Multichannel Semi-blind Sparse Deconvolution of Seismic Signals

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Presentation outline

- Introduction
- Problem Formulation
- Multichannel Semi-Blind Deconvolution – MSBD
- Experimental Results
- Conclusions
- Future work

Introduction

Seismic waves

- Seismic waves are a form of energy that travel through the earths' layers.
- The velocity of the wave is affected by the density and elasticity of the medium.
- The elasticity of the medium is measured by a parameter called acoustic impedance.
- When a seismic wave traveling through the earth encounters an interface between two areas with different acoustic impedance, some of its energy is reflected.

Reflection seismology

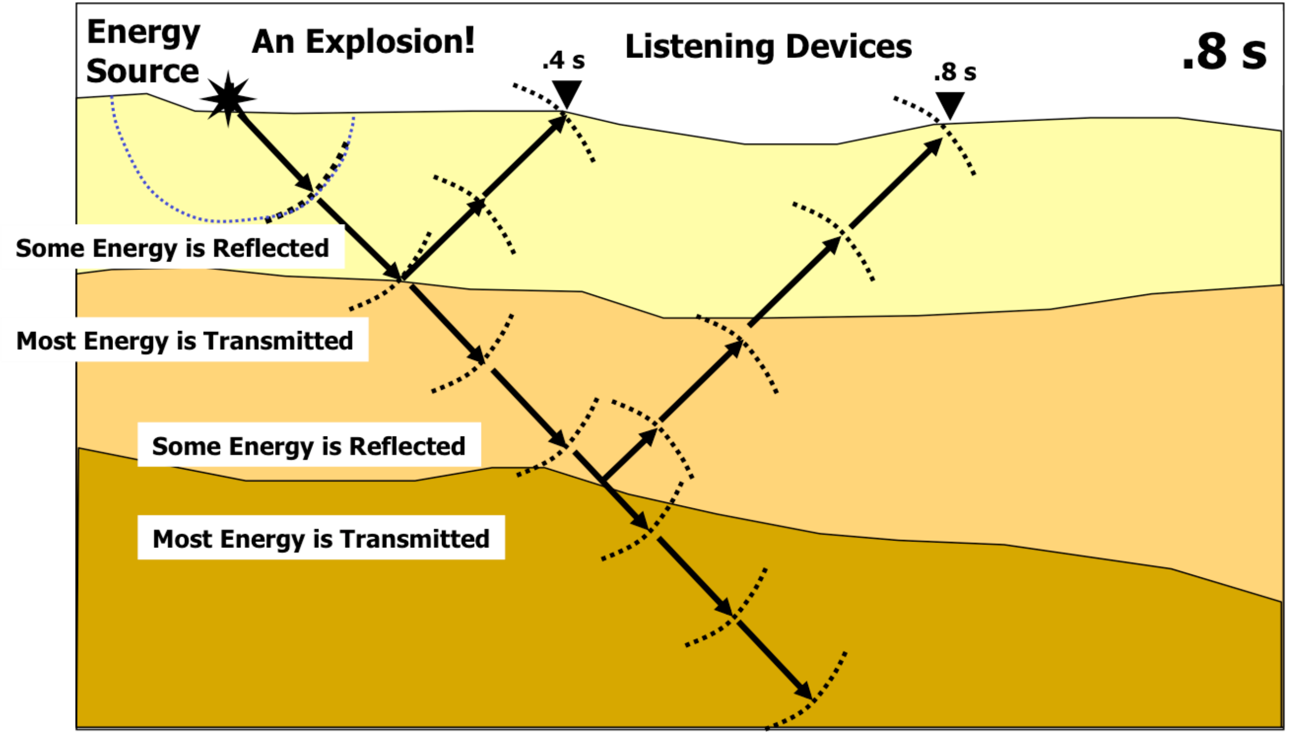
- Reflection seismology is a field which uses the properties of seismic waves to explore the subsurface and reconstruct an image of it.

- Seismic waves are generated using an energy source such as dynamite explosion.

- Signals reflected from the interfaces are received by geophones and reflection times are measured.

- Pathways reconstruction is made by geophysicists using acoustic velocity and reflection times from the surface to various receivers.

Reflection seismology



Seismic deconvolution

- Seismic data is the product of data recorded by the geophones. Usually after a process of stack and migration.

- However, the seismic data does not truly represent the subsurface because each reflected signal was distorted while propagating from the surface and back.

- Seismic deconvolution is the process where we take the seismic data and remove the distortion created by the propagation in the medium.

- The distortion is modeled as a convolution between the desired reflectivity image and a signal defining the medium, called wavelet.

Seismic deconvolution

- Non-Blind deconvolution methods assume to know the wavelet and can apply the deconvolution process even on one trace.

- Blind deconvolution methods assume nothing on the wavelet and use a multichannel setup to recover both the wavelet and the reflectivity signal.

- We address the problem of multichannel semi-blind seismic deconvolution, where the wavelet is known up to some error.

Problem Formulation

Definitions

- We denote the earths' impulse response, the wavelet, by $w[n]$
- The reflectivity series and the seismic data are denoted by $r[n]$ and $s[n]$, respectively.
- The input-output relation between the reflectivity series, the wavelet and the seismic data are given by the following matrix-vector form,

$$\mathbf{s}_i = \mathbf{W}\mathbf{r}_i + \mathbf{v}_i, \quad 1 \le i \le N$$

where,

$$\mathbf{s}_i \in \mathbb{R}^{N_r+N_w-1}, \quad \mathbf{r}_i \in \mathbb{R}^{N_r}, \quad \mathbf{W} \in \mathbb{R}^{(N_r+N_w-1)\times N_r}, \quad \mathbf{v}_i \in \mathbb{R}^{N_r+N_w-1}$$

The index i represents the channel number in the multichannel case

The problem

- In the semi blind case, we would like to recover the reflectivity signal, $\boldsymbol{r_i}$, from the seismic data, $\boldsymbol{s_i}$, when holding a noisy version of the wavelet, $w'[n]$.

- As a side result, we will also recover the true wavelet, $w[n]$.

Assumptions

- We hold $\boldsymbol{w'}$ which is a noisy version of $\boldsymbol{w}$:

$$\boldsymbol{w'} = \boldsymbol{w} - \boldsymbol{w_n}$$

  - We assume an additive uncertainty in the wavelet, which is represented by $\boldsymbol{w_n}$.
  - We also assume to know the statistics of the elements of $\boldsymbol{w_n}$ up to 2nd order.
- We assume normally distributed channel noise, $v[n]$, with known statistics.
- We assume the reflectivity signal is sparse.

Multichannel Semi-Blind Deconvolution – MSBD

MSBD - Introduction

- We present a novel, two-stage iterative algorithm that recovers both the reflectivity and the wavelet.
- While the reflectivity series is recovered using sparse modeling of the signal, the wavelet is recovered using L2 minimization, exploiting the fact that all channels share the same wavelet.
- The method we propose is an iterative method, with two steps in each iteration, as follows:
    1. Assume to know the wavelet and use the sparse deconvolution method to recover the reflectivity signal.
    2. Assume to know the reflectivity and use the L2 minimization method to recover the wavelet.

Reflectivity recovery - SSI

- The first step of MSBD is the reflectivity recovery process where we assume to know the wavelet.
- This step is done by the SSI (Sparse Spike Inversion) algorithm, which is a well known deconvolution method for sparse signals.
- SSI solves the following optimization problem

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \frac{1}{2} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \lambda \|\boldsymbol{x}\|_1$$

- Where we know that $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$
- Minimization of the term $\|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2$ maintains fidelity to the observations
- Minimization of the term $\|\boldsymbol{x}\|_1$ signal maintains sparsity of the recovered signal.

Reflectivity recovery - SSI

- The above optimization problem can be also presented in the following way:

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \|\boldsymbol{x}\|_1 \quad s.t. \quad \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2 < \varepsilon$$

- We will use the last form, where ε determines the trade-off between fidelity to the observations and fidelity to the sparseness of the reflectivity signal.

- The important thing in SSI is choosing wisely the parameter ε.

- When discussing the multichannel case, we will use SSI on each channel separately, choosing the appropriate ε for each channel.

Reflectivity recovery – modified equations

- Now we impose SSI on our problem.

- As mentioned before, in the beginning of the process we hold a noisy version of the true wavelet. Analytically, we assume additive noise.

- We rewrite the equations:

$$\boldsymbol{s_i} = \boldsymbol{W r_i} + \boldsymbol{v_i} = (\boldsymbol{W'} - \boldsymbol{W_n})\boldsymbol{r_i} + \boldsymbol{v_i} = \boldsymbol{W' r_i} + \boldsymbol{v_i} - \boldsymbol{W_n r_i}$$

- Looking at the relation $\boldsymbol{s_i} = \boldsymbol{W' r_i} + \boldsymbol{v_i} - \boldsymbol{W_n r_i}$ we can mark $\boldsymbol{W'}$ as our "known" wavelet and $\boldsymbol{v_i} - \boldsymbol{W_n r_i}$ as the term that represents the noise or uncertainty in the problem.

Reflectivity recovery – trade-off parameter

- For a wise choice of the trade-off parameter, variance analysis must be performed for that term.

- A major issue we have identified is that in each iteration the variance of the uncertainty term can be changed and a wise adaptation to that trade-off parameter needs to be made.

- We denote the new noise term as $\boldsymbol{v'}_{\boldsymbol{i}} = \boldsymbol{v_i} - \boldsymbol{W_n r_i}$

- We propose two ways of calculating the trade-off parameter from the noise term.

Reflectivity recovery – trade-off parameter

- The first way is:
  - Define the total noise in the problem as

$$\bar{v}'_i = \sum_{n=1}^{N_r+N_w-1} v'_i[n]$$

  - Calculate the trade off parameter:

$$\varepsilon_i = \sigma_{\bar{v}'_i} = \sqrt{E(\bar{v}'_i - E\bar{v}'_i)^2}$$

- The second way is:

$$\varepsilon_i = \sqrt{\sum_{j=1}^{Nr+N_w-1} \sigma^2_{v'_i[n]}}$$

Reflectivity recovery – trade-off parameter

- The main difference is that in the first form we look at different elements of the noise vector with common elements constructing them, hence there is a strong correlation between the sources of noise from different elements.
- In the second form we look at the total noise as the sum of independent noise sources in each element.
- The selection of a specific form comes more from intuition and empirical processes and less from an analytical proof that the chosen form is the only correct one.
- Different forms can be suggested, the important thing is to maintain a logical connection to the model of the problem and to take into account and quantify all the noise sources in the problem.

Wavelet recovery – dictionary learning

- The second step of MSBD is recovering the wavelet while assuming to know the reflectivity signal.
- Unlike in the first step, here we cannot apply SSI, or any other sparse inversion method for that matter, because the wavelet signal is not sparse.
- We look at this problem from another point of view, dictionary learning.
- The seismic data can be considered a linear combination of the columns of **W**, the dictionary, where the reflectivity series can be treated as the coefficients.
- With that in mind, finding the wavelet when the reflectivity series is known can be treated by methods from the field of dictionary learning.

Wavelet recovery – dictionary learning

- We use a method of dictionary update based on the Signature Dictionary as described in [M. Elad, "*Sparse and Redundant Representations*", 2010].
- Specifically, we would like to minimize the L2 expression

$$\sum_{i=1}^{N} \|\boldsymbol{s_i} - \boldsymbol{W}\boldsymbol{r_i}\|_2^2$$

- The k-th column of $\boldsymbol{W}$, denoted by $\boldsymbol{w_k}$, is a shifted version of $\boldsymbol{w}$.
- We can write that $\boldsymbol{w_k} = \boldsymbol{R_k}\boldsymbol{w}$ where

$$R_k = \begin{pmatrix} 0_{k-1 \times N_w} \\ I_{N_w \times N_w} \\ 0_{N_r - k \times N_w} \end{pmatrix}$$

Wavelet recovery – dictionary learning

- The presented minimization problem was solved by M. Elad to obtain the optimal $\boldsymbol{w}$.

- Although solved for different $\boldsymbol{R_k}$ matrices we can use the obtained solution as is with the appropriate matrices.

$$\boldsymbol{w}^{opt} = \left(\sum_{k=1}^{N_r}\sum_{j=1}^{N_r}\left[\sum_{i=1}^{N} r_i[k]r_i[j]\right]\boldsymbol{R}_k^T\boldsymbol{R}_j\right)^{-1}\sum_{i=1}^{N}\sum_{k=1}^{N_r} r_i[k]\boldsymbol{R}_k^T\boldsymbol{s_i}$$

Use cases – experimental results

Use cases

- In this part we review two use cases of wavelet uncertainty
  - Wavelet AWGN
  - Wavelet parametric change

Wavelet AWGN – problem definition

- In this case we discuss AWGN contamination of the wavelet.
- The model we are assuming:

$$\boldsymbol{w}' = \boldsymbol{w} + \boldsymbol{w_n}$$

- Where

$$i.i.d \quad \boldsymbol{w_n}[k] \sim N(0, \sigma_w^2) \quad 1 \leq k \leq N_w$$

- And we assume to know σ_w.
- We recall that the new noise term is $\boldsymbol{v'}_{\boldsymbol{i}} = \boldsymbol{v_i} - \boldsymbol{W_n r_i}$, where $\boldsymbol{W_n}$ is the convolution matrix of $\boldsymbol{w_n}$.

Wavelet AWGN – Noise analysis

- Substitute $v'_i[n]$ with corresponding term:

$$\boldsymbol{v'_i} = \boldsymbol{v_i} - \boldsymbol{W_n}\boldsymbol{r_i} =$$

$$= \begin{bmatrix} v_i[1] \\ v_i[2] \\ \cdot \\ \cdot \\ \cdot \\ v_i[N_r + N_w - 1] \end{bmatrix} - \begin{pmatrix} w_n[1] & 0 & \cdot & \cdot & \cdot & 0 \\ w_n[2] & w_n[1] & 0 & \cdot & \cdot & 0 \\ \cdot & w_n[2] & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ w_n[N_w] & \cdot & & & & 0 \\ 0 & w_n[N_w] & & & & w_n[1] \\ 0 & 0 & \cdot & & & w_n[2] \\ \cdot & 0 & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & & & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & w_n[N_w] \end{pmatrix} \cdot \begin{bmatrix} r_i[1] \\ r_i[2] \\ \cdot \\ \cdot \\ \cdot \\ r_i[N_r] \end{bmatrix}$$

- So the general element in $\boldsymbol{v'_i}$ can be written as:

$$v'_i[k] = v_i[k] - \sum_{j=1}^{k} w_n[k-j+1]r_i[j]$$

Wavelet AWGN – ε calculation

- Here we will choose to calculate the trade-off parameter according to the first form. We can see that there are common noise sources in elements from different vectors.

- Define the total noise in the problem according to the first form

$$\bar{v}'_i = \sum_{n=1}^{N_r+N_w-1} v'_i[n]$$

- And after some algebra we obtain:

$$\bar{v}'_i = \sum_{l=1}^{N_r+N_w-1} v_i[l] - \sum_{j=1}^{N_w}\left(\sum_{k=1}^{N_r} r_i[k]\right) w_n[j]$$

Wavelet AWGN – ε calculation

- We can see that $\bar{v}'_i$ is a linear combination of independent normally distributed random variables, so we can directly obtain the variance and the standard deviation of it.
- Again, some algebra and we get:

$$\varepsilon_i = \sigma_{\bar{v}'_i} = \sqrt{(N_r + N_w - 1)\sigma_v^2 + N_w \left(\sum_{k=1}^{N_r} r_i[k] \right)^2 \sigma_w^2}$$

- Notice the dependence on σ_v and σ_w, which changes from iteration to iteration.

Wavelet AWGN – Noise STD update

- Updating $\boldsymbol{\sigma_v}$ is not trivial and has no analytical solution to date.
- We show an easy way to update $\boldsymbol{\sigma_w}$ in each iteration to best fit ε to the current iteration.
- The initial wavelet, $\boldsymbol{w}_{init}$, and the current wavelet, $\boldsymbol{w}_{curr}$, can be modeled as:

$$\boldsymbol{w}_{init} = \boldsymbol{w} + \boldsymbol{w}_n$$
$$\boldsymbol{w}_{curr} = \boldsymbol{w} + \boldsymbol{w}_n'$$

Where we assume normal distribution of $\boldsymbol{w}_n$ and $\boldsymbol{w_n'}$.

- We analyze:

$$\|\boldsymbol{w}_{init} - \boldsymbol{w}_{curr}\|_2^2 = \|\boldsymbol{w} + \boldsymbol{w}_n - (\boldsymbol{w} + \boldsymbol{w_n'})\|_2^2 = N_w[\sigma_w^2 + \sigma'^2_w]$$

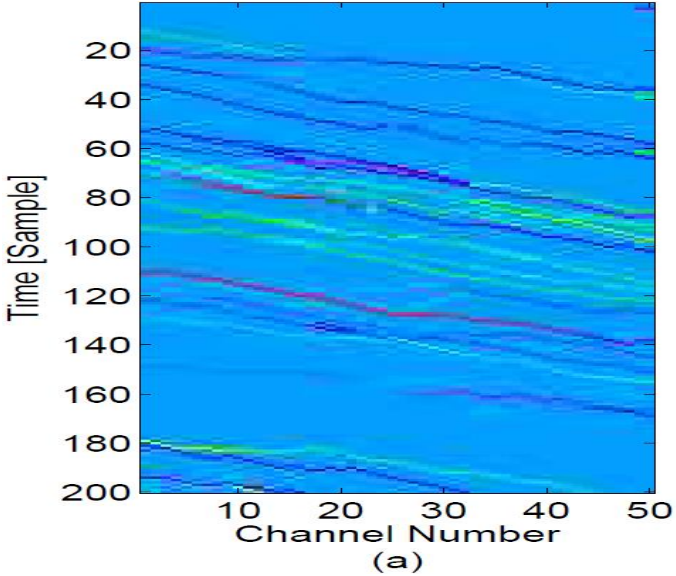
And obtain:

$$\sigma'_w = \sqrt{\frac{1}{N_w}\|\boldsymbol{w}_{init} - \boldsymbol{w}_{curr}\|_2^2 - \sigma_w^2}$$

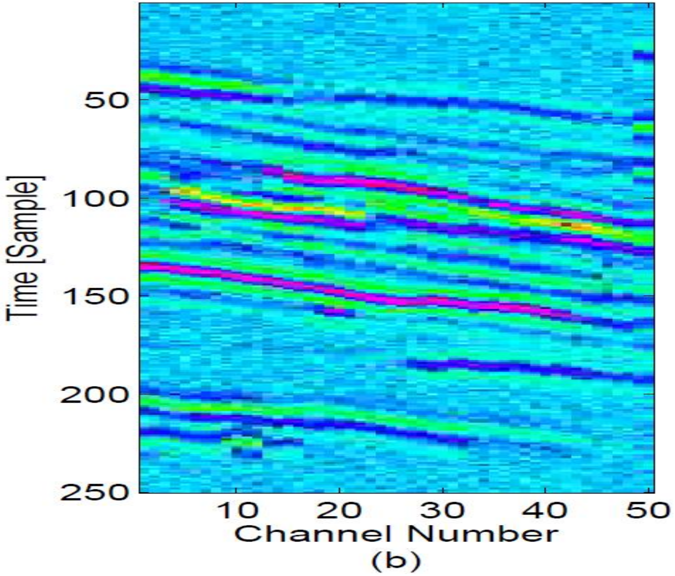
Wavelet AWGN – Experimental results

- We start with a specific example
- $SNR = 10dB, \sigma_w = 0.15$
- 20 Iterations

Wavelet AWGN – Experimental results, seismic data

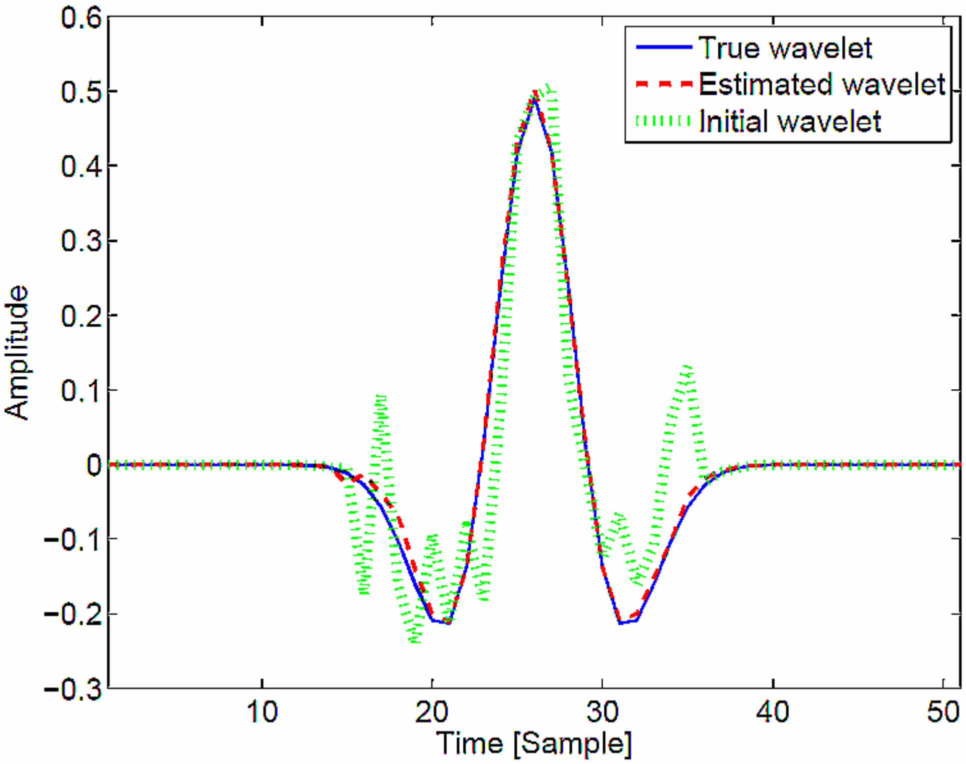


(a) true (original) reflectivity

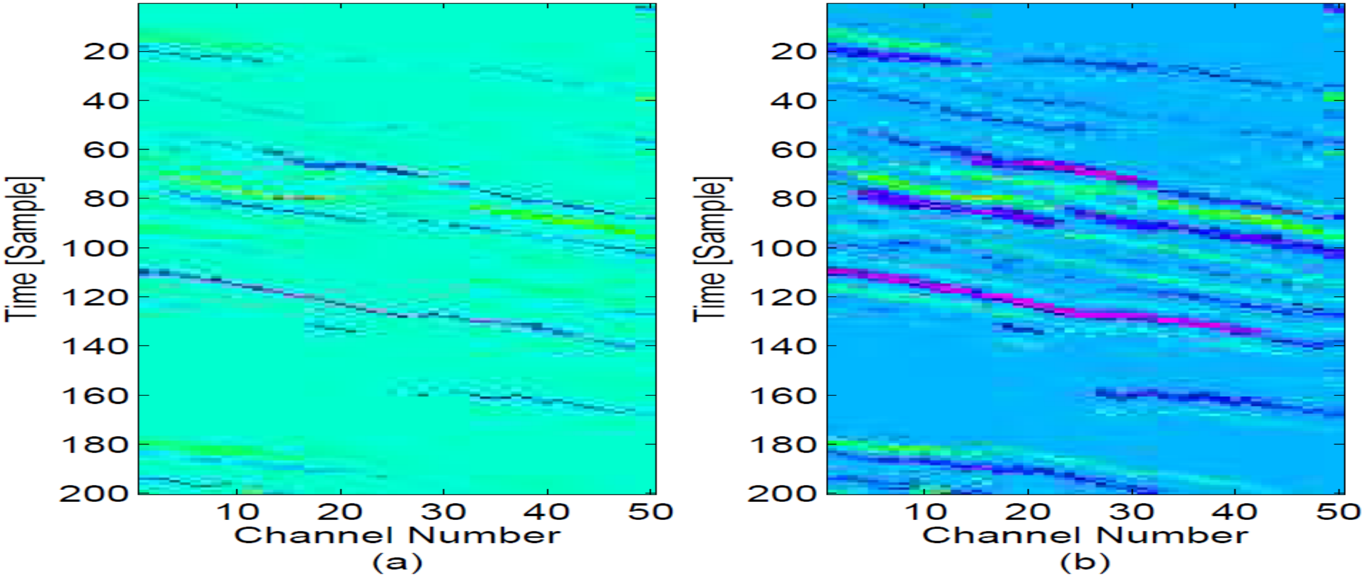


(b) seismic data

Wavelet AWGN – Experimental results, wavelet recovery



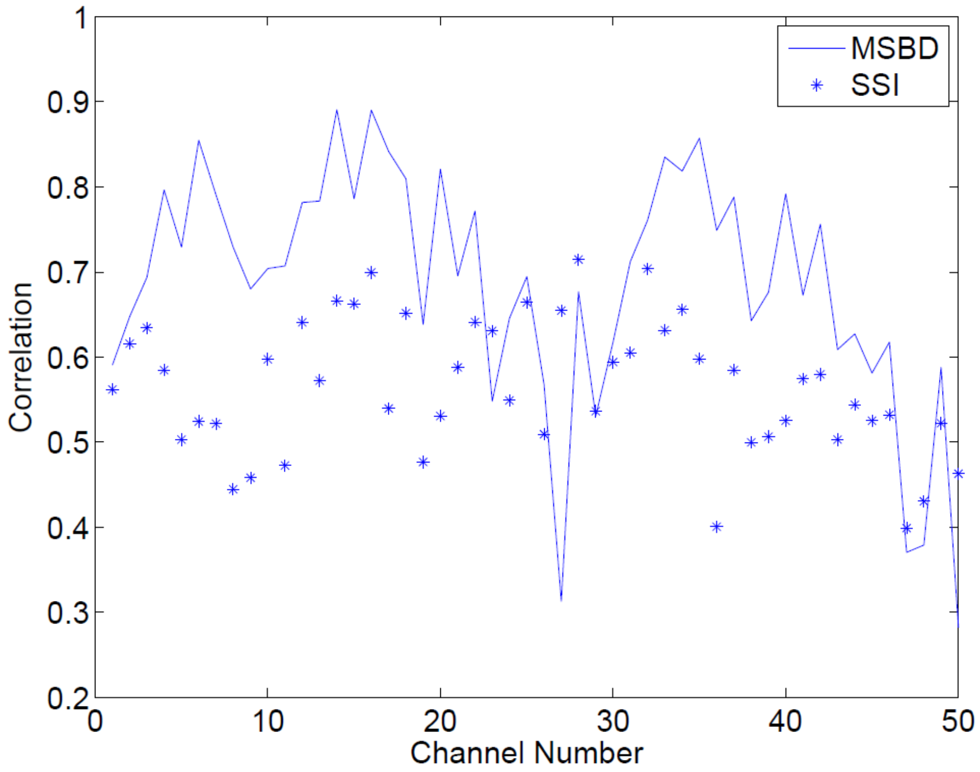
Wavelet AWGN – Experimental results, reflectivity recovery



(a) MSBD recovered reflectivity

(b) SSI Recovered Reflectivity

Wavelet AWGN – Experimental results, Correlation measure



Wavelet parametric change– Problem definition

- In this case we address presence of noise in one of the parameters defining the wavelet.
- Specifically, we assume the Ricker wavelet which is defined by:

$$w(t;f) = (1 - 2\pi^2 f^2 t^2)e^{-\pi^2 f^2 t^2}$$

- Where f is a parameter that represents the frequency of the wavelet.
- In our case, we assume that the seismic data results from a wavelet defined by a frequency f_0, $w(t;f_0)$, but the wavelet we are initially given is defined by the parameter f, $f \neq f_0$.
- We assume normal distribution of f, $f \sim N(f_0, \sigma_f^2)$

Wavelet parametric change– Make it additive

- This modeling of the wavelet uncertainty cannot be treated as an additive noise to the wavelet.
- We present the problem as an additive noise to the wavelet by representing $w(t; f)$ as a Taylor series around f_0.
- We develop the Taylor series up to its 2nd order. In the case of Ricker wavelet we didn't see any differences when 3rd order Taylor development was used.

Wavelet parametric change– Taylor series

- Calculate first 2 derivatives of the wavelet:

$$\frac{\partial w(t;f)}{\partial f} = (4\pi^4 f^3 t^4 - 6\pi^2 f t^2)e^{-\pi^2 f^2 t^2}$$

$$\frac{\partial^2 w(t;f)}{\partial f^2} = (24\pi^4 f^2 t^4 - 8\pi^6 f^4 t^6 - 6\pi^2 t^2)e^{-\pi^2 f^2 t^2}$$

- Represent the wavelet as its Taylor series:

$$w(t;f) \approx w(t;f_0) + \frac{\partial w(t;f)}{\partial f}|_{f=f_0}(f - f_0) + \frac{1}{2}\frac{\partial^2 w(t;f)}{\partial f^2}|_{f=f_0}(f - f_0)^2$$

Wavelet parametric change– Taylor series

- Put the above in a vector-matrix form:

$$\boldsymbol{W} = \boldsymbol{W_0} + \boldsymbol{DW_1}|_{f=f_0}\Delta f + \frac{1}{2}\boldsymbol{DW_2}|_{f=f_0}\Delta f^2$$

where:

- $\boldsymbol{W}$ and $\boldsymbol{W_0}$ are the convolution matrices of the initial and the true wavelet respectively.
- $\boldsymbol{DW_1}$ and $\boldsymbol{DW_2}$ are the convolution matrices of the first and second derivatives of the wavelet, respectively.
- $\Delta f = f - f_0$

Wavelet parametric change– Additive modeling

- We can see that we can impose the initial model, $\boldsymbol{W} = \boldsymbol{W}' - \boldsymbol{W_n}$, by choosing:

$$\boldsymbol{W_n} = -\left(\boldsymbol{DW_1}|_{f=f_0}\Delta f + \frac{1}{2}\boldsymbol{DW_2}|_{f=f_0}\Delta f^2\right)$$

- Now we can write the new noise term, $\boldsymbol{v'_i} = \boldsymbol{v_i} - \boldsymbol{W_v r_i}$, as:

$$\boldsymbol{v'_i} = \boldsymbol{v_i} + \left(\boldsymbol{DW_1}|_{f=f_0}\Delta f + \frac{1}{2}\boldsymbol{DW_2}|_{f=f_0}\Delta f^2\right)\boldsymbol{r_i}$$

- We assume normal distribution of Δf, $\Delta f \sim N(0, \sigma_f^2)$

Wavelet parametric change– ε calculation

- Here we found the second form of ε calculation more suitable.

$$\varepsilon_i = \sqrt{\sum_{j=1}^{Nr+N_w-1} \sigma^2_{v'_{i}[n]}}$$

- Standard deviation analysis of the elements of $\boldsymbol{v'_i}$ yields:

$$\sigma^2_{v'_{i}[k]} = \sigma_v^2 + (\boldsymbol{DW_1}|_{f=f_0}\boldsymbol{r_i})^2[k]\sigma_f^2 + \frac{1}{2}(\boldsymbol{DW_2}|_{f=f_0}\boldsymbol{r_i})^2[k]\sigma_f^4$$

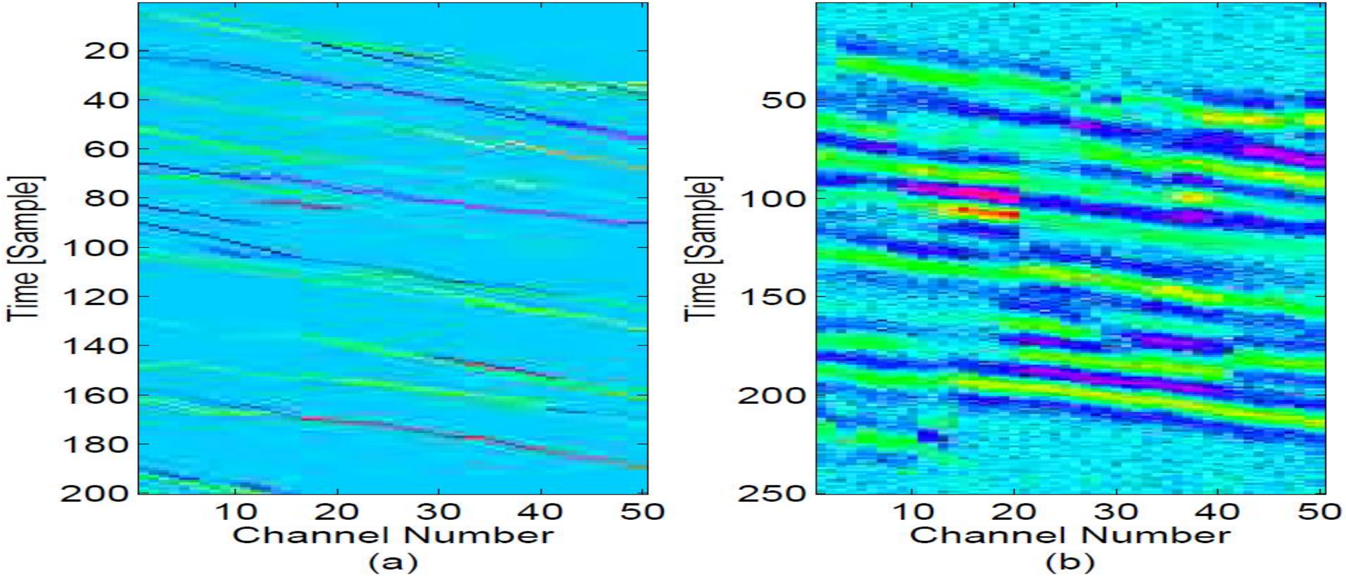
$$\varepsilon_i = \sqrt{N_r\sigma_v^2 + \sigma_f^2\sum_{k=1}^{N_r}(\boldsymbol{DW_1}|_{f=f_0}\boldsymbol{r_i})^2[k] + \frac{1}{2}\sigma_f^4\sum_{k=1}^{N_r}(\boldsymbol{DW_2}|_{f=f_0}\boldsymbol{r_i})^2[k]}$$

- We note here that the derivatives must be recalculated at each iteration.

Wavelet parametric change– Experimental results

- We start with a specific example
- $SNR = 10dB, f_0 = 2, \sigma_f = 1$
- 20 Iterations

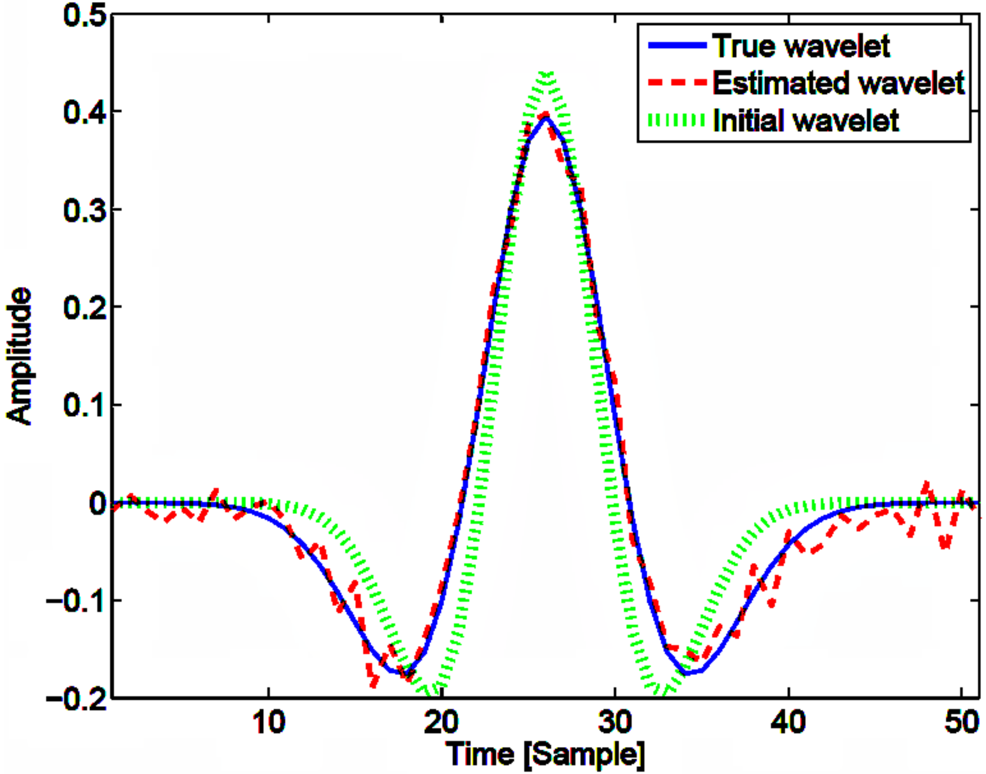
Wavelet parametric change– Experimental results, seismic data



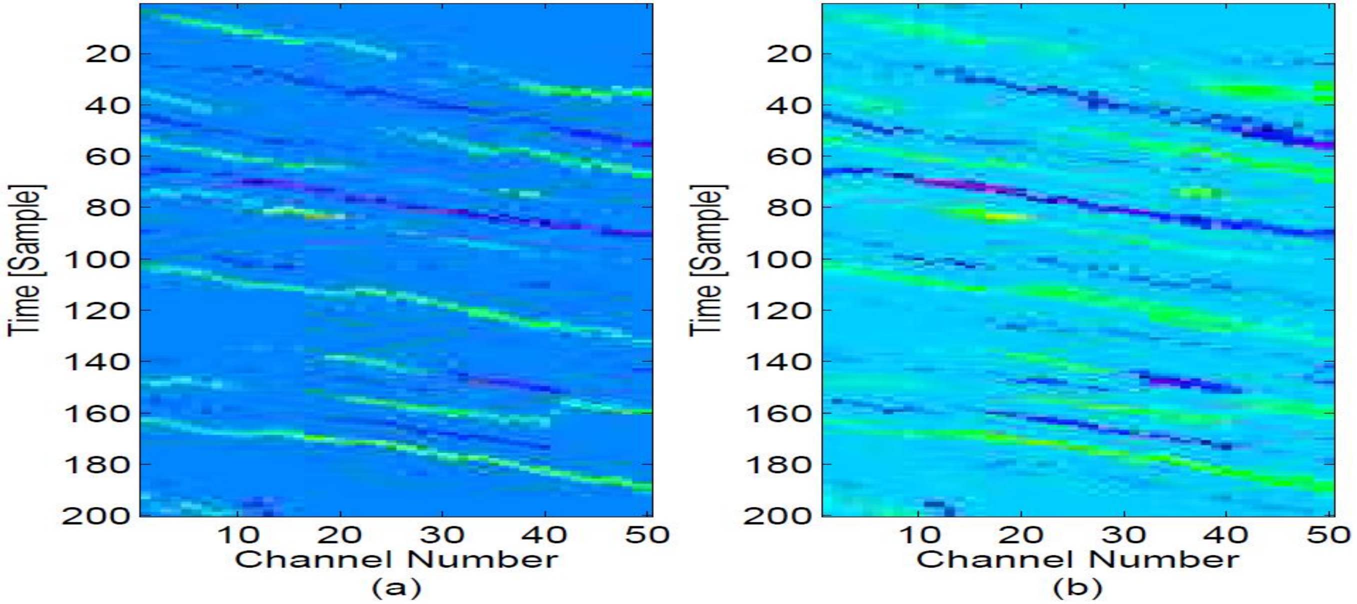
(a) true (original) reflectivity

(b) seismic data.

Wavelet parametric change– Experimental results, wavelet recovery

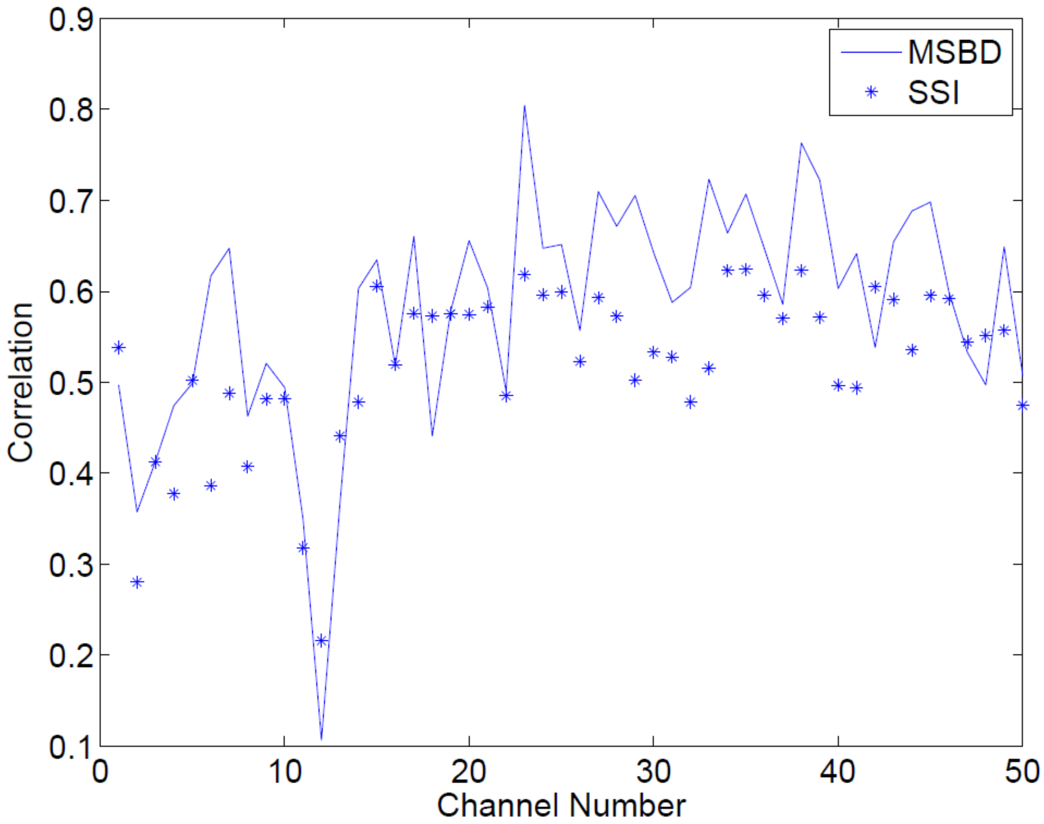


Wavelet parametric change– Experimental results, reflectivity recovery



(a) MSBD recovered reflectivity (b) SSI Recovered Reflectivity

Wavelet parametric change–Experimental results, Correlation measure



Conclusions

Conclusions

- We presented a general two-stage method, where the first stage assumes known wavelet and aims to recover the reflectivity signal and the second stage assumes known reflectivity signal and aims to recover the wavelet.

- In this study we have presented two different cases in which we analytically calculated the trade-off parameter between sparsity of the recovered signal and fidelity of it to the measured data.

- The results clearly show the advantage of MSBD over SSI.

Future work

Future work

- Searching for the best point to stop the iterative process.

- Finding a more general form for choosing the trade-off parameter.

- Further adaption of the wavelet recovery stage to take into account a-priori information regarding the wavelet.

THANKS!

Any questions?