, we derive the time-domain equivalent design of Nth-order DMAs
- The design is general and can be applied directly to any order and any number of sensors
- A simple closed-form solution is derived which does not involve any additional constraints in the frequency-domain
- In addition, we establish the time-domain version of the performance measures used for the assessment of beamformers
- We show that under the DMA's assumption, the proposed design converged to the traditional theoretical model of DMAs

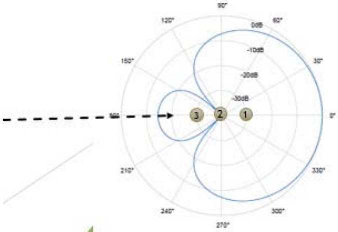
From linear to circular DMAs

- Linear DMAs can provide FIBP and are suitable for signals like speech
- Yet, desired signal is confined to the endfire direction
- Circular DMAs (CDMAs) serve as a good alternative CompLinearCircular
- CDMAs can provide similar response in all azimuthal directions
- More suitable for 2D/3D sound recording applications like teleconferencing

- Previous works impose constraints in the frequency domain to get the FIBP property [Chan, 07'][Yang 03'][Zhang 10']
- Benesty et al., [2015] has applied the general design proposed for linear DMAs, also to the circular geometry RecentWorkCircular

Asymmetric CDMAs - motivation

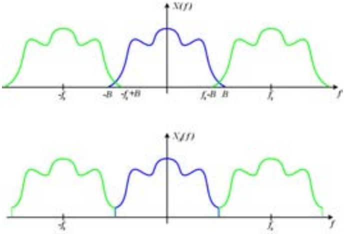
- The traditional design of DMAs was focused on linear geometry
- Linear geometry enforces symmetric beampatterns w.r.t. the array axis
- Herein, a new general theoretical model for circular DMAs is derived
- It includes the extension of the theoretical Nth-order DMAs directivity pattern into a more general model which includes both the symmetric and the asymmetric case
- Asymmetric versions for hypercardioid and supercardioid are derived, yielding better performance due to a more flexible design



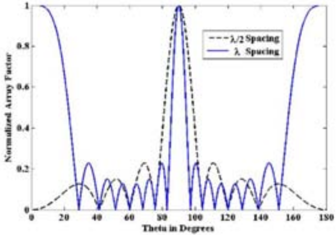
Element spacing trade-off on uniform arrays

- For $\delta > \frac{\lambda}{2}$: grating lobes appear

temporal aliasing: $f_s < 2f_{max}$



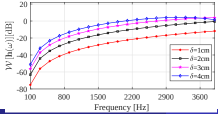
spatial aliasing: $\delta > \frac{\lambda_{min}}{2}$



- For the case that $\delta \ll \frac{\lambda}{2}$, white noise amplification is occurred since $\mathbf{h}(\omega) = f(\mathbf{R}^{-1}(\mathbf{D}(\omega,\Theta)), \mathbf{D}^{-1}(\omega,\Theta))$
- where the steering matrix $\mathbf{D}(\omega,\Theta)$ is typically an ill-conditioned

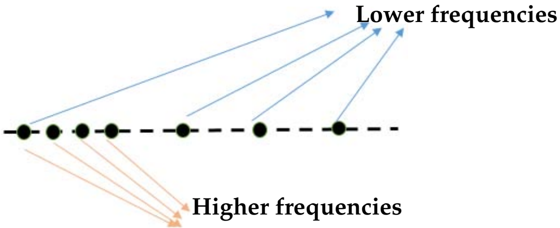
$$\mathbf{D}(\omega,\Theta) = [\mathbf{d}(\omega,\theta_1), \mathbf{d}(\omega,\theta_2), \mathbf{d}(\omega,\theta_3), ..., \mathbf{d}(\omega,\theta_L)]$$

- especially at low frequencies



Sparse design - motivation

- The aforementioned trade-off dictates small element spacing for broadband signals and uniform arrays
- Instead, nonuniform array design may relax this trade-off



- Sparse approaches optimize both the number of sensors and their locations
- Yielding robust arrays with reduced amount of sensors and without grating lobes
- Sparsity may lead to more flexibility in the design of FI beamformers

Joint sparsity

- One essential constraint that should be considered during a sparse design of broadband beamformer is the **joint-sparse** one:

$$\mathbf{H}(\omega) = \begin{bmatrix} H_1(\omega_1) & H_1(\omega_2) & \cdots & H_1(\omega_J) \\ H_2(\omega_1) & H_2(\omega_2) & \cdots & H_2(\omega_J) \\ H_3(\omega_1) & H_3(\omega_2) & \cdots & H_3(\omega_J) \\ \vdots & \vdots & \ldots & \vdots \\ H_M(\omega_1) & H_M(\omega_2) & \cdots & H_M(\omega_J) \end{bmatrix}$$

- Then,

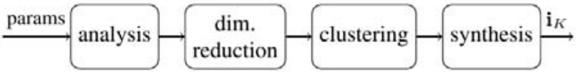
$$\mathbf{H}_{\text{JS}}(\omega) = \begin{bmatrix} H_1(\omega_1) & H_1(\omega_2) & \cdots & H_1(\omega_J) \\ 0 & 0 & \cdots & 0 \\ H_3(\omega_1) & H_3(\omega_2) & \cdots & H_3(\omega_J) \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Previous work on sparse design

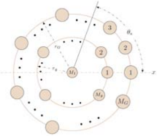
- Sparse design for the **narrowband** case [Trucco et al., 97′, Liu et al., 10′, Fuchs, B. , 14′,...]
- Harmonic nested arrays [Chou, T., 95′, Zheng et al., 04′] - A deterministic design with no consideration to other constraints like robustness
- Simulated annealing based approach [Crocco & Trucco, 12′] which involves a multidimensional searching algorithm and assuming a-priori knowledge regarding the number of sensors
- Recently, a convex optimization based approach was applied to design of FI beamformers [Liu et al., 15′]
- It assumes an initial grid of sensors, and involves an ℓ_1-norm optimization under multiple convex constraints, one of them is the joint-sparse one
- As the optimization is performed **simultaneously** for all the frequencies, we refer to this approach as a **coherent** sparse design

Incoherent Design of FI Sparse Arrays - Motivation

- The **coherent** approach involves the optimization over the entire bandwidth **simultaneously**
- This approach may be infeasible when the number of candidate sensors is high
- We propose a new four-step **incoherent** sparse design where the sensors locations are sparsely optimized $\forall \omega_j \in \Omega$ **separately**
- Then, all the layouts are fused together yielding an array layout which is joint to all the frequency bins
- This approach is more computational efficient and may be feasible to array with a large number of candidate sensors



Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern - Motivation



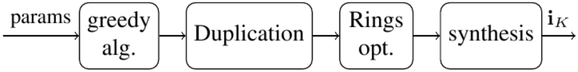
- Some practical applications like video conference systems and voice service devices, require variable steering direction while preserving similar performance
- Concentric arrays can provide similar and steerable beampattern for 360° azimuthal coverage
- The **incoherent** sparse approach presented before obtains superior performance with a reasonable computational effort
- Yet, it suffers from the limitation that its mainbeam can not be steered to other directions while preserving similar properties
- Herein, we extend the incoherent approach, and propose a new greedy joint-sparse design for concentric arrays with rotationally- and frequency- invariant beampattern

Previous work on sparse concentric array design

- Sparse concentric array design for the **narrowband** case includes genetic algorithms (GA) approach [Haupt, R. L. , 08′, Chen et al., 15′], differential evolution (DE) [Chatterjee et al., 10′], deterministic approaches [Morabito et al., 16′, Angeletti et al., 13′], and more
- Broadband sparse concentric array design approaches can be found in [Zhang et al. 12′, Gregory et al., 12′] - optimization of the sidelobes with no consideration of the FI property and only for the broadside
- An analytical approach for design of FI concentric arrays is presented in [Li et al. 06′], yet it does not provide enough frequency range coverage over-which the beampattern is FI, and no optimization of the total number of sensors is performed

Greedy Joint Sparse Design for Concentric Arrays

- A greedy based joint-sparse design is derived
- It optimizes both the number of sensors and the number of rings while taking into consideration the requirement regarding the rotationally-invariant property
- It also supports the case that one is interested only in a sector or a discrete number of azimuthal directions
- The proposed design contains four main steps



Journal Publications

- Y. Buchris, I. Cohen, and J. Benesty, "On the design of time-domain differential microphone arrays," *Applied Acoustics*, vol. 148, pp. 212-222, May 2019. TD
- Y. Buchris, I. Cohen, and J. Benesty, "Frequency-domain design of asymmetric circular differential microphone arrays," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, no. 4, pp. 760-773, Apr. 2018. ASY
- Y. Buchris, A. Amar, I. Cohen, and J. Benesty, "Incoherent Synthesis of Sparse Arrays for Frequency-Invariant Beamforming," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482-495, March 2019. SP1
- Y. Buchris, I. Cohen, J. Benesty, and A. Amar, "Joint Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern," submitted to *IEEE/ACM Transactions on Audio, Speech, and Language Processing*. SP2

FutureResDir

Journal Publications

- Y. Buchris, I. Cohen, and J. Benesty, "On the design of time-domain differential microphone arrays," *Applied Acoustics*, vol. 148, pp. 212-222, May 2019.
- Y. Buchris, I. Cohen, and J. Benesty, "Frequency-domain design of asymmetric circular differential microphone arrays," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, no. 4, pp. 760-773, Apr. 2018.
- Y. Buchris, A. Amar, I. Cohen, and J. Benesty, "Incoherent Synthesis of Sparse Arrays for Frequency-Invariant Beamforming," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482-495, March 2019.
- Y. Buchris, I. Cohen, J. Benesty, and A. Amar, "Joint Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern," submitted to *IEEE/ACM Transactions on Audio, Speech, and Language Processing*.

Time-domain design for DMAs - a signal model

- A broadband far-field signal $s(n)$
- A uniform linear array with M omnidirectional microphones
- The signal at the mth microphone is:

$$y_m(n) = s\left[n - \Delta - f_{\mathrm{s}}\tau_m(\theta)\right] + v_m(n) = x_m(n) + v_m(n),$$

where

$$\tau_m(\theta) = (m-1)\frac{\delta \cos\theta}{c}$$

Time-domain design for DMAs - a signal model (2)

- Using Shannon's sampling theorem, $y_m(n)$ can be expressed as:

$$y_m(n) = \sum_{l=-\infty}^{\infty} s\left[n - \Delta - l\right] \operatorname{sinc}\left[l - f_\mathrm{s}\tau_m(\theta)\right] + v_m(n)$$
$$\approx \sum_{l=-P}^{P+\mu L_h} s\left[n - \Delta - l\right] \operatorname{sinc}\left[l - f_\mathrm{s}\tau_m(\theta)\right] + v_m(n)$$

  where $P \gg f_\mathrm{s}\tau_m(\theta)$, μ is a fractional number, and L_h is the FIR length.

- Hence, the signal $y_m(n)$ can also be expressed as

$$y_m(n) = \mathbf{g}_m^T(\theta)\mathbf{s}\left(n - \Delta\right) + v_m(n)$$

Time-domain design for DMAs - a signal model (3)

- Taking L_h successive time samples of the signal

$$\mathbf{y}_m(n) = \mathbf{G}_m(\theta)\mathbf{s}\left(n - \Delta\right) + \mathbf{v}_m(n)$$

where $\mathbf{G}_m(\theta)$ is a $L_h \times L$ Toeplitz matrix:

$$\left[\mathbf{G}_m(\theta)\right]_{i,j} = \text{sinc}\left[-P - i + j - f_s\tau_m(\theta)\right], \;\; i = 1, ..., L_h, \;\; j = 1, ..., L,$$

and $L = 2P + \mu L_h$

- We get a separable model in the time-domain, similarly to the frequency-domain model:

$$\mathbf{Y}_m(f) = \mathbf{d}(f)X(f) + V(f)$$

where $\mathbf{d}(f)$ is the steering vector

Time-domain design for DMAs - a signal model (4)

- Finally, by concatenating all microphones:

$$\begin{aligned}\underline{\mathbf{y}}(n) &= \left[\begin{array}{cccc} \mathbf{y}_1^T(n) & \mathbf{y}_2^T(n) & \cdots & \mathbf{y}_M^T(n) \end{array} \right]^T \\ &= \underline{\mathbf{G}}(\theta)\mathbf{s}_L\left(n-\Delta\right) + \underline{\mathbf{v}}(n)\end{aligned}$$

where

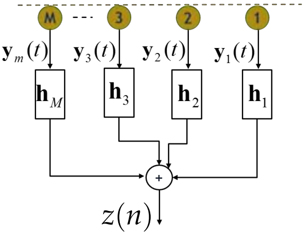
$$\underline{\mathbf{G}}(\theta) = \begin{bmatrix} \mathbf{G}_1(\theta) \\ \mathbf{G}_2(\theta) \\ \vdots \\ \mathbf{G}_M(\theta) \end{bmatrix}$$

is the equivalent time-domain steering matrix of size $ML_h \times L$

Time-domain design for DMAs - broadband beamforming

- Define the filters vector

$$\underline{\mathbf{h}} = \begin{bmatrix} \mathbf{h}_1^T & \mathbf{h}_2^T & \cdots & \mathbf{h}_M^T \end{bmatrix}^T$$



- The output of the beamformer is:

$$\begin{aligned} z(n) &= \sum_{m=1}^{M} \mathbf{h}_m^T \mathbf{y}_m(n) \\ &= \underline{\mathbf{h}}^T \underline{\mathbf{y}}(n) \\ &= \underline{\mathbf{h}}^T \underline{\mathbf{G}}(\theta) \mathbf{s}_L (n - \Delta) + \underline{\mathbf{h}}^T \underline{\mathbf{v}}(n) \end{aligned}$$

Time-domain design for DMAs - general solution for Nth order

The design of Nth-order DMAs involves at least $M \geq N+1$ microphones, where N is the DMA's order, and $N+1$ constraints.

- A distortionless constraint (assuming endfire direction):

$$\underline{\mathbf{h}}^T \underline{\mathbf{G}}(0) = \mathbf{i}^T(D)$$

- N constraints of the form:

$$\underline{\mathbf{h}}^T \underline{\mathbf{G}}(\theta_n) = \alpha_n \mathbf{i}^T(D), \quad n = 1, 2, ..., N,$$

Time-domain design for DMAs - general solution for Nth order (2)

Combining all these constraints into a matrix:

$$\mathbf{C}_{N,M}(\boldsymbol{\theta})\underline{\mathbf{h}} = \underline{\mathbf{i}}_N(\boldsymbol{\alpha}, D)$$

where

$$\mathbf{C}_{N,M}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{G}_1^T(0) & \mathbf{G}_2^T(0) & \cdots & \mathbf{G}_M^T(0) \\ \mathbf{G}_1^T(\theta_1) & \mathbf{G}_2^T(\theta_1) & \cdots & \mathbf{G}_M^T(\theta_1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}_1^T(\theta_N) & \mathbf{G}_2^T(\theta_N) & \cdots & \mathbf{G}_M^T(\theta_N) \end{bmatrix}$$

and

$$\underline{\mathbf{i}}_N(\boldsymbol{\alpha}, D) = \begin{bmatrix} \mathbf{i}^T(D) & \alpha_1\mathbf{i}^T(D) & \cdots & \alpha_N\mathbf{i}^T(D) \end{bmatrix}^T$$

Time-domain design for DMAs - general solution for Nth order (3)

- The beamformer can be derived as

$$\underline{\mathbf{h}}_{N,M;\text{Pinv}} = \mathbf{P}^{\dagger}_{\mathbf{C}_{N,M}}(\boldsymbol{\theta})\underline{\mathbf{i}}_N(\boldsymbol{\alpha}, D)$$

where

$$\mathbf{P}^{\dagger}_{\mathbf{C}_{N,M}}(\boldsymbol{\theta}) = \left[\mathbf{C}^T_{N,M}(\boldsymbol{\theta})\mathbf{C}_{N,M}(\boldsymbol{\theta}) + \eta\mathbf{I}\right]^{-1}\mathbf{C}^T_{N,M}(\boldsymbol{\theta})$$

- The optimal delay D can be estimated by:

$$D^{\text{opt}} = \min_D ||\mathbf{C}_{N,M}(\boldsymbol{\theta})\underline{\mathbf{h}}_{N,M;\text{Pinv}} - \underline{\mathbf{i}}_N(\boldsymbol{\alpha}, D)||^2$$

Time-domain design for DMAs - performance measures

- The gain in SNR can be expressed as

$$\mathcal{G}\left(\underline{\mathbf{h}}\right) = \frac{\text{oSNR}\left(\underline{\mathbf{h}}\right)}{\text{iSNR}} = \frac{\underline{\mathbf{h}}^T \underline{\mathbf{G}}(0)\underline{\mathbf{G}}^T(0)\underline{\mathbf{h}}}{\underline{\mathbf{h}}^T \Gamma_{\underline{\mathbf{v}}} \underline{\mathbf{h}}}$$

- The white noise gain (WNG) can be derived by taking $\Gamma_{\underline{\mathbf{v}}} = \mathbf{I}$
- The time-domain broadband beampattern may be defined as

$$\left|\mathcal{B}\left(\underline{\mathbf{h}}, \theta\right)\right|^2 = \underline{\mathbf{h}}^T \underline{\mathbf{G}}(\theta)\underline{\mathbf{G}}^T(\theta)\underline{\mathbf{h}}$$

- The directivity factor (DF) is

$$\mathcal{D}\left(\underline{\mathbf{h}}\right) = \frac{2}{\int_0^\pi \left|\mathcal{B}\left(\underline{\mathbf{h}}, \theta\right)\right|^2 \sin\theta d\theta}$$

Time-domain design for DMAs - relation to the theoretical model

It can be shown theoretically that the following relation exists:

$$\left|\mathcal{B}\left(\underline{\mathbf{h}},\theta\right)\right|^2 \xrightarrow[\delta\ll\lambda]{} \mathcal{B}_N^2\left(\theta\right)$$

where

$$\mathcal{B}_N\left(\theta\right)=\sum_{n=0}^{N} a_{N,n}\cos^n\theta$$

The time-domain model converges to the theoretical DMAs' model

Time-domain design for DMAs - simulation results

First-order design:

- $M = 2$
- $\delta = 1\text{cm}$
- $L_h = 16$ taps

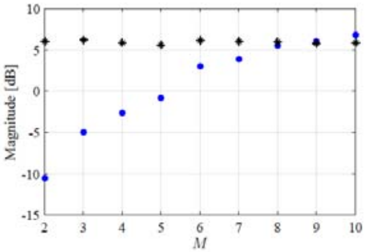
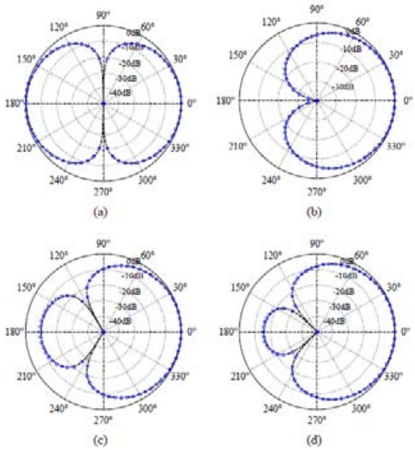
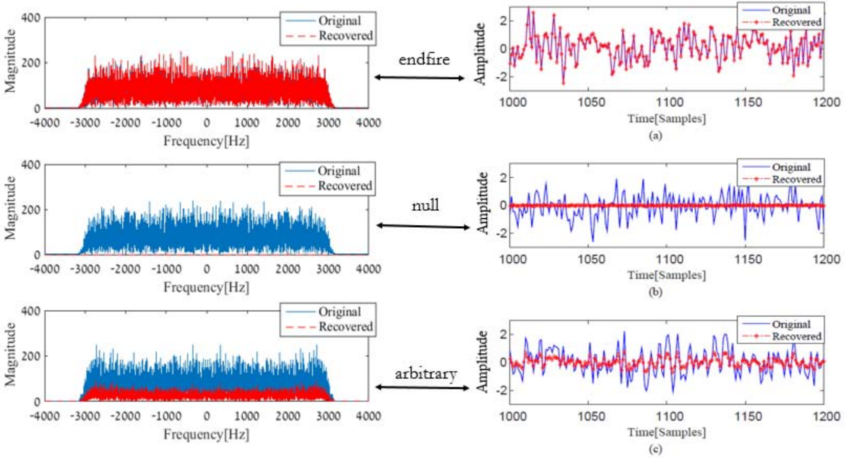


Fig. 5: WNG (circles) and DF (stars) vs. M, for the case of a first-order hypercardioid.



Pattern	$\mathcal{W}(\underline{\mathbf{h}})$ [dB]	$\mathcal{D}(\underline{\mathbf{h}})$ [dB]	E_{BP}
Dipole	−14.4	4.90	0.040
Cardioid	−8.68	4.75	0.007
Hypercardioid	−11.00	5.84	0.010
Supercardioid	−10.00	5.42	0.007

Spectrums and waveforms of output signals in various directions:



Time-domain design for DMAs - simulation results(3)

Influence of large δ:

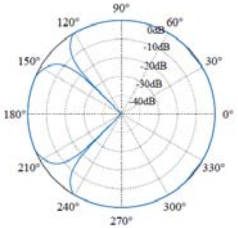


Fig. 8: Beampattern of a first-order hypercardioid ($M = 2$) for the case of $\delta = 15$ cm.

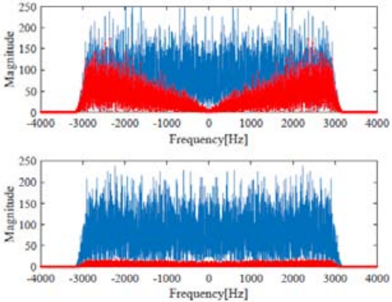


Fig. 9: Waveform for the case of $\delta = 15$ cm (top) and for the case of $\delta = 1$ cm (bottom) for a signal arrived from 145°. The dark blue spectrum is the original spectrum and the light red one is the output signal spectrum.

Time-domain design for DMAs - simulation results(4)

Third-order design:

- $M = 4$
- $\delta = 1\text{cm}$
- $L_h = 20$ taps

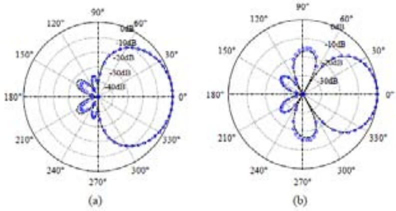


Fig. 13: Beampatterns for two cases of third-order DMAs with 3 distinct nulls, produced by the time-domain implementation (dark dashed line): (a) case 1, and (b) case 2. The theoretical patterns are also presented (blue circles line).

Pattern	$\mathcal{W}(\underline{h})$ [dB]	$\mathcal{D}(\underline{h})$ [dB]	E_{BP}
Case 1	-29.32	10.16	0.007
Case 2	-36.78	11.78	0.030

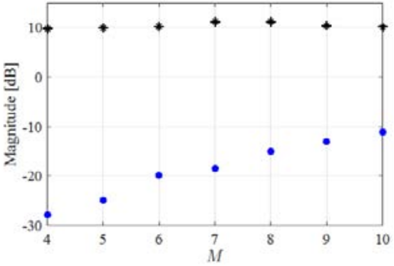


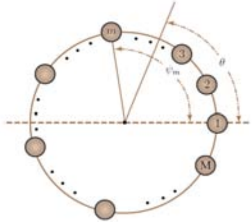
Fig. 14: WNG (circles) and DF (stars) vs. M, for the case of a third-order DMA (case 1).

Back

Journal Publications

- Y. Buchris, I. Cohen, and J. Benesty, ”On the design of time-domain differential microphone arrays,” *Applied Acoustics*, vol. 148, pp. 212-222, May 2019.
- Y. Buchris, I. Cohen, and J. Benesty, ”Frequency-domain design of asymmetric circular differential microphone arrays,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, no. 4, pp. 760-773, Apr. 2018.
- Y. Buchris, A. Amar, I. Cohen, and J. Benesty, ”Incoherent Synthesis of Sparse Arrays for Frequency-Invariant Beamforming,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482-495, March 2019.
- Y. Buchris, I. Cohen, J. Benesty, and A. Amar, ”Joint Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern,” submitted to *IEEE/ACM Transactions on Audio, Speech, and Language Processing*.

CDMAs - signal model



- An acoustic far-field source signal $s(\omega)$
- A uniform circular array (UCA) of radius r
- M omnidirectional microphones
- The distance between two sensors:

$$\delta = 2r \sin\left(\frac{\pi}{M}\right) \approx \frac{2\pi r}{M}$$

- The delay between the mth microphone and the array center is

$$\tau_m = \frac{r}{c} \cos(\theta_s - \psi_m), \quad m = 1, 2, ..., M,$$

- The angular position of the mth array element is $\psi_m = \frac{2\pi(m-1)}{M}$
- The steering vector is

$$\mathbf{d}\left(\omega, \theta\right) = \left[e^{j\omega r c^{-1} \cos(\theta - \psi_1)} \quad \ldots \quad e^{j\omega r c^{-1} \cos(\theta - \psi_M)} \right]^T$$

CircularDefs

Asymmetric CDMAs - theoretical beampattern

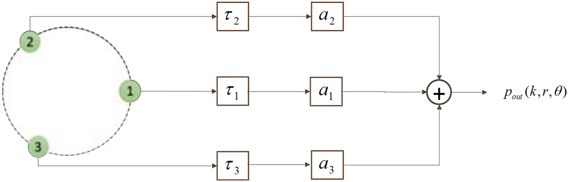
First-order circular asymmetric case:

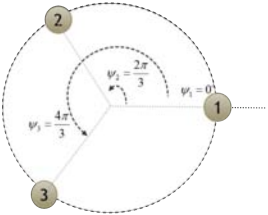
- Assuming a 2D propagation model (i.e. $\phi = \pi/2$), the pressure field is:

$$p\left(k, r, \theta, \psi_l\right) = P_0 e^{j\varpi \cos(\theta - \psi_l)}, l = 1, 2, 3, \quad \varpi = \frac{\omega r}{c}$$

- The output signal is:

$$p_{out}\left(k, r, \theta\right) = P_0 \sum_{l=1}^{3} a_l e^{j\omega\tau_l} e^{j\varpi \cos(\theta - \psi_l)}$$





Asymmetric CDMAs - theoretical beampattern (2)

- The last expression can be approximated as:

$$p_{out}(k,r,\theta) \approx 1 + a_2 + a_3 \\ + j\omega \sum_{l=1}^{3} a_l \left[\tau_l + \frac{r}{c}\left(\cos\theta\cos\psi_l + \sin\theta\sin\psi_l\right)\right]$$

- Imposing $a_2 + a_3 = -1$, assuming $a_1 = 1, \tau_1 = 0$, and define:

$$\alpha_1 = \frac{\sum_{l=1}^{3} a_l \tau_l}{\sum_{l=1}^{3} a_l \left(\tau_l + \frac{r}{c}\cos\psi_l\right)}, \quad 1 - \alpha_1 = \frac{\sum_{l=1}^{3} a_l \frac{r}{c}\cos\psi_l}{\sum_{l=1}^{3} a_l \left(\tau_l + \frac{r}{c}\cos\psi_l\right)},$$

and

$$\beta_1 = \frac{\sum_{l=1}^{3} a_l \frac{r}{c}\sin\psi_l}{\sum_{l=1}^{3} a_l \left(\tau_l + \frac{r}{c}\cos\psi_l\right)}.$$

Asymmetric CDMAs - theoretical beampattern (3)

- We can now write the normalized response as

$$\mathcal{B}_1(\theta) = \frac{p_{out}(k,r,\theta)}{p_{out}(k,r,0)} = \alpha_1 + (1-\alpha_1)\cos\theta + \beta_1 \sin\theta$$

- Notice that $p_{out}(k,r,\theta)$ can be written as

$$p_{out}(k,r,\theta) = x\left(e^{j\varpi\cos(\theta-\psi_1)} - e^{j\omega\tau_2}e^{j\varpi\cos(\theta-\psi_2)}\right) + (1-x)\left(e^{j\varpi\cos(\theta-\psi_1)} - e^{j\omega\tau_3}e^{j\varpi\cos(\theta-\psi_3)}\right)$$

where $-1 \leq x \leq 0$

- Therefore, $p_{out}(k,r,\theta)$ can be interpreted as a weighted sum of two first-order linear DMAs

Asymmetric CDMAs - theoretical beampattern (4)

- The second-order asymmetric CDMA's beampattern can be written as a product of two first-order terms, i.e.

$$\mathcal{B}_2(\theta) = \prod_{i=1}^{2} [\alpha_i + (1 - \alpha_i) \cos\theta + \beta_i \sin\theta]$$

- From the last equation one can easily derive the following general form of second order asymmetric CDMA:

$$\mathcal{B}_2(\theta) = \gamma_0 + \gamma_1 \cos\theta + \gamma_2 \cos^2\theta + \gamma_3 \sin\theta \cos\theta + \gamma_4 \sin\theta$$

Asymmetric CDMAs - theoretical beampattern (5)

- Generalizing the last result, the nth-order asymmetric CDMA beampattern is:

$$\mathcal{B}_n(\theta-\theta_s)=\sum_{i=0}^{n}\xi_i\cos^i(\theta-\theta_s)+\sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor}\mu_i\sin^{2i+1}(\theta-\theta_s)$$
$$+\sum_{i=1}^{\lfloor\frac{n}{2}\rfloor}\zeta_i\cos(\theta-\theta_s)\sin^{2i-1}(\theta-\theta_s)$$

- In order to prove the last result, we use the Fourier Theorem stating that each aperiodic function in $[-\pi,\pi]$ can be represented by an infinite series of sine and cosine functions, i.e:

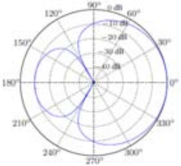
$$\mathcal{B}_N^f(\theta)=\sum_{n=0}^{N}(a_n\cos(n\theta)+b_n\sin(n\theta))$$

- Proof: in the paper ...

Optimal asymmetric hypercardioid beampatterns

- In the symmetric case, hypercardioid maximizes the directivity factor:

$$\mathcal{B}_1(\theta) = \frac{1}{4} + \frac{3}{4}\cos\theta$$



- The equivalent "asymmetric hypercardioid" can be also derived
- General definition of the directivity factor (DF):

$$\mathcal{D} = \frac{\left|\mathcal{B}_n(\theta_s, \phi_s)\right|^2}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left|\mathcal{B}_n(\theta, \phi)\right|^2 \sin\phi d\phi d\theta}$$

- Herein, we focus on the case:

$$\mathcal{B}_n(\theta - \theta_s) = \mathcal{B}_n(\theta - \theta_s, \phi = \pi/2)$$

and $\theta_s = 0°$, i.e:

$$\mathcal{B}_n(\theta) = \sum_{i=0}^{n} \xi_i \cos^i(\theta) + \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor} \mu_i \sin^{2i+1}(\theta) + \sum_{i=1}^{\lfloor\frac{n}{2}\rfloor} \zeta_i \cos(\theta)\sin^{2i-1}(\theta)$$

Optimal asymmetric hypercardioid beampatterns (2)

- It is obvious that

$$\mathcal{D}^{-1} \propto \int_0^{2\pi} \left|\mathcal{B}_n(\theta)\right|^2 d\theta$$

- In addition

$$\int_0^{2\pi} \left|\mathcal{B}_n(\theta)\right|^2 d\theta = \mathbf{a}^T \mathbf{H} \mathbf{a}$$

where

$$\mathbf{a} = \left[\xi_0, ..\xi_n, \mu_1, ..\zeta_2, ..\right]^T$$

Optimal asymmetric hypercardioid beampatterns (3)

- It can be shown that **H** is a block-diagonal matrix
- Therefore, $\{\xi_i\}_{i=0}^{n}$, $\{\mu_i\}_{i=1,i\ odd}^{n}$ and $\{\zeta_i\}_{i=2,i\ even}^{n}$ are independent
- Additional constraints should be imposed for a reasonable solution
- Distortionless constraint:

$$\mathcal{B}_n\left(\theta = \theta_s\right) = 1$$

leads to

$$\sum_{i=0}^{n} \xi_i = 1$$

- Up to $2n$ more additional attenuation constraints:

$$\mathcal{B}_n\left(\theta = \theta_l\right) = \alpha_l, \;\; l = 1, 2, ..2n$$

Optimal asymmetric hypercardioid beampatterns (4)

- Arrange all these constraints in a matrix form:

$$\mathbf{H_c a} = \mathbf{b}$$

- We can solve the following problem:

$$\min_{\mathbf{a}} \quad \mathbf{a}^T \mathbf{H} \mathbf{a}$$
$$\text{subject to} \quad \mathbf{H_c a} = \mathbf{b}$$

- LCMV solution

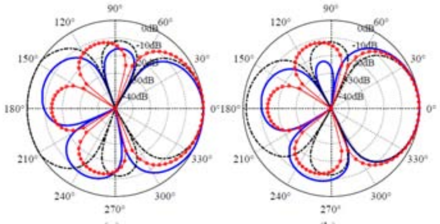
$$\mathbf{a}_{opt}^T = \mathbf{b}^T \left[\mathbf{H_c}^T \mathbf{H}^{-1} \mathbf{H_c} \right]^{-1} \mathbf{H_c}^T \mathbf{H}^{-1}$$

Optimal asymmetric hypercardioid beampatterns - a second-order design example

$\theta_s = 0^\circ$

$\theta_n^{(1)} = 60^\circ$

$\theta_n^{(2)} = 110^\circ$



$\theta_s = 0^\circ$

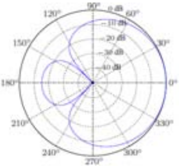
$\theta_n^{(1)} = 120^\circ$

$\theta_n^{(2)} = 295^\circ$

Fig. 3: Beampatterns for the second-order asymmetric hypercardioid CDMAs (blue solid line) and its symmetric version (black dashed line). The red circles line is the second-order unconstrained symmetric hypercardioid [6]. (a) $\theta_1 = 60^\circ$, $\theta_2 = 110^\circ$. (b) $\theta_1 = 120^\circ$, $\theta_2 = 295^\circ$.

DI [dB]	(a)	(b)
Asymmetric (blue solid)	6.2	6.7
Symmetric (black dashed)	2.6	5.5
Theoretical optimal (red circles)	7.0	7.0

Optimal asymmetric supercardioid beampatterns



- In the symmetric case, supercardioid maximizes the front-to-back-ratio (FBR):

$$\mathcal{B}_1(\theta) = \sqrt{2} - 1 + (2 - \sqrt{2})\cos\theta$$

- The equivalent "asymmetric hypercardioid" can be also derived
- General definition of the front-to-back-ratio:

$$\mathcal{F} = \frac{\int_{-\pi/2}^{\pi/2} |\mathcal{B}_n(\theta)|^2 \, d\theta}{\int_{\pi/2}^{3\pi/2} |\mathcal{B}_n(\theta)|^2 \, d\theta}$$

Practical design of asymmetric CDMAs

- A distortionless constraint

$$\mathbf{d}^H(\omega, \theta_s)\,\mathbf{h}(\omega) = 1,$$

- Up to $2N$ additional constraints

$$\mathbf{d}^H(\omega, \theta_s + \theta_i)\,\mathbf{h}(\omega) = \nu_i, \quad i = 1, ..., 2N,$$

- In matrix formulation

$$\mathbf{D}_{N,M}(\omega, \theta_s, \boldsymbol{\theta})\mathbf{h}(\omega) = \boldsymbol{\nu},$$

Practical design of asymmetric CDMAs (2)

- Practically, it is desired to add a constraint on the squared norm of the solution vector $\mathbf{h}(\omega)$, which is inversely proportional to the WNG and minimizes the objective function

$$J(\mathbf{h}(\omega)) = \left\| \boldsymbol{\nu} - \mathbf{D}_{N,M}(\omega, \theta_s, \boldsymbol{\theta})\mathbf{h}(\omega) \right\|_2^2 + \eta \left\| \mathbf{h}(\omega) \right\|_2^2 ,$$

- Using the method of Lagrange multipliers we can obtain the regularized pseudo-inverse solution:

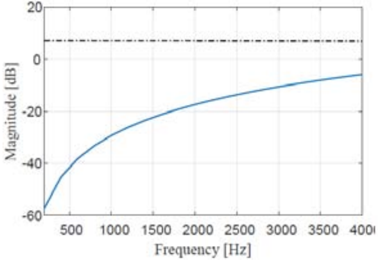
$$\mathbf{h}(\omega) = \mathbf{P}^{\dagger}_{\mathbf{D}_{N,M}}(\omega, \theta_s, \boldsymbol{\theta})\boldsymbol{\nu},$$

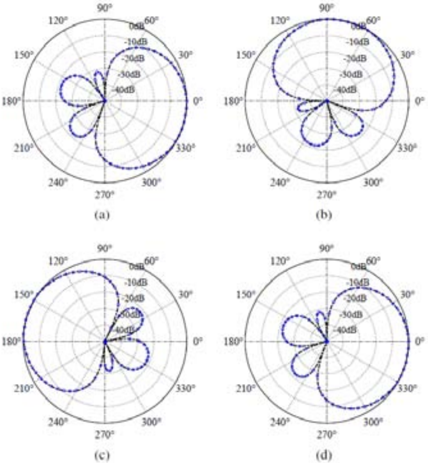
where

$$\mathbf{P}^{\dagger}_{\mathbf{X}} = \left[\mathbf{X}^H \mathbf{X} + \eta \mathbf{I} \right]^{-1} \mathbf{X}^H$$

Experimental results

- Asymmetric second-order hypercardioid
- $N = 2$
- $M = 5$





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Journal Publications

- Y. Buchris, I. Cohen, and J. Benesty, ”On the design of time-domain differential microphone arrays,” *Applied Acoustics*, vol. 148, pp. 212-222, May 2019.
- Y. Buchris, I. Cohen, and J. Benesty, ”Frequency-domain design of asymmetric circular differential microphone arrays,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, no. 4, pp. 760-773, Apr. 2018.
- Y. Buchris, A. Amar, I. Cohen, and J. Benesty, ”Incoherent Synthesis of Sparse Arrays for Frequency-Invariant Beamforming,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482-495, March 2019.
- Y. Buchris, I. Cohen, J. Benesty, and A. Amar, ”Joint Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern,” submitted to *IEEE/ACM Transactions on Audio, Speech, and Language Processing*.

Signal model

- A far-field **desired** beampattern $\mathcal{B}_{\mathrm{d}}(\boldsymbol{\rho})$, $\boldsymbol{\rho} = (\theta, \phi)$
- M candidate sensors, located at $\mathbf{p}_m,\ m = 1, 2, ..., M$
- The steering vector is

$$\mathbf{d}\left[\mathbf{k}_{\omega}(\boldsymbol{\rho})\right] = \left[e^{-\jmath \mathbf{k}_{\omega}^T(\boldsymbol{\rho})\mathbf{p}_0}\ \ e^{-\jmath \mathbf{k}_{\omega}^T(\boldsymbol{\rho})\mathbf{p}_1}\ \ \ldots\ \ e^{-\jmath \mathbf{k}_{\omega}^T(\boldsymbol{\rho})\mathbf{p}_M}\right]^T$$

where $\mathbf{k}_{\omega}(\boldsymbol{\rho}) = -\frac{\omega}{c}\left[\cos\theta\sin\phi \quad \sin\theta\sin\phi \quad \cos\phi\right]^T$

- The **synthesized** beampattern is

$$\mathcal{B}\left[\mathbf{h}(\omega)\right] = \mathbf{h}^H(\omega)\mathbf{d}\left[\mathbf{k}_{\omega}(\boldsymbol{\rho})\right]$$

where $\mathbf{h}(\omega) = \left[H_1(\omega)\ \ H_2(\omega)\ \ \cdots\ \ H_M(\omega)\right]^T$

Problem formulation

- Goal: select a set of $K \ll M$ **joint-sparse** positions, $\{\mathbf{p}_{i_k}\}_{k=1}^{K}, \{i_k\}_{k=1}^{K} \in [1, 2, ..., M]$, such that $\forall \omega \in \Omega$, the synthesized beampattern

$$\mathcal{B}\left[\mathbf{h}(\omega), \mathbf{T}_{\text{sc}}\left(\mathbf{i}_K\right)\right] = \mathbf{h}^H(\omega)\mathbf{T}_{\text{sc}}^T\left(\mathbf{i}_K\right)\mathbf{T}_{\text{sc}}\left(\mathbf{i}_K\right)\mathbf{d}\left[\mathbf{k}_\omega\left(\boldsymbol{\rho}\right)\right]$$

  will be as close as possible to $\mathcal{B}_{\text{d}}\left(\boldsymbol{\rho}\right)$, under some design constraints
- Where $\mathbf{i}_K = [i_1, i_2, ..., i_K]^T$ and $\mathbf{T}_{\text{sc}}\left(\mathbf{i}_K\right)$ is a $K \times M$ **frequency-invariant (FI)** selection matrix, e.g.,

$$M = 8, \quad K = 4, \quad \mathbf{i}_4 = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 7 \end{bmatrix}, \quad \mathbf{T}_{\text{sc}}\left(\mathbf{i}_4\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Typical design constraints

- Ω,Θ - frequency and angle range of interest,respectively
- Uniform discretization: $\{\omega_j\}_{j=1}^{J} \in \Omega$, $\left\{\boldsymbol{\rho}_p\right\}_{p=1}^{P} \in \Theta$
- Mainlobe directions set $\boldsymbol{K}_j^{\mathrm{m}} = \left\{\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_1), \mathbf{k}_{\omega_j}(\boldsymbol{\rho}_2), \cdots, \mathbf{k}_{\omega_j}(\boldsymbol{\rho}_L)\right\}$
- Sidelobe directions set $\boldsymbol{K}_j^{\mathrm{s}} = \left\{\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_{L+1}), \mathbf{k}_{\omega_j}(\boldsymbol{\rho}_{L+2}), \cdots, \mathbf{k}_{\omega_j}(\boldsymbol{\rho}_P)\right\}$
- **Mainlobe constraints:** $\forall\omega_j \in \Omega$

$$\mathcal{C}_1 : \left\|(\mathbf{b}_{\mathrm{d}}^{\mathrm{m}})^T - \mathbf{h}^H\left(\omega_j\right)\mathbf{T}_{\mathrm{s}}^T\left(\mathbf{i}_K\right)\mathbf{T}_{\mathrm{s}}\left(\mathbf{i}_K\right)\mathbf{D}\left(\boldsymbol{K}_j^{\mathrm{m}}\right)\right\|_2^2 \leq \epsilon_1(\omega_j)$$

- The matrix

$$\mathbf{D}\left(\boldsymbol{K}_j^{\mathrm{m}}\right) = \left[\mathbf{d}\left[\mathbf{k}_{\omega_j}\left(\boldsymbol{\rho}_1\right)\right], \mathbf{d}\left[\mathbf{k}_{\omega_j}\left(\boldsymbol{\rho}_2\right)\right], \cdots, \mathbf{d}\left[\mathbf{k}_{\omega_j}\left(\boldsymbol{\rho}_L\right)\right]\right]$$

  is an $M \times L$ matrix
- And $\mathbf{b}_{\mathrm{d}}^{\mathrm{m}} = \left[\mathcal{B}_{\mathrm{d}}\left(\boldsymbol{\rho}_1\right), \mathcal{B}_{\mathrm{d}}\left(\boldsymbol{\rho}_2\right), \cdots, \mathcal{B}_{\mathrm{d}}\left(\boldsymbol{\rho}_L\right)\right]^T$

Typical design constraints (2)

- **Sidelobe constraints:** $\forall \omega_j \in \Omega$

$$\mathcal{C}_2 : \left\| \left(\mathbf{b}_{\mathrm{d}}^{\mathrm{s}}\right)^T - \mathbf{h}^H\left(\omega_j\right)\mathbf{T}_{\mathrm{s}}^T\left(\mathbf{i}_K\right)\mathbf{T}_{\mathrm{s}}\left(\mathbf{i}_K\right)\mathbf{D}\left(\boldsymbol{K}_j^{\mathrm{s}}\right) \right\|_2^2 \leq \epsilon_2(\omega_j)$$

- **Distortionless constraints:** $\forall \omega_j \in \Omega$

$$\mathcal{C}_3 : \ \mathbf{h}^H\left(\omega_j\right)\mathbf{T}_{\mathrm{s}}^T\left(\mathbf{i}_K\right)\mathbf{T}_{\mathrm{s}}\left(\mathbf{i}_K\right)\mathbf{d}\left[\mathbf{k}_{\omega_j}\left(\boldsymbol{\rho}_{\mathrm{s}}\right)\right] = 1$$

- **White noise output limitation:** $\forall \omega_j \in \Omega$

$$\mathcal{C}_4 : \ \mathbf{h}^H\left(\omega_j\right)\mathbf{T}_{\mathrm{s}}^T\left(\mathbf{i}_K\right)\mathbf{T}_{\mathrm{s}}\left(\mathbf{i}_K\right)\mathbf{h}\left(\omega_j\right) \leq \gamma(\omega_j)$$

The Optimization Problem

- Combining constraints $\mathcal{C}_1$,$\mathcal{C}_2$,$\mathcal{C}_3$,$\mathcal{C}_4$, we can formulate the general joint-sparse problem of interest as

$$\text{minimize}\{\text{number of sensors} - K\}$$
$$\text{subject to } \mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4, \ \forall \omega_j \in \Omega,$$

whose solution yields the jointly-sparse filters

$$\mathbf{h}_K\left(\omega_j\right) = \mathbf{T}_{\mathrm{s}}\left(\mathbf{i}_K\right)\mathbf{h}\left(\omega_j\right), \ \forall \omega_j \in \Omega.$$

Theoretical (ℓ_0) and practical (ℓ_1) optimization problems

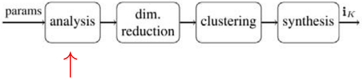
- Theoretically, sparse solutions are obtained by minimizing the ℓ_0-norm of the vector
- which is, however, an NP-hard combinatorial optimization problem
- A practical alternative is to solve an ℓ_1-norm optimization problem instead

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}, \quad \|\mathbf{x}\|_0 = \{\text{number of } x_i \neq 0, i = 1, ..., N\}, \quad \|\mathbf{x}\|_1 = \sum_{i=1}^{N} |x_i|$$

Incoherent Design of FI Sparse Arrays - Analysis

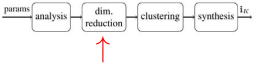
- First, we solve **separately** $\forall \omega_j \in \Omega$ the following

$$\begin{aligned}
&\text{minimize} \quad \sum_{m=1}^{M} |H_m(\omega_j)| \\
&\text{subject to} \\
&\quad \mathbf{h}^H(\omega_j)\, \mathbf{d}\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_\mathrm{s})\right] = 1 \\
&\quad \mathbf{h}^H(\omega_j)\, \mathbf{h}(\omega_j) \le \gamma(\omega_j) \\
&\quad \left\| (\mathbf{b}_\mathrm{d}^\mathrm{m})^T - \mathbf{h}^H(\omega_j)\, \mathbf{D}\left(K_j^\mathrm{m}\right) \right\|_2^2 \le \epsilon_1(\omega_j) \\
&\quad \left\| (\mathbf{b}_\mathrm{d}^\mathrm{s})^T - \mathbf{h}^H(\omega_j)\, \mathbf{D}\left(K_j^\mathrm{s}\right) \right\|_2^2 \le \epsilon_2(\omega_j)
\end{aligned}$$



- We get the vectors $\mathbf{h}_\mathrm{A}(\omega_j)$, $j = 1, 2, ..., J$ of length M

Incoherent Design of FI Sparse Arrays - Dimentionally Reduction and clustering



- Define the $M \times J$ matrix $\mathbf{H}_\mathrm{A}$

$$\mathbf{H}_\mathrm{A} = \left[\mathbf{h}_\mathrm{A}(\omega_1), \mathbf{h}_\mathrm{A}(\omega_2), ..., \mathbf{h}_\mathrm{A}(\omega_J)\right]$$

- Formulate the problem of determining the K most dominant sensors positions as a clustering problem

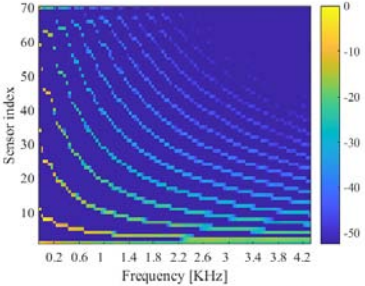


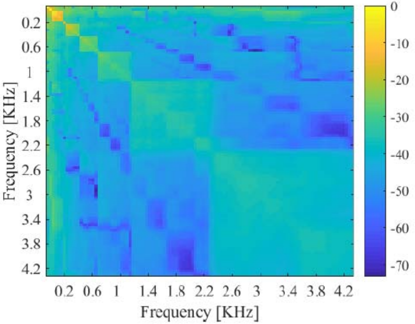
Fig. 2: Absolute values of the elements of the matrix $\mathbf{H}_\mathrm{A}$ on a logarithmic scale.

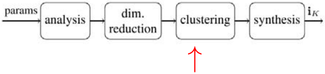
- Therefore, we treat the matrix $\mathbf{H}_\mathrm{A}$ as a measurements matrix of M observations, each with J features
- In other words, each of the M potential microphones is represented by a feature vector in $\mathbb{C}^J$

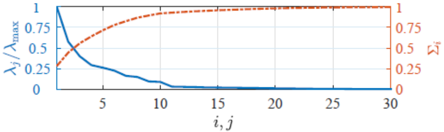
Incoherent Design of FI Sparse Arrays - Dimentionally Reduction and Clustering (2)

- The matrix $\mathbf{H}_{\mathrm{A}}$ is effectively rank-deficient with a decaying singular values spectrum
- Meaning that it contains some redundancy
- Thus, we apply the PCA algorithm in order to reduce dimensionality
- Finally, applying the k-means clustering algorithm

$$\mathbf{R}_{\mathrm{A}} = \mathbf{H}_{\mathrm{A}}^{H}\mathbf{H}_{\mathrm{A}}$$

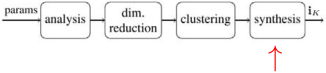






Incoherent Design of FI Sparse Arrays - Synthesis

- The previous step provides the K indices $\{i_k\}_{k=1}^{K}$ of the sensors
- In order to synthesis the desired FI beampattern, $\forall \omega_j \in \Omega$,we solve



$$\begin{aligned} & \text{minimize} \quad \left\| \mathbf{h}_K^H(\omega_j) \right\|_2^2 \\ & \text{subject to} \\ & \qquad \mathbf{h}_K^H(\omega_j)\, \mathbf{d}_K\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_\mathrm{s})\right] = 1 \\ & \qquad \left\| \left(\mathbf{b}_\mathrm{d}^\mathrm{m}\right)^T - \mathbf{h}_K^H(\omega_j)\, \mathbf{D}_K\left(\boldsymbol{K}_j^\mathrm{m}\right) \right\|_2^2 \leq \epsilon_1(\omega_j) \\ & \qquad \left\| \left(\mathbf{b}_\mathrm{d}^\mathrm{s}\right)^T - \mathbf{h}_K^H(\omega_j)\, \mathbf{D}_K\left(\boldsymbol{K}_j^\mathrm{s}\right) \right\|_2^2 \leq \epsilon_2(\omega_j) \end{aligned}$$

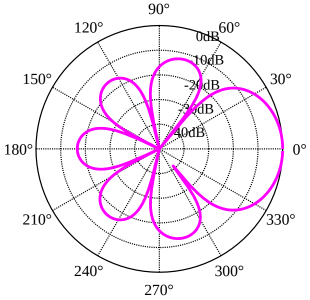
where $\mathbf{d}_K\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_\mathrm{s})\right] = \mathbf{T}_\mathrm{s}(\mathbf{i}_K)\, \mathbf{d}\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_\mathrm{s})\right]$,
$\mathbf{D}_K\left(\boldsymbol{K}_j^\mathrm{m}\right) = \mathbf{T}_\mathrm{s}(\mathbf{i}_K)\, \mathbf{D}\left(\boldsymbol{K}_j^\mathrm{m}\right)$, and $\mathbf{D}_K\left(\boldsymbol{K}_j^\mathrm{s}\right) = \mathbf{T}_\mathrm{s}(\mathbf{i}_K)\, \mathbf{D}\left(\boldsymbol{K}_j^\mathrm{s}\right)$

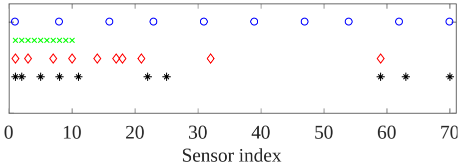
Numerical Simulations - One Dimensional Array Design

- A third-order hypercardioid DMA :

$$\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos^n \theta$$

- $M = 70, K = 10, J = 108,$
- Minimal spacing $\delta = 1$cm
- Nulls: $\theta_1 = 51°, \theta_2 = 103°, \theta_3 = 154°$



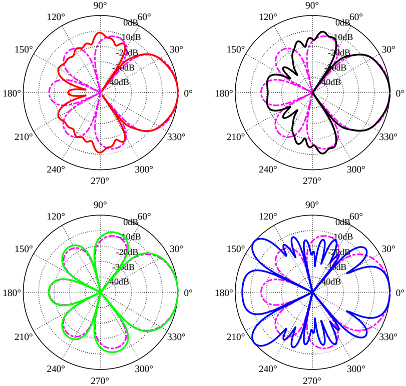


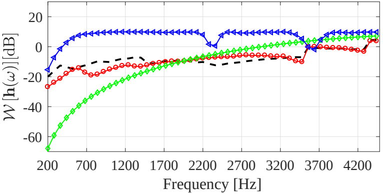
black - incoherent **green - small uniform**
red - coherent **blue - large uniform**

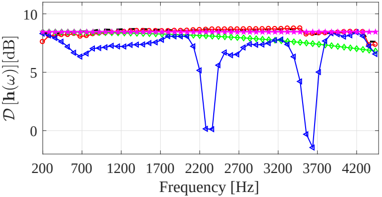
magenta - theoretical

Numerical Simulations - One Dimensional Array Design (2)

Processing time (i7-5600U CPU @ 2.6Ghz):
- Coherent approach: 160 min.
- Incoherent approach: 3 min.

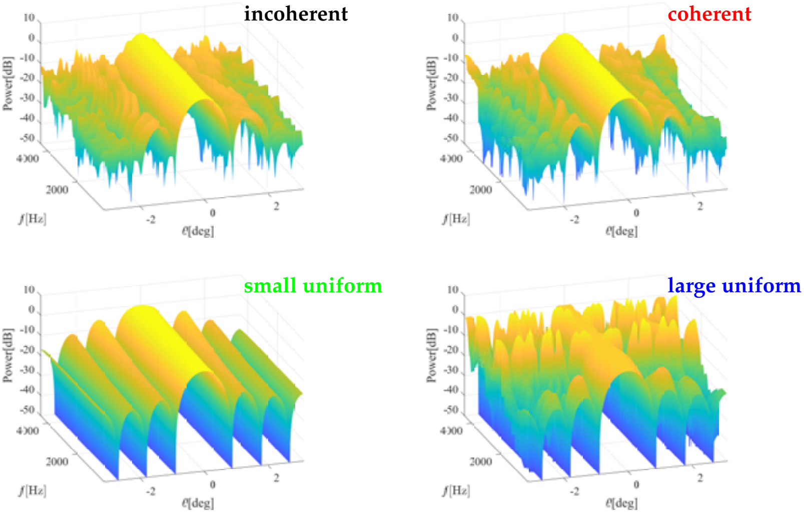






black - incoherent **green - small uniform**
red - coherent **blue - large uniform**
magenta - theoretical

Numerical Simulations - One Dimensional Array Design (3)



Numerical Simulations - Two Dimensional Array Design

Sparse Superdirective Plannar Array:

$$\mathbf{h}_{\mathrm{SD}}(\omega_j) = \frac{\Gamma^{-1}_{\mathrm{dn},\epsilon}(\omega_j)\mathbf{d}\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_{\mathrm{s}})\right]}{\mathbf{d}^H\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_{\mathrm{s}})\right]\Gamma^{-1}_{\mathrm{dn},\epsilon}(\omega_j)\mathbf{d}\left[\mathbf{k}_{\omega_j}(\boldsymbol{\rho}_{\mathrm{s}})\right]},$$

- $M = 256$, $K = 36$, $J = 58$,
- Minimal spacing $\delta = 1.5$cm

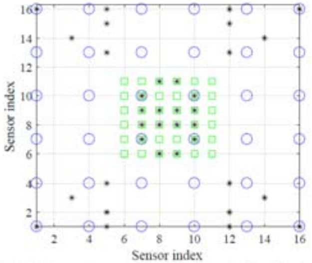


Fig. 8: The array layout of the planar array obtained by the incoherent proposed approach (black stars), the SAP uniform approach (green squares), and the LAP approach (blue circles).

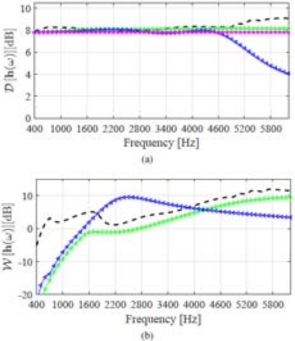


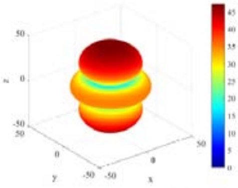
Fig. 10: (a) DF and (b) WNG vs. frequency obtained by the incoherent sparse design (black dashed line), the LAP design (blue triangles line), and the SAP design (green diamonds line). Also presented is the DF of the desired beampattern (magenta stars line).

For such a high number of candidate sensors and frequency bins, the coherent processing is infeasible when using standard hardware

Numerical Simulations - Two Dimensional Array Design (2)

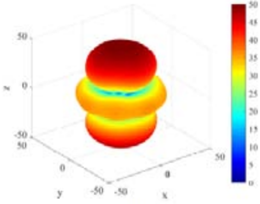
Sparse Superdirective Plannar Array:

theoretical



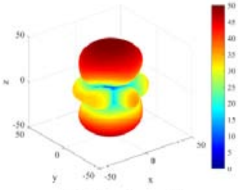
(a) Desired beampattern

small uniform



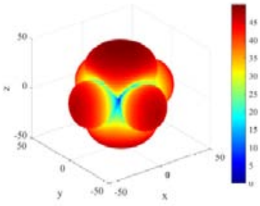
(c) SPA design

incoherent



(b) Incoherent sparse design

large uniform



(d) LPA design

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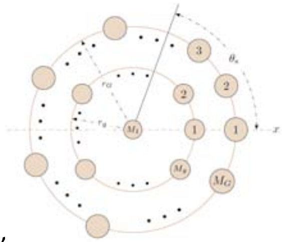
Journal Publications

- Y. Buchris, I. Cohen, and J. Benesty, "On the design of time-domain differential microphone arrays," *Applied Acoustics*, vol. 148, pp. 212-222, May 2019.
- Y. Buchris, I. Cohen, and J. Benesty, "Frequency-domain design of asymmetric circular differential microphone arrays," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 26, no. 4, pp. 760-773, Apr. 2018.
- Y. Buchris, A. Amar, I. Cohen, and J. Benesty, "Incoherent Synthesis of Sparse Arrays for Frequency-Invariant Beamforming," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 27, no. 3, pp. 482-495, March 2019.
- Y. Buchris, I. Cohen, J. Benesty, and A. Amar, "Joint Sparse Concentric Array Design for Frequency and Rotationally Invariant Beampattern," submitted to *IEEE/ACM Transactions on Audio, Speech, and Language Processing*.

Signal model and problem formulation



- A 2D scenario
- A concentric array with G candidate rings
- The total number of candidate sensors is $M = \sum_{g=1}^{G} M_g$
- Let $\mathcal{B}_{\mathrm{d}}\left(\theta, \theta_{\mathrm{s}}\right), \theta, \theta_{\mathrm{s}} \in \Theta$ be a desired far-field FI beampattern in the bandwidth of interest Ω, with a mainlobe steered to θ_{s}
- Uniform discretization: $\left\{\omega_j\right\}_{j=1}^{J} \in \Omega, \ \left\{\theta_p\right\}_{p=1}^{P} \in \Theta$
- Let $\mathcal{B}_K\left(\omega_j, \theta_p\right), \forall \omega_j \in \Omega$ be the synthesized beampattern
- Goal: select a set of $K \ll M$ **joint-sparse** positions, concentrated in $G' < G$ rings, such that $\forall \omega \in \Omega$, $\mathcal{B}_K\left(\omega_j, \theta_p\right)$ will be as close as possible to $\mathcal{B}_{\mathrm{d}}\left(\theta, \theta_{\mathrm{s}}\right)$, under some design constraints
- Additionally, the selected sensors should support the rotationally-invariant property

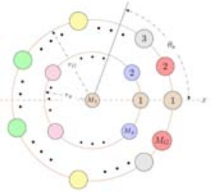
Typical design constraints

- The first four constraints are similar to those derived in the previous work ($\mathcal{C}_1 - \mathcal{C}_4$)
- In this work, two additional constraints should be defined
- The first is a symmetry of the designed beampattern w.r.t. $\theta_s = 0^\circ$
- **Symmetry constraint:** $\forall \omega_j \in \Omega$

$$\mathcal{C}_5 : \left\| \mathbf{S}(1 : M'', :)\mathbf{h}\left(\omega_j\right) \right\|_2^2 \leq \epsilon_3(\omega_j)$$

- where M'' is the number of pairs of sensors that do not lay on the main axis
- and $\mathbf{S}$ has a typical structure like

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Typical design constraints(2)

- The second additional constraint is related to the rotationally-invariant property
- Thus, we may formulate the **rotationally-invariant** constraint, $\forall \theta_{\rm s} \in \Theta$

$$\mathcal{C}_6 : \sum_{j=1}^{J} \sum_{p=1}^{P} |\mathcal{B}_{\rm d}\left(\theta_p, \theta_{\rm s}\right) - \mathcal{B}_K\left(\omega_j, \theta_p\right)|^2 \leq \epsilon_{\rm t}^2$$

The Optimization Problem

- Combining constraints $\mathcal{C}_1$,$\mathcal{C}_2$,$\mathcal{C}_3$,$\mathcal{C}_4$,$\mathcal{C}_5$,$\mathcal{C}_6$, we can formulate the general joint-sparse rotationally-invariant problem of interest as

$$\text{minimize}\{K, G'\}$$

$$\text{subject to } \ \mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4, \mathcal{C}_5, \mathcal{C}_6, \ \forall \omega_j \in \Omega$$

whose solution yields the jointly-sparse filters

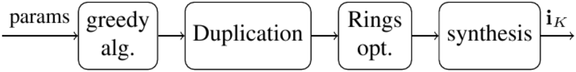
$$\mathbf{h}_K\left(\omega_j\right) = \mathbf{T}_{\mathrm{s}}\left(\mathbf{i}_K\right)\mathbf{h}\left(\omega_j\right), \ \forall \omega_j \in \Omega$$

- where $\mathbf{i}_K = [i_1, i_2, ..., i_K]^T$ and $\mathbf{T}_{\mathrm{sc}}\left(\mathbf{i}_K\right)$ is a $K \times M$ **frequency-invariant (FI)** selection matrix, e.g.,

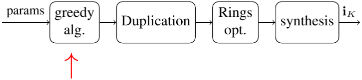
$$M = 8, \ \ K = 4, \ \ \mathbf{i}_4 = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 7 \end{bmatrix}, \ \ \mathbf{T}_{\mathrm{sc}}\left(\mathbf{i}_4\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Proposed Approach: Greedy-Based Design of FI Sparse Concentric Arrays

- Find greedily an optimal array layout for steering from endfire (i.e., $\theta_s = 0^\circ$)
- Based on a **joint-sparse** version of the OMP algorithm, $K' < K$ sensors are selected which comply $\mathcal{C}_1 - \mathcal{C}_5$
- $\mathcal{C}_6$ is obtained by duplicating the array layout to other directions
- Then, an ℓ_{12}-optimization is applied for removing redundancies and optimize both the number of sensors and rings
- The obtained array layout which is **joint-sparse** both in the number of sensors and in the number of rings is used in the final step of synthesis



Determining the array layout for steering from $\theta_\mathrm{s} = 0^\circ$



- Initialization step:

$$\mathbf{r}^{(0)}\left(\omega_j\right) = \mathbf{b}_\mathrm{d}(\theta_\mathrm{s}),\ \forall \omega_j \in \Omega$$

$$\mathbf{b}_\mathrm{g}^{(0)}\left(\omega_j\right) = \mathbf{0}_P,\ \forall \omega_j \in \Omega$$

$$\mathbf{i}_{K'}^{(0)} = [\,]$$

- where

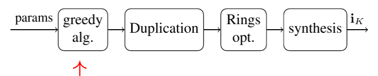
$$\mathbf{b}_\mathrm{d}(\theta_\mathrm{s}) = \begin{bmatrix} \mathbf{b}_\mathrm{d}^\mathrm{m}(\theta_\mathrm{s}) \\ \mathbf{b}_\mathrm{d}^\mathrm{s}(\theta_\mathrm{s}) \end{bmatrix}$$

- and

$$\mathbf{b}_\mathrm{d}^\mathrm{m}(\theta_\mathrm{s}) = \left[\mathcal{B}_\mathrm{d}\left(\theta_1, \theta_\mathrm{s}\right), \mathcal{B}_\mathrm{d}\left(\theta_2, \theta_\mathrm{s}\right), \ldots, \mathcal{B}_\mathrm{d}\left(\theta_{P'}, \theta_\mathrm{s}\right)\right]^T$$

- $P' < P$ is the number of directions covering the mainlobe region
- $\mathbf{b}_\mathrm{d}^\mathrm{s}(\theta_\mathrm{s})$ defined similarly to $\mathbf{b}_\mathrm{d}^\mathrm{m}(\theta_\mathrm{s})$

Determining the array layout for steering from $\theta_s = 0^\circ$ (2)



- An iterative greedy algorithm: For each iteration we may find the symmetric pair of sensors that maximizes the function:

$$m_l^{\text{JSBB}} = \underset{m=1,2,\ldots,M'}{\text{argmax}} \left[\mathbf{W}^{(l-1)} \mathbf{S} \sum_{j=1}^{J} \left| \mathbf{D}_j \mathbf{r}^{(l)} \left(\omega_j \right) \right| \right]_m$$

- where

$$\mathbf{D}_j = \begin{bmatrix} \mathbf{D}_{M,\Theta_\mathrm{m}}^H \left(\omega_j \right) \\ \mathbf{D}_{M,\Theta_\mathrm{s}}^H \left(\omega_j \right) \end{bmatrix}$$

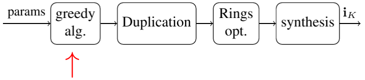
is the dictionary

- and

$$\mathbf{D}_{M,\Theta_\mathrm{m}}(\omega_j) = \left[\mathbf{d} \left(\omega_j, \theta_1 \right), \mathbf{d} \left(\omega_j, \theta_2 \right), \ldots, \mathbf{d} \left(\omega_j, \theta_{P'} \right) \right]$$

- $\mathbf{S}$ is a symmetry matrix, and $\mathbf{W}^{(l-1)}$ is a masking matrix

Determining the array layout for steering from $\theta_s = 0^\circ$ (3)



- In each iteration up to two additional sensors are selected, and the following is updated:

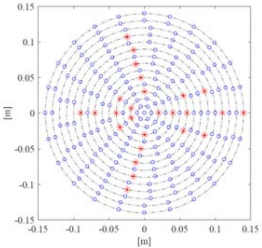
$$\mathbf{i}_{K'}^{(l)} = \left[\left(\mathbf{i}_{K'}^{(l-1)} \right)^T \quad \left(\mathbf{i}(m_l^{\mathrm{JSBB}}) \right)^T \right]^T$$

$$\mathbf{D}_j(\mathbf{i}_{K'}^{(l)}) = \mathbf{T}_\mathrm{s}\left(\mathbf{i}_{K'}^{(l)} \right) \mathbf{D}_j$$

$$\mathbf{h}_{L^{(l)}}^{\mathrm{uc}}(\omega_j) = \left[\mathbf{D}_j^H(\mathbf{i}_{K'}^{(l)}) \mathbf{D}_j(\mathbf{i}_{K'}^{(l)}) \right]^{-1} \mathbf{D}_j^H(\mathbf{i}_{K'}^{(l)}) \mathbf{b}_\mathrm{d}(\theta_\mathrm{s})$$

$$\mathbf{b}_\mathrm{g}^{(l)}(\omega_j) = \mathbf{D}_j^H(\mathbf{i}_{K'}^{(l)}) \mathbf{h}_{L^{(l)}}^{\mathrm{uc}}(\omega_j)$$

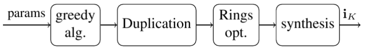
$$\mathbf{r}^{(l)}(\omega_j) = \mathbf{r}^{(l-1)}(\omega_j) - \mathbf{b}_\mathrm{g}^{(l)}(\omega_j)$$



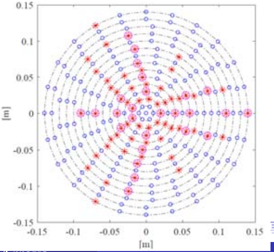
← $\theta_\mathrm{s} = 0^\circ$

- As a stopping criteria, we check whether the array layout, constructed from all the selected sensors up to the current iteration, complies $\mathcal{C}_1 - \mathcal{C}_4$

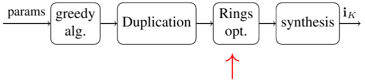
Duplicating the array layout



- The previous step yields the $K' < K$ indices of sensors used to construct the sparse array steered to $\theta_s = 0°$
- In order to get a rotationally- invariant beampattern we first duplicate the array layout to $Q - 1$ additional directions
- We get $Z \leq QK'$ sensors in the array layout, whose indices are denoted by $\{i_z\}_{z=1}^{Z} \in \mathbf{i}_Z$
- Now we have to remove the redundancy
- This can be obtained by an optimization of both the number of sensors and the number of rings



Optimizing the number of rings



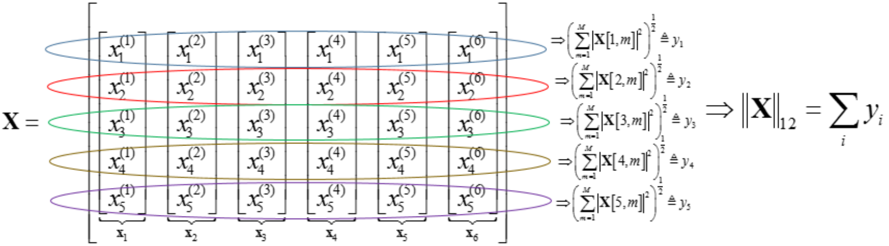
- In this step, it is desired to optimize both the number of sensors and rings and obtain a joint-sparse array layout
- Joint-sparsity can be obtained by minimizing the ℓ_{12}-norm instead of the ℓ_1-norm: Let $\mathbf{x}_m, m = 1, 2, ..., M$, be vectors of length n, and define $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_M]$, then:

$$\|\mathbf{X}\|_{12} \triangleq \sum_{i=1}^{n} \left(\sum_{m=1}^{M} |\mathbf{X}[i,m]|^2 \right)^{\frac{1}{2}}$$

where $\mathbf{X}[i,m]$ being the entry corresponds to the ith row and the mth column of $\mathbf{X}$

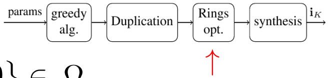
ℓ_{12} illustration

- Assume for example $M = 6$ and $n = 5$, then ℓ_{12} is defined as

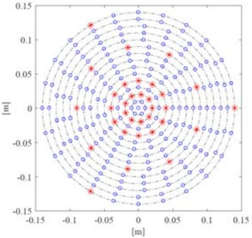


- and assume that a constrained optimization of $\|\mathbf{X}\|_{12} = \sum y_i$ yields the vector $\mathbf{y} = [y_1, y_2, ..., y_5]^T = [\alpha_1, 0, 0, \alpha_4, \alpha_5]^T$, then:

$$\bar{\mathbf{X}} = \Bigg[\underbrace{\begin{bmatrix} x_1^{(1)} \\ 0 \\ 0 \\ x_4^{(1)} \\ x_5^{(1)} \end{bmatrix}}_{\mathbf{x}_1} \underbrace{\begin{bmatrix} x_1^{(2)} \\ 0 \\ 0 \\ x_4^{(2)} \\ x_5^{(2)} \end{bmatrix}}_{\mathbf{x}_2} \underbrace{\begin{bmatrix} x_1^{(3)} \\ 0 \\ 0 \\ x_4^{(3)} \\ x_5^{(3)} \end{bmatrix}}_{\mathbf{x}_3} \underbrace{\begin{bmatrix} x_1^{(4)} \\ 0 \\ 0 \\ x_4^{(4)} \\ x_5^{(4)} \end{bmatrix}}_{\mathbf{x}_4} \underbrace{\begin{bmatrix} x_1^{(5)} \\ 0 \\ 0 \\ x_4^{(5)} \\ x_5^{(5)} \end{bmatrix}}_{\mathbf{x}_5} \underbrace{\begin{bmatrix} x_1^{(6)} \\ 0 \\ 0 \\ x_4^{(6)} \\ x_5^{(6)} \end{bmatrix}}_{\mathbf{x}_6} \Bigg]$$



- Define the set $\left\{\omega_j'\right\}_{j=1}^{J'} = \left\{\omega_j \mid j \bmod J_\mathrm{d} = 0\right\} \in \ \Omega$
- and define $\mathbf{h}_Z(\omega_j') = [H_{i_1}(\omega_j'), H_{i_2}(\omega_j'), ..., H_{i_Z}(\omega_j')]^T, \omega_j' \in \Omega$
- Then, the following ℓ_{12}-norm iterative optimization problem may be solved



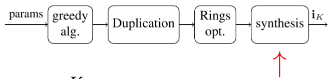
$$
\begin{aligned}
&\underset{\{\mathbf{h}_Z(\omega_j')\}}{\text{minimize}} && \sum_{p=1}^{P} \alpha_p^k \eta_p \\
&\text{subject to} && \eta_p \geq \left\|\bar{\mathbf{h}}_p\right\|_2 \\
&\text{and } \ \forall \omega_j' \in \Omega && \\
& && \mathbf{h}_Z^H\left(\omega_j'\right)\mathbf{d}_{L_Z}\left(\omega_j', \theta_\mathrm{s}'\right) = 1 \\
& && \mathbf{h}_Z^H\left(\omega_j'\right)\mathbf{h}_Z\left(\omega_j'\right) \leq \gamma(\omega_j') \\
& && \left\|\left(\mathbf{b}_\mathrm{d}^\mathrm{m}(\theta_\mathrm{s})\right)^T - \mathbf{h}_Z^H\left(\omega_j'\right)\mathbf{D}_{L_Z,\Theta_\mathrm{m}'}\left(\omega_j'\right)\right\|_2^2 \leq \epsilon_1(\omega_j') \\
& && \left\|\left(\mathbf{b}_\mathrm{d}^\mathrm{s}(\theta_\mathrm{s})\right)^T - \mathbf{h}_Z^H\left(\omega_j'\right)\mathbf{D}_{L_Z,\Theta_\mathrm{s}'}\left(\omega_j'\right)\right\|_2^2 \leq \epsilon_2(\omega_j')
\end{aligned}
$$

$$
\bar{\mathbf{h}}_p = \left\{\left\{\left\{H_{i_z}(\omega_j')\right\}_{z=1}^{Z}\right\}_{j=1}^{J'} \in \ p\text{th ring}\right\}, \quad p = 1, 2, \ldots, P.
$$

$$
\alpha_m^k = 1/\left(\eta_m^{k-1} + \epsilon\right), \mathbf{d}_{L_Z}\left(\omega_j', \theta_\mathrm{s}'\right) = \mathbf{T}_\mathrm{s}\left(\mathbf{i}_Z\right)\mathbf{d}\left(\omega_j', \theta_\mathrm{s}'\right),
$$

$$
\mathbf{D}_{L_Z,\Theta_\mathrm{m}'}\left(\omega_j'\right) = \mathbf{T}_\mathrm{s}\left(\mathbf{i}_Z\right)\mathbf{D}_{M,\Theta_\mathrm{m}'}\left(\omega_j'\right), \theta_\mathrm{s}' = \theta_\mathrm{s} + \frac{\pi}{Q}, \Theta_\mathrm{m}' = \Theta_\mathrm{m} + \frac{\pi}{Q}, \Theta_\mathrm{s}' = \Theta_\mathrm{s} + \frac{\pi}{Q}
$$

Synthesis



- The previous step provides the K indices $\{i_k\}_{k=1}^{K}$ of the sensors, concentrated in a minimal number of rings, G'
- In order to synthesis the desired FI beampattern, $\forall \omega_j \in \Omega$,we solve

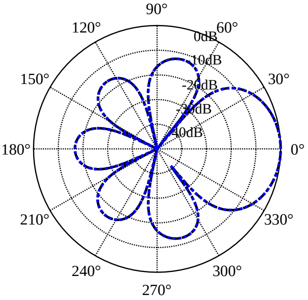
$$
\begin{aligned}
&\underset{\mathbf{h}_K(\omega_j)}{\text{minimize}} \quad \left\| \mathbf{h}_K^H\left(\omega_j\right) \right\|_2^2 \\
&\text{subject to} \\
&\qquad \mathbf{h}_K^H\left(\omega_j\right)\mathbf{d}_K\left(\omega_j, \theta_\mathrm{s}\right) = 1 \\
&\qquad \left\| \left(\mathbf{b}_\mathrm{d}^\mathrm{m}(\theta_\mathrm{s})\right)^T - \mathbf{h}_K^H\left(\omega_j\right)\mathbf{D}_{K,\Theta_\mathrm{m}}\left(\omega_j\right) \right\|_2^2 \leq \epsilon_1(\omega_j) \\
&\qquad \left\| \left(\mathbf{b}_\mathrm{d}^\mathrm{s}(\theta_\mathrm{s})\right)^T - \mathbf{h}_K^H\left(\omega_j\right)\mathbf{D}_{K,\Theta_\mathrm{s}}\left(\omega_j\right) \right\|_2^2 \leq \epsilon_2(\omega_j)
\end{aligned}
$$

where $\mathbf{d}_K\left(\omega_j, \theta_\mathrm{s}\right) = \mathbf{T}_\mathrm{s}\left(\mathbf{i}_K\right)\mathbf{d}\left(\omega_j, \theta_\mathrm{s}\right)$, $\mathbf{D}_{K,\Theta_\mathrm{m}}\left(\omega_j\right) = \mathbf{T}_\mathrm{s}\left(\mathbf{i}_K\right)\mathbf{D}_{\Theta_\mathrm{m}}\left(\omega_j\right)$, and $\mathbf{D}_{K,\Theta_\mathrm{s}}\left(\omega_j\right) = \mathbf{T}_\mathrm{s}\left(\mathbf{i}_K\right)\mathbf{D}_{\Theta_\mathrm{s}}\left(\omega_j\right)$

Numerical Simulations - Concentric DMAs

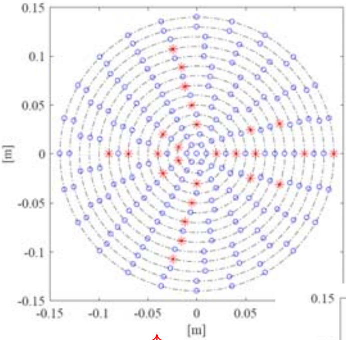
- A third-order hypercardioid DMA :

$$\mathcal{B}_{\mathrm{d}}\left(\theta, \theta_{\mathrm{s}}\right) = \mathcal{B}_N(\theta_{\mathrm{s}} - \theta) = \sum_{n=0}^{N} a_{N,n} \cos^n(\theta_{\mathrm{s}} - \theta)$$



- $G = 15$, $M = 234$, $J = 202$, $Q = 3$
- Minimal spacing $\delta = 1\mathrm{cm}$
- Nulls: $\theta_1 = 51°$, $\theta_2 = 103°$, $\theta_3 = 154°$
- Compare to uniform and random design approaches

Numerical Simulations - Concentric DMAs (2)

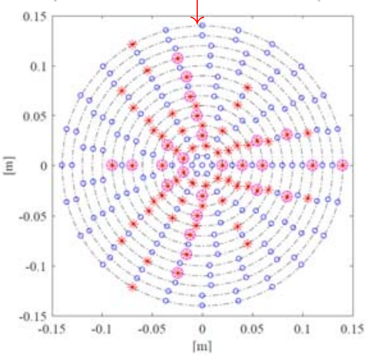


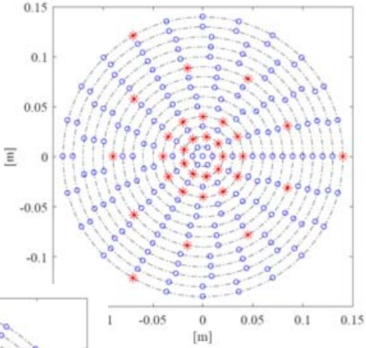
Greedy algorithm

Output $K' = 26$

after duplication

$Z = 72$



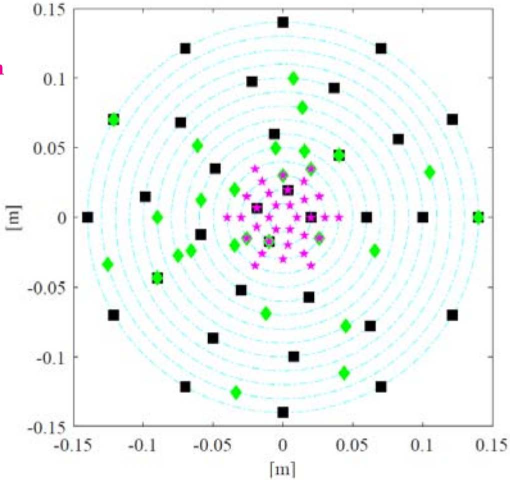


after ring optimization

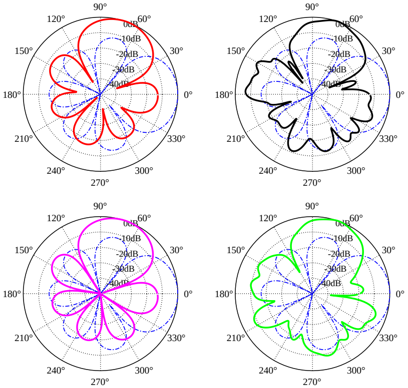
$K = 33, G' = 4$

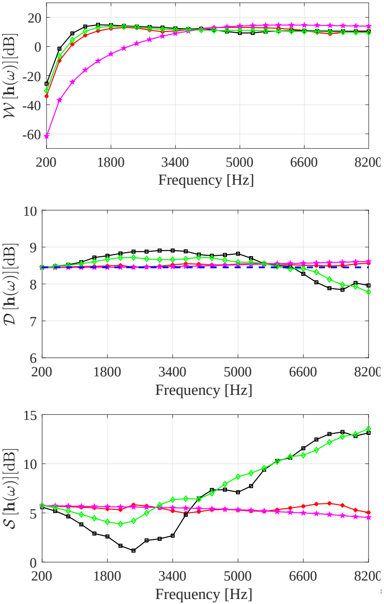
black - large uniform
magenta - small uniform

green - random



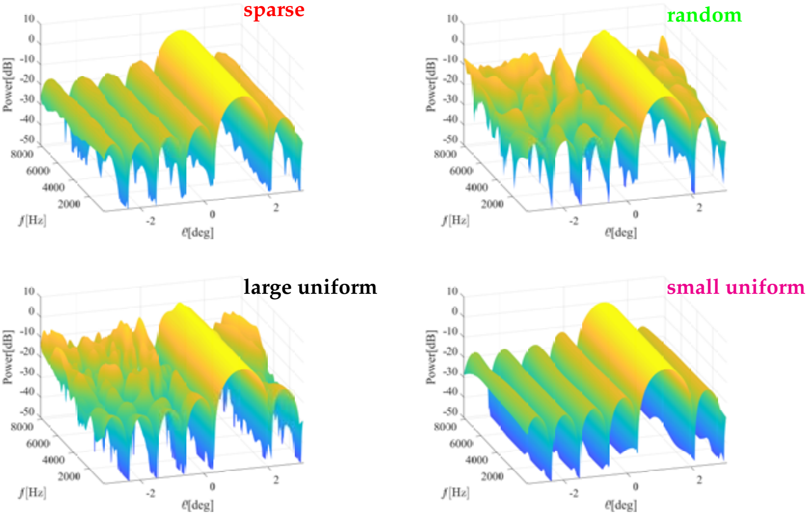
Numerical Simulations - Concentric DMAs (4)





red - greedy sparse
black - large uniform
magenta - small uniform
green - random

blue - theoretical


sparse
random
large uniform
small uniform
Power[dB]
f[Hz]
θ[deg]

Back

Possible future research directions

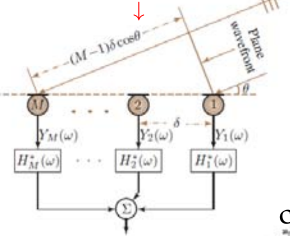
- Throughout this thesis, we developed various approaches to obtained better DMAs design in terms of robustness, directivity, design flexibility, and computational complexity
- In spite of the encouraging improvements, there are still open issues for a future research which are expected to take the obtained performance one step further
    - Extension from 2D to 3D model
    - Volumetric arrays design
    - Other tools and algorithms of data fusion and clustering that can obtain more superior and optimal performance
    - Sparse and asymmetric time-domain DMAs
    - Development of adaptive versions of DMAs used for more complicated scenarios like reverberations

Thanks for listening

Narrowband beamformer:

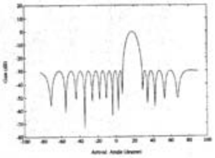
$$Y_m(\omega) = X_m(\omega) + V_m(\omega)$$
$$= X(\omega)e^{-j\omega(t+\tau_m)} + V_m(\omega),$$
$$\tau_m = \frac{(m-1)\delta\cos\theta}{c}$$
$$m = 1, 2, ..., M$$

Broadside ($\theta = 90°$) ↓



← Endfire ($\theta = 0°$)

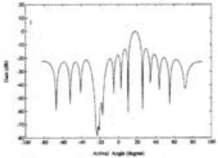
conventional



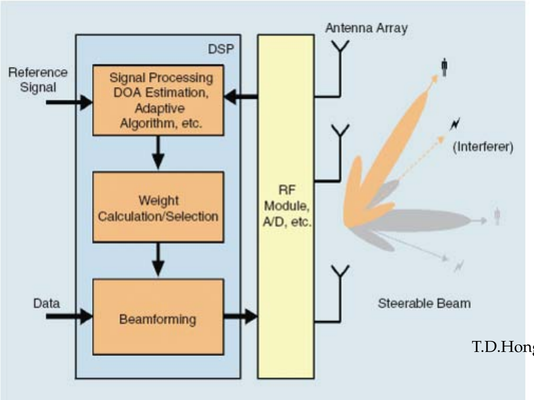
In matrix formulation:

$$\mathbf{y}(\omega) = \begin{bmatrix} Y_1(\omega) & \cdots & Y_M(\omega) \end{bmatrix}^T$$
$$= \mathbf{d}(\omega, \theta)X(\omega)$$
$$\mathbf{h}(\omega) = \begin{bmatrix} H_1(\omega) \cdots & H_M(\omega) \end{bmatrix}^H$$
$$\mathrm{BP}(\omega, \theta) = \left|\mathbf{h}^H(\omega)\mathbf{d}(\omega, \theta)\right|^2_{dB}$$

Adaptive



Smart antennas for high speed wireless data communications

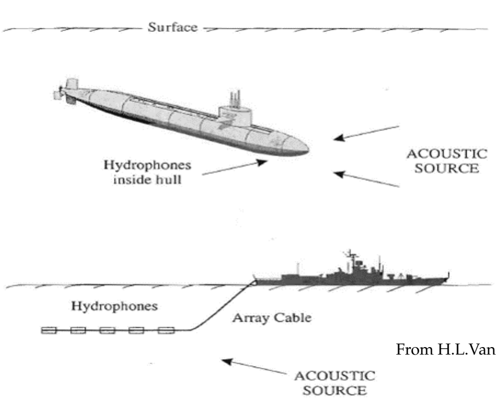


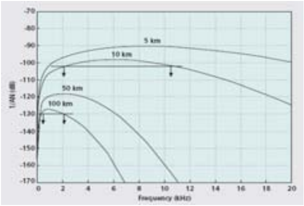
T.D.Hong, P.Russer, 2004

Increasing the signal bandwidth along with the use of antenna arrays is an effective way to increase the data rate in future wireless communication systems

Other broadband beamforming applications(2)

Underwater acoustic communications



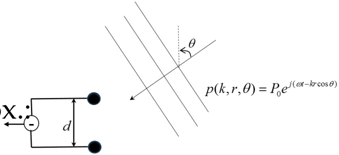


$$SNR(r,f) = \frac{S(f)}{A(r,f)N(f)}$$

From H.L.Van Trees, 2002

The limited available bandwidth and the relatively low carrier frequency result in underwater communication system which is inherently broadband

DMAs: optimality of the endfire direction



The differential of the pressure signal is approx.:

$$\frac{\Delta p}{\Delta r} \approx \frac{p\left(k, r+\frac{d}{2}, \theta\right) - p\left(k, r-\frac{d}{2}, \theta\right)}{d} \propto \exp^{-jkr\cos\theta} \sin\left(\frac{kd\cos\theta}{2}\right)$$

The output power is prop. to the squared magnitude of the differential:

$$\mathrm{P}_{out} \propto \sin^2\left(\frac{kd\cos\theta}{2}\right)$$

The last term gets its maximal value for the case that $\theta = 0°$.

Time-domain design - pros and cons

Pros:

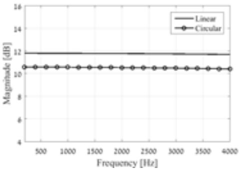
- Important for some applications in which minimal delay is required, such as real-time communications
- Circumvents the edge effects between successive snapshots
- Reduced computational complexity in the design and filtering of time domain when **short filters are sufficient**

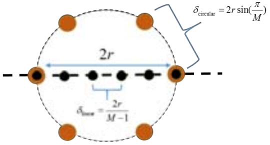
Cons:

- Less suitable for frequency dependent processing algorithms like frequency-selective null-steering
- Less efficient calculation of sample matrix inversion (SMI)
- Less suitable when long filters are required

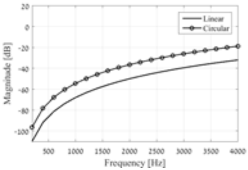
CDMAs Vs. LDMAs - gain in signal to noise ratio

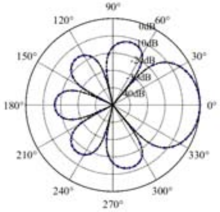
DI

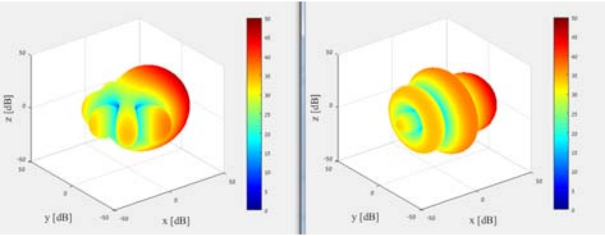




WNG

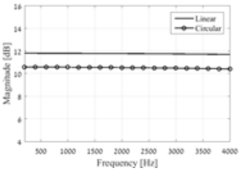


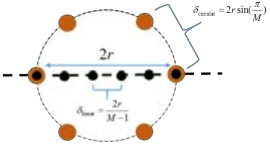




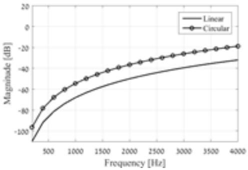
CDMAs Vs. LDMAs - gain in signal to noise ratio

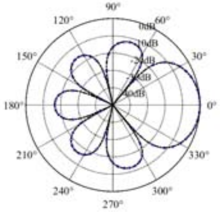
DI

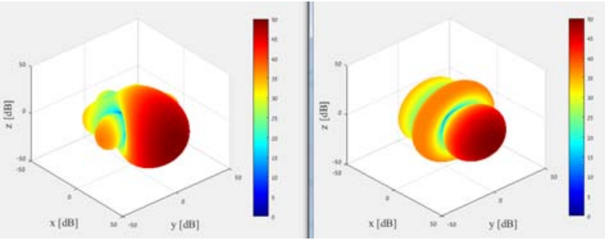




WNG

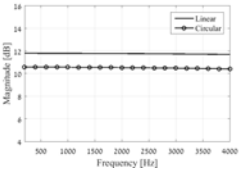


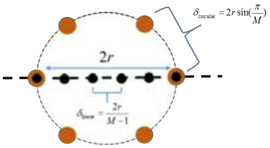




CDMAs Vs. LDMAs - gain in signal to noise ratio

DI





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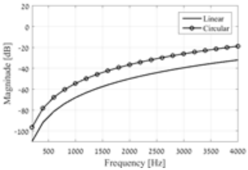

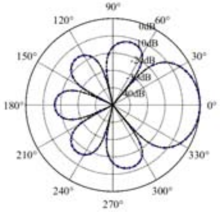

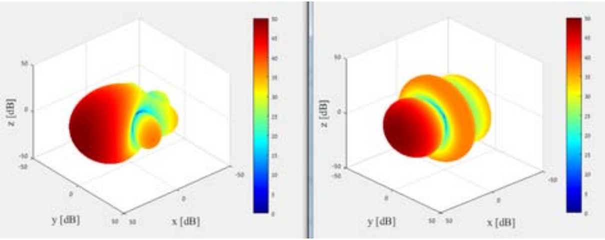

CDMAs Vs. LDMAs - gain in signal to noise ratio

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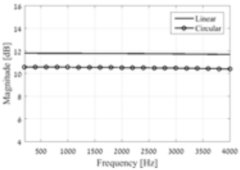

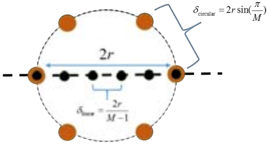

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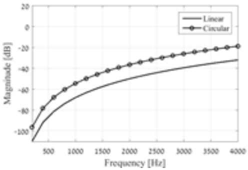

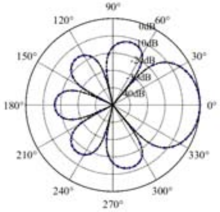

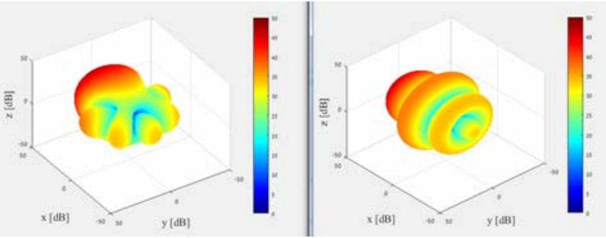

CDMAs Vs. LDMAs - gain in signal to noise ratio

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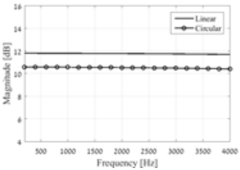

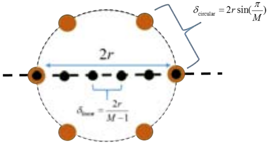

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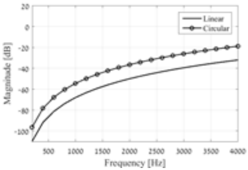

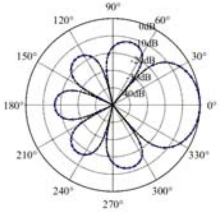

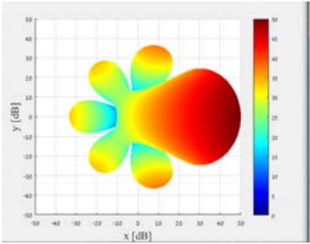

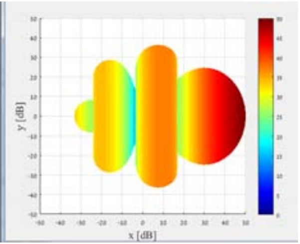

CDMAs Vs. LDMAs - gain in signal to noise ratio

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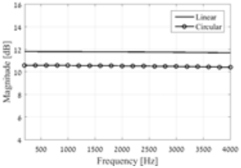

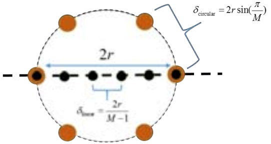

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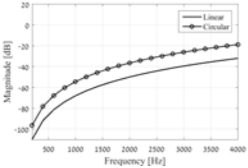
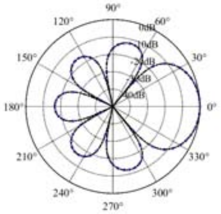
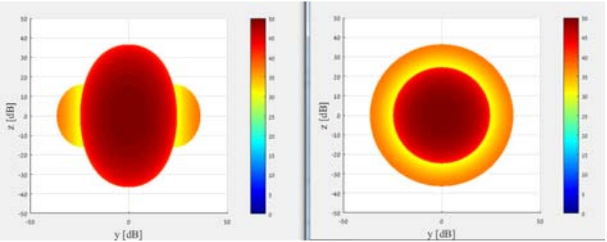
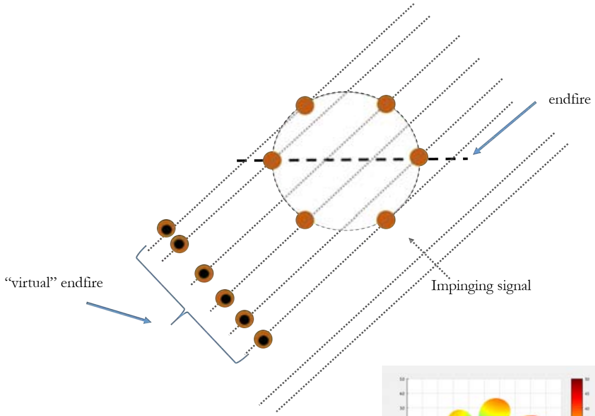
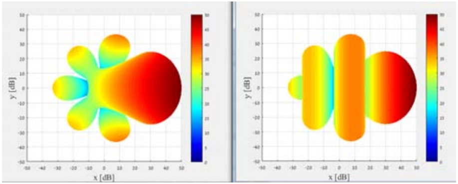






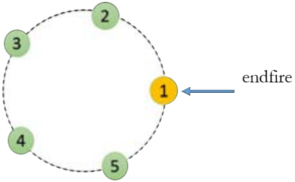
CDMAs - "virtual" endfire





CDMAs - recent work

- Recently, the textbook "Design of CDMAs" by Benesty, Chen, and Cohen
- Fundamental theory and algorithms for the design of general Nth-order CDMAs were established
- Very simple design which exploit the symmetry of the array
  - Design for $\theta_s = 0^\circ$
  - The filters can be used directly in $\psi_m, \quad m = 1, 2, ..., M$ by flipping between the appropriate coefficients



CDMAs - beampattern

- Similar to the linear case:

$$\mathcal{B}\left[\mathbf{h}(\omega,\theta_s),\theta\right] = \mathbf{h}^H(\omega,\theta_s)\mathbf{d}(\omega,\theta) = \sum_{m=1}^{M} H_m^*(\omega,\theta_s)e^{j\varpi\cos(\theta-\psi_1)}, \ \varpi = \frac{\omega r}{c}$$

- where

$$\mathbf{h}(\omega,\theta_s) = \left[H_1(\omega,\theta_s) \ \ H_2(\omega,\theta_s) \ \ \ldots \ \ H_M(\omega,\theta_s)\right]$$

- The theoretical beampattern:

$$\mathcal{B}_N\left(\theta-\theta_s\right) = \sum_{n=0}^{N} a_{N,n}\cos^n\left(\theta-\theta_s\right).$$

CDMAs - gain in signal to noise ratio

- The gain in SNR is

$$\mathcal{G}\left[\mathbf{h}(\omega,\theta_s)\right] = \frac{\left|\mathbf{h}^H(\omega,\theta_s)\mathbf{d}(\omega,\theta_s)\right|^2}{\mathbf{h}^H(\omega,\theta_s)\Gamma_{\text{v}}(\omega)\mathbf{h}(\omega,\theta_s)}$$

- The WNG can be derived by setting $\Gamma_{\text{v}}(\omega) = \mathbf{I}$
- The directivity index is accepted by substituting

$$\left[\Gamma_{\text{v}}(\omega)\right]_{ij} = \text{sinc}\left(\frac{\omega\delta_{ij}}{c}\right)$$

where

$$\delta_{ij} = 2r\left|\sin\left[\frac{\pi(i-j)}{M}\right]\right|$$