Multichannel Deconvolution of Layered Media Using MCMC methods

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2. Blind Seismic Deconvolution Using MCMC Methods
3. Multichannel Seismic Deconvolution
4. Blind Multichannel MCMC Deconvolution
5. Deconvolution By Smoothing
6. Conclusions

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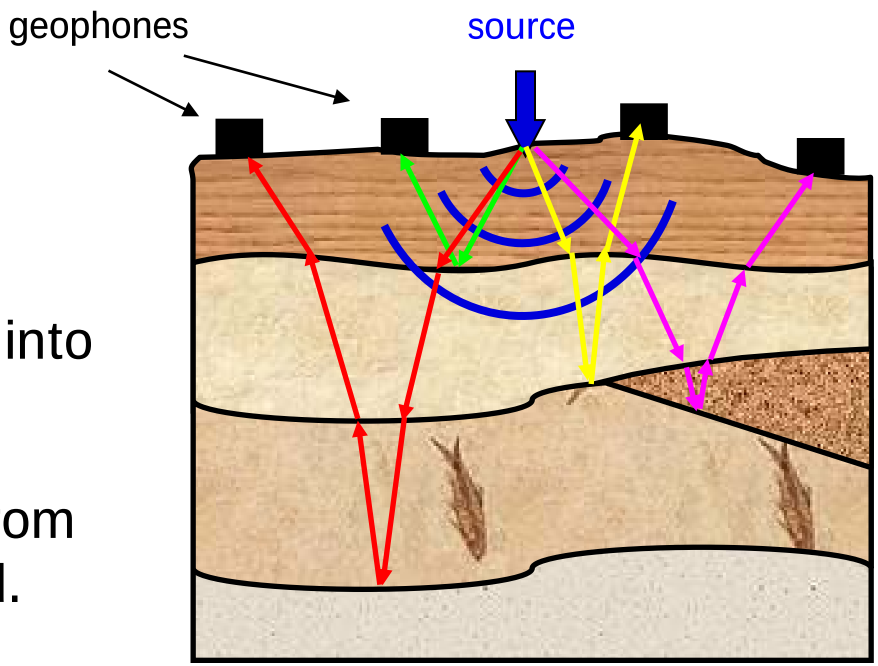
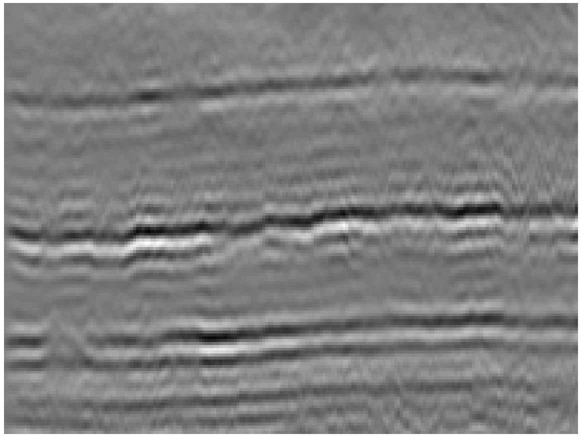
Introduction

- An acoustic wave is transmitted into the ground.
- The reflected energy resulting from impedance changes is measured.
- The observed trace can be modeled as a noisy convolution between a 2D reflectivity and an unknown wavelet.
- Deconvolution is used to remove the effect of the wavelet

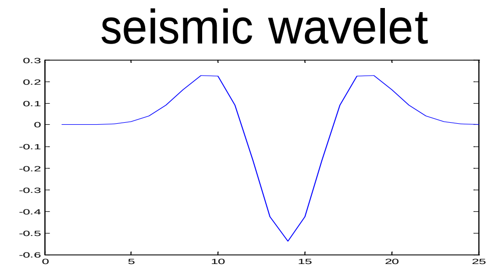


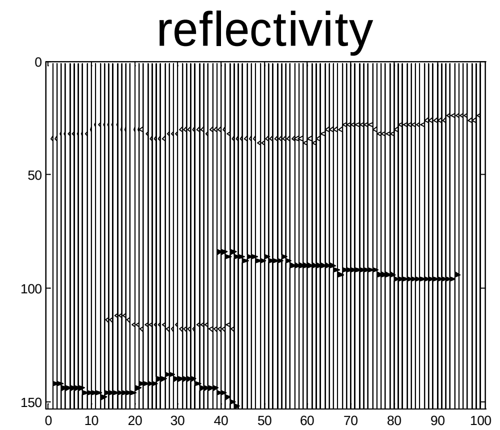
seismic trace



Problem Formulation

- The seismic trace $\mathbf{Y}$ is modeled as the convolution: $\mathbf{Y}=\mathbf{h}*\mathbf{R}+\mathbf{W}$.
- $\mathbf{h}$ - unknown 1D seismic wavelet, invariant in both horizontal and vertical directions.
- $\mathbf{R}$ - 2D reflectivity section, consist of continuous, smooth and mostly horizontal layer boundaries.
- $\mathbf{W}$ - white Gaussian noise independent from $\mathbf{R}$ with zero mean and variance σ_w^2.





The blind deconvolution problem consists in recovering the unknown seismic wavelet $\mathbf{h}$ and the 2D reflectivity section $\mathbf{R}$ from the observed seismic trace $\mathbf{Y}$.

Goals

- Previous works:
  - “Blind Marine Seismic Deconvolution Using Statistical MCMC Methods”

    O. Rosec, J. M. Bouceher, B. Nsiri, and T. Chonavel
  - “Multichannel seismic deconvolution”

    J. Idier and Y. Goussard
- Goals:
  1. Combine the two methods above into a blind stochastic multichannel deconvolution scheme.
  2. Create a smoothing version of the proposed multichannel scheme.

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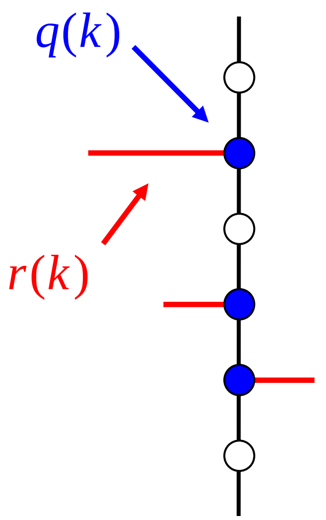
Single Channel Deconvolution

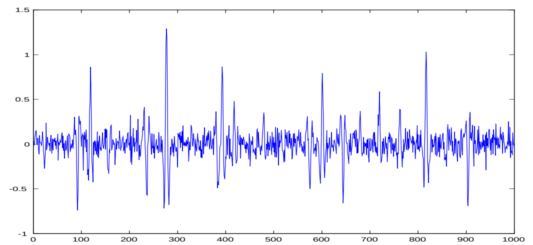
- The multichannel deconvolution problem can be broken into independent 1D vertical deconvolution problems.
- Single channel blind deconvolution consists in recovering the 1D reflectivity sequence $\mathbf{r}$ and the wavelet $\mathbf{h}$ from a 1D observed trace $\mathbf{y}$.
- In the vertical direction, a 1D reflectivity signal appears as a sparse spike train.
- The reflectivity sequence can be modeled as a Bernoulli-Gaussian (BG) process:

$$p(q(k)=1)=\lambda, \quad p(r(k))=\lambda N(0,\sigma_1^2)+(1-\lambda)\delta(r(k))$$

1D reflectivity



1D trace



ML Parameter Estimation

- The parameters $\boldsymbol{\theta} = (\mathbf{h}, \lambda, \sigma_1^2, \sigma_w^2)$ need to be estimated.
- This is carried out using ML estimation:

$$\hat{\boldsymbol{\theta}}_{ML} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} \ln p(\mathbf{y} \mid \boldsymbol{\theta})$$

- This is an incomplete data problem:

$$\hat{\boldsymbol{\theta}}_{ML} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} \ln p(\mathbf{r}, \mathbf{q}, \mathbf{y} \mid \boldsymbol{\theta})$$

- This maximization problem is solved using the stochastic expectation maximization (SEM) algorithm.

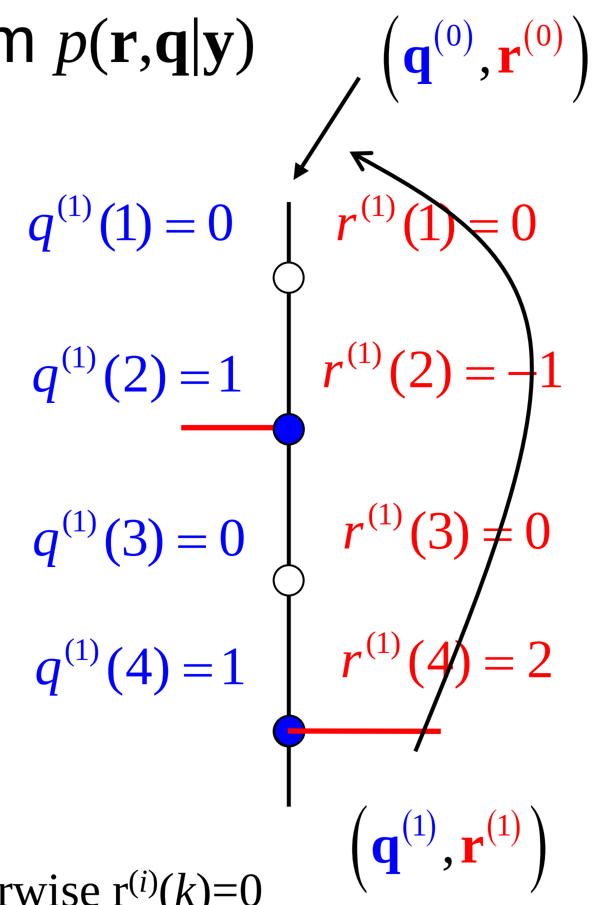
The Gibbs Sampler

- Suppose we wish to sample a random vector $\mathbf{x}=(x_1,\ldots,x_n)$ according to $f(\mathbf{x})$
- Gibbs Sample algorithm:
  A. For a given $\mathbf{x}_t$, generate $\mathbf{y}=(y_1,\ldots,y_n)$ as follows:
     1) Draw y_1 from the conditional pdf $f(x_1|x_{t,2},\ldots, x_{t,n})$.
     2) Draw y_i from the conditional pdf $f(x_i|y_1,\ldots,y_{i-1},x_{t,i+1},\ldots, x_{t,n})$.
     3) Draw y_n from the conditional pdf $f(x_n|y_1,\ldots,y_{t,n-1})$.
  B. Let $\mathbf{x}_{t+1}=\mathbf{y}$.
- Under mild conditions, the limiting dist. of the process $\{\mathbf{x}_t, t=1,2,\ldots\}$ is precisely $f(\mathbf{x})$

The Gibbs Sampler (cont.)

- Simulates observations of $\mathbf{q}$ and $\mathbf{r}$ from $p(\mathbf{r},\mathbf{q}|\mathbf{y})$
- The algorithm samples from:
  - $p(r(k),q(k)\,|\,\mathbf{y},\mathbf{q}_{-k},\mathbf{r}_{-k}) \sim Bi(\lambda_k)\cdot N(m_1,V_1)$
- Gibbs Sampler algorithm:

1. Initialization: choice of $\mathbf{q}^{(0)}$ and $\mathbf{r}^{(0)}$.
2. For $i \leq I$ and for $k=1,\ldots N_r$
   - Detection step:
     - compute $\lambda_k = p(q(k)=1|\boldsymbol{y},\boldsymbol{q}_{-k},\boldsymbol{r}_{-k})$
     - simulate $q^{(i)}(k) \sim Bi(\lambda_k)$
   - Estimation step:
     - simulate $\mathrm{r}^{(i)}(k) \sim N(m_1,V_1)$ if $q^{(i)}(k)=1$, otherwise $\mathrm{r}^{(i)}(k)=0$



The SEM algorithm

The SEM algorithm follows the steps:

1. Initialization: choice of $\mathbf{r}^{(0)},\mathbf{q}^{(0)},\boldsymbol{\theta}^{(0)}$
2. For $i=1,\ldots,I$:
   - E step: simulation of $\mathbf{r}^{(i)},\mathbf{q}^{(i)}$ by the Gibbs sampler according to $p(\mathbf{r},\mathbf{q}|\mathbf{y},\boldsymbol{\theta}^{(i\text{-}1)})$
   - M step: parameters estimation:

     $$\hat{\boldsymbol{\theta}}^{(i)} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\, p(\mathbf{r}^{(i)},\mathbf{q}^{(i)},\mathbf{y}\,|\,\boldsymbol{\theta})$$
3. $$\hat{\boldsymbol{\theta}} = \frac{1}{I-I_0}\sum_{i=I_0+1}^{I}\boldsymbol{\theta}^{(i)}$$

The Deconvolution Process

- Map estimation: $(\hat{\mathbf{r}}, \hat{\mathbf{q}}) = \underset{\mathbf{r},\mathbf{q}}{\operatorname{argmax}}\ p(\mathbf{r}, \mathbf{q} \mid \mathbf{y})$
- This maximization problem can be solved in two steps:
  - Detection: $\hat{\mathbf{q}} = \underset{\mathbf{q}}{\operatorname{argmax}}\ p(\mathbf{q} \mid \mathbf{y})$
  - Estimation: $\hat{\mathbf{r}} = \underset{\mathbf{r}}{\operatorname{argmax}}\ p(\mathbf{r} \mid \mathbf{y}, \hat{\mathbf{q}})$
- The detection problem is hard because $\mathbf{q}$ has 2^{N_r} discrete configurations.
- A simpler criterion called maximum posterior mode (MPM) which maximizes $p(r(k),q(k)|\mathbf{y})$ is used instead.

The MPM algorithm

The MPM algorithm follows the steps:

1. For i=1,…,I simulate ($\mathbf{r}^{(i)}$, $\mathbf{q}^{(i)}$) using the Gibbs sampler
2. For i=1,…,N_r
   - detection step:

$$\hat{q}(k) = \begin{cases} 1 & \text{if } \dfrac{1}{I - I_0} \sum_{i=I_0+1}^{I} q^{(i)}(k) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

   - estimation step:

$$\hat{r}(k) = \begin{cases} \dfrac{\sum_{i=I_0+1}^{I} q^{(i)}(k) r^{(i)}(k)}{\sum_{i=I_0+1}^{I} q^{(i)}(k)}, & \text{if } \hat{q}(k) = 1 \\ 0, & \text{otherwise} \end{cases}$$

The MPM algorithm (cont.)

$\mathbf{q}^{(1)}$	$\mathbf{q}^{(2)}$	$\mathbf{q}^{(3)}$	$\mathbf{q}^{(4)}$	$\mathbf{q}^{(5)}$
0	1	0	0	0
1	0	1	1	1
0	0	0	0	0
1	1	1	0	1
0	0	0	0	0

$\Rightarrow$

$\hat{\mathbf{q}}$
0
1
0
1
0

$\mathbf{r}^{(1)}$	$\mathbf{r}^{(2)}$	$\mathbf{r}^{(3)}$	$\mathbf{r}^{(4)}$	$\mathbf{r}^{(5)}$
0	4	0	0	0
4	0	4.5	4	3.5
0	0	0	0	0
-2	-2.5	-2	0	-1.5
0	0	0	0	0

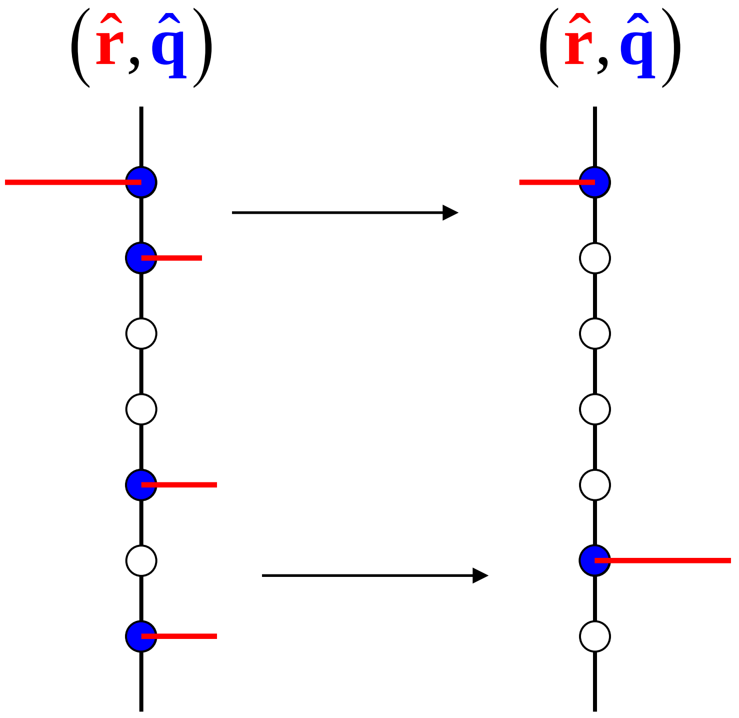
$\Rightarrow$

$\hat{\mathbf{r}}$
0
4
0
-2
0

Reflectivity Post Processing

Fuse and replace by their gravity center:

1. 2 successive impulses
2. 2 impulses separated by one sample



Advantages And Limitations

- Advantages:
  1. Estimates the wavelet and BG model's missing parameters.
  2. Produces good estimates for single channel traces.
  3. Uses stochastic methods which are less likely to converge to local maximum points.
- Limitations
  1. Does not account for the medium's stratified structure in the deconvolution process.

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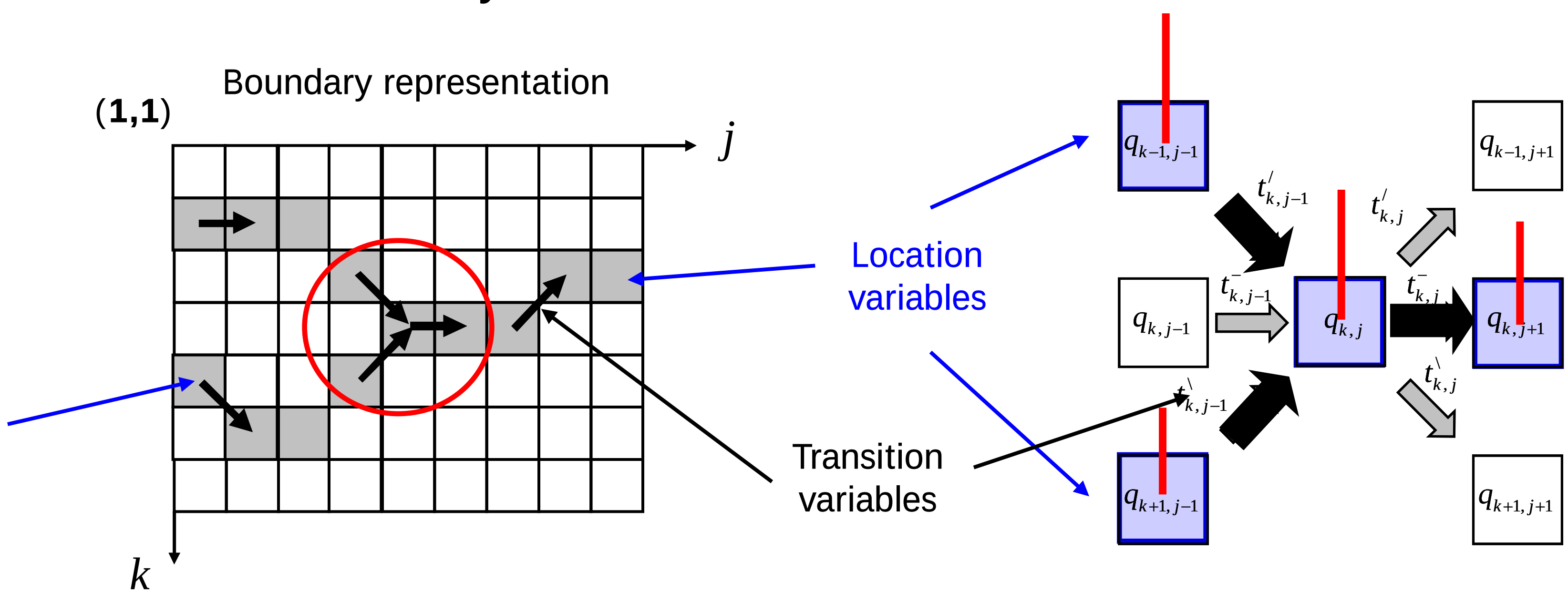
MBG-I Model

Exact 2D extension of the BG representation. Comprised of:

1. MBRF $p(\mathbf{T},\mathbf{Q})$ – models the geometric properties of the reflectivity using location and transition variables.
2. $p(\mathbf{R}\,|\,\mathbf{T},\mathbf{Q})$ - white Gaussian reflectivity amplitude model, defined conditionally to the MBRF.



Markov Bernoulli Random Field Characteristics

- MBRF characteristics:
  1. $p(t^{/}_{k,j}, t^{-}_{k,j}, t^{\backslash}_{k,j}) = p(t^{/}_{k,j})\, p(t^{-}_{k,j})\, p(t^{\backslash}_{k,j})$
  2. $q_{k,j} \sim Bi(\lambda),\ t^{/}_{k,j} \sim Bi(\mu^{/}),\ t^{-}_{k,j} \sim Bi(\mu^{-}),\ t^{\backslash}_{k,j} \sim Bi(\mu^{\backslash})$
  3. $\lambda = 1-(1-\mu^{/})(1-\mu^{-})(1-\mu^{\backslash})(1-\varepsilon)$
  4. $p(t^{/}_{k,j} = 0, t^{-}_{k,j} = 0, t^{\backslash}_{k,j} = 0 \mid q_{k,j} = 0) = 1$
  5. $p(q_{k,j} = 1 \mid t^{/}_{k-1,j} = 0, t^{-}_{k,j} = 0, t^{\backslash}_{k+1,j} = 0) = \varepsilon$
- Amplitude field characteristics:
  1. $r_{k,j} = 0$
  2. $r_{k,j} \sim N(0, \sigma_1^2)$
  3. $r_{k,j} = a r_{k+d,j-1} + w_r,\ w_r \sim N(0,(1-a^2)\sigma_1^2)$

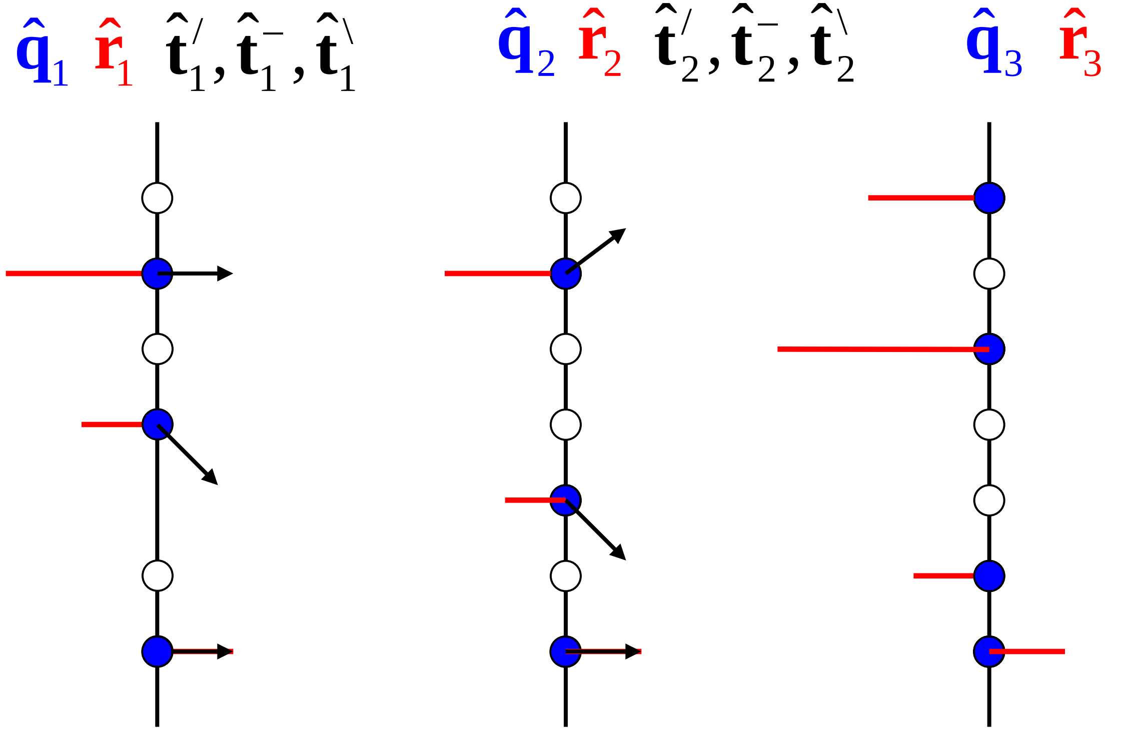
Deconvolution Scheme

- MAP estimation: $(\hat{\mathbf{R}},\hat{\mathbf{Q}},\hat{\mathbf{T}}^{/},\hat{\mathbf{T}}^{-},\hat{\mathbf{T}}^{\backslash}) = \underset{\mathbf{R},\mathbf{Q},\mathbf{T}^{/},\mathbf{T}^{-},\mathbf{T}^{\backslash}}{\operatorname{argmax}}\ p(\mathbf{R},\mathbf{Q},\mathbf{T}^{/},\mathbf{T}^{-},\mathbf{T}^{\backslash} \mid \mathbf{Y})$
- Computing the exact MAP solution is practically impossible.
- The following suboptimal recursive maximization procedure is used instead:
  1. First column: $(\hat{\mathbf{r}}_1,\hat{\mathbf{q}}_1) = \underset{\mathbf{r}_1,\mathbf{q}_1}{\operatorname{argmax}}\ p(\mathbf{r}_1,\mathbf{q}_1,\mathbf{y}_1)$
  2. $2 \le j \le J$: $(\hat{\mathbf{r}}_j,\hat{\mathbf{q}}_j,\hat{\mathbf{t}}^{/}_{j-1},\hat{\mathbf{t}}^{-}_{j-1},\hat{\mathbf{t}}^{\backslash}_{j-1})$

$$= \underset{\mathbf{r}_j,\mathbf{q}_j,\mathbf{t}^{/}_{j-1},\mathbf{t}^{-}_{j-1},\mathbf{t}^{\backslash}_{j-1}}{\operatorname{argmax}}\ p(\mathbf{r}_j,\mathbf{q}_j,\mathbf{t}^{/}_{j-1},\mathbf{t}^{-}_{j-1},\mathbf{t}^{\backslash}_{j-1},\mathbf{y}_j \mid \hat{\mathbf{r}}_{j-1},\hat{\mathbf{q}}_{j-1})$$

- Each partial criterion is maximized using a suboptimal SMLR type algorithm.

Deconvolution Scheme (cont.)



Advantages And Limitations

- Advantages:
  1. Produces good estimates of the 2D reflectivity
- Limitations:
  1. Non blind.
  2. Each partial criterion is maximized only with respect to $\mathbf{r}_j, \mathbf{q}_j, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1}$.
  3. $\mathbf{r}_j$ is determined based on observations only up to $\mathbf{y}_j$.
  4. The SMLR-type algorithm may converge to a local optimum.
  5. The first reflectivity column is assumed to be known.

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Blind Multichannel MCMC Deconvolution

- Uses the MBG-I reflectivity model.

- Uses the following suboptimal recursive maximization procedure :

  1. First column: $(\hat{\mathbf{r}}_1, \hat{\mathbf{q}}_1) = \operatorname{argmax} p(\mathbf{r}_1, \mathbf{q}_1 \mid \mathbf{y}_1)$

  2. $2 \le j \le J$: $(\hat{\mathbf{r}}_j, \hat{\mathbf{q}}_j, \hat{\mathbf{t}}^{/}_{j-1}, \hat{\mathbf{t}}^{-}_{j-1}, \hat{\mathbf{t}}^{\backslash}_{j-1})$

$$= \underset{\mathbf{r}_j, \mathbf{q}_j, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1}}{\operatorname{argmax}} p(\mathbf{r}_j, \mathbf{q}_j, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1} \mid \mathbf{y}_j, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1})$$

- The SMLR type algorithm is replaced by an extended version of the MPM algorithm which maximizes:

$$p(t^{/}_{k,j-1}, t^{-}_{k,j-1}, t^{\backslash}_{k,j-1}, q_{k,j}, r_{k,j} \mid \mathbf{y}_j, \hat{\mathbf{q}}_{j-1}, \hat{\mathbf{r}}_{j-1})$$

MBG I Parameters Estimation

- The parameters $\boldsymbol{\theta} = (\mathbf{h}, \lambda, \sigma_1^2, \sigma_w^2)$ are estimated using the SEM algorithm.
- The following method is used to estimate $\boldsymbol{\theta}_{MBG} = (a, \mu^{/}, \mu^{-}, \mu^{\backslash}, \varepsilon)$:
  1. apply single channel deconvolution to each of $\mathbf{Y}$'s columns.
  2. remove all the isolated reflectors from the obtained reflectivity section.
  3. calculate:
     - $\mu^{/}$ = #boundary upward transitions/# samples in $\mathbf{T}^{/}$
     - μ^{-} = #boundary upward transitions/# samples in $\mathbf{T}^{-}$
     - $\mu^{\backslash}$ = #boundary upward transitions/# samples in $\mathbf{T}^{\backslash}$
     - a = average attenuation ratio between neighboring reflectors

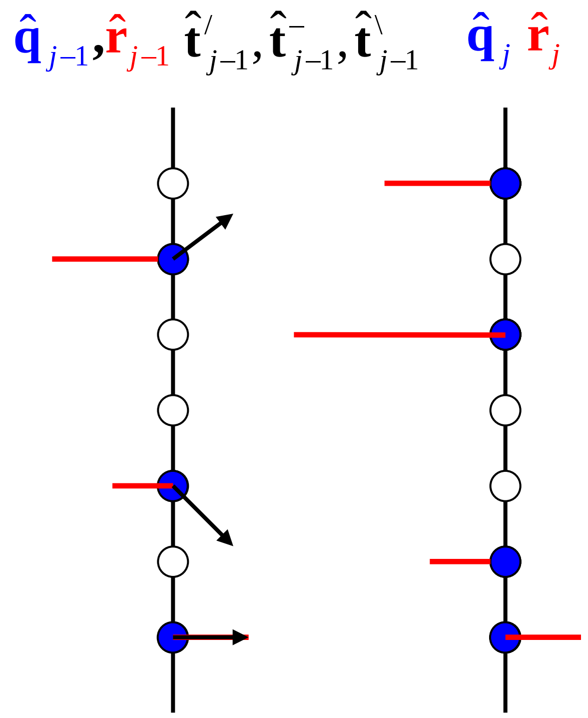
Multichannel Deconvolution's Gibbs Sampler

- Used by the extended MPM algorithm.
- Simulates observations of $\mathbf{r}_j, \mathbf{q}_j, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1}$ from $p(\mathbf{r}_j, \mathbf{q}_j, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1} \mid \mathbf{y}_j, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1})$.
- The algorithm samples from:
  - $p(r_{k,j}, q_{k,j} \mid \mathbf{y}_j, \mathbf{r}_{-k,j}, \mathbf{q}_{-k,j}, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1}, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1}) \sim Bi(\lambda^{b}_{k,j}) \cdot N\left(m_b, V_b\right)$
  - $p(t^{/}_{k,j-1} \mid \mathbf{t}^{/}_{-k,j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1}, \mathbf{q}_j, \mathbf{r}_j, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1}, \mathbf{y}_j) \sim Bi\left(\mu^{/}\right)$
  - $p(t^{-}_{k,j-1} \mid \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{-k,j-1}, \mathbf{t}^{\backslash}_{j-1}, \mathbf{q}_j, \mathbf{r}_j, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1}, \mathbf{y}_j) \sim Bi\left(\mu^{-}\right)$
  - $p(t^{/}_{k,j-1} \mid \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{-k,j-1}, \mathbf{q}_j, \mathbf{r}_j, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1}, \mathbf{y}_j) \sim Bi\left(\mu^{\backslash}\right)$

Multichannel Deconvolution's Gibbs Sampler's Algorithm

1. Initialization: choice of $\mathbf{r}_j^{(0)}, \mathbf{q}_j^{(0)}, \mathbf{t}_{j-1}^{/\,(0)}, \mathbf{t}_{j-1}^{-\,(0)}, \mathbf{t}_{j-1}^{\backslash\,(0)}$
2. For $i \leq I$ and for $k=1,\ldots N_r$
   - Detection step:
     - compute $\lambda_{k,j}^{b}, \mu^{/}, \mu^{-}, \mu^{\backslash}$
     - simulate $t_{k,j}^{/\,(i)} \sim Bi(\mu^{/}), t_{k,j}^{-\,(i)} \sim Bi(\mu^{-}),$ $t_{k,j}^{\backslash\,(i)} \sim Bi(\mu^{\backslash}), q_{k,j}^{(i)} \sim Bi(\lambda_{k,j}^{b})$
   - Estimation step:
     - simulation of $r_{k,j}^{(i)} \sim N(m_b, V_b)$ if $q_{k,j}^{(i)} = 1$ and $r_{k,j}^{(i)} = 0$ if $q_{k,j}^{(i)} = 0$

transition variables



Multichannel Deconvolution's MPM Algorithm

The extended MPM algorithm follows the steps:

1. For i=1,…,I simulate $\mathbf{r}_j^{(i)}, \mathbf{q}_j^{(i)}, \mathbf{t}_{j-1}^{/\,(i)}, \mathbf{t}_{j-1}^{-\,(i)}, \mathbf{t}_{j-1}^{\backslash\,(i)}$ using the Gibbs sampler
2. For i=1,…,N_r
   - detection step:

$$\hat{t}_{k,j-1}^{/} = \begin{cases} 1 & \text{if } \dfrac{1}{I-I_0}\displaystyle\sum_{i=I_0+1}^{I} t_{k,j-1}^{/\,(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}, \quad \hat{t}_{k,j-1}^{-} = \begin{cases} 1 & \text{if } \dfrac{1}{I-I_0}\displaystyle\sum_{i=I_0+1}^{I} t_{k,j-1}^{-\,(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{t}_{k,j-1}^{\backslash} = \begin{cases} 1 & \text{if } \dfrac{1}{I-I_0}\displaystyle\sum_{i=I_0+1}^{I} t_{k,j-1}^{\backslash\,(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}, \quad \hat{q}_{k,j} = \begin{cases} 1 & \text{if } \dfrac{1}{I-I_0}\displaystyle\sum_{i=I_0+1}^{I} q_{k,j}^{(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Multichannel Deconvolution's MPM Algorithm (cont.)

- estimation step:

$$\hat{r}_{k,j} = \begin{cases} \dfrac{\sum_{i=I_0+1}^{I} q_{k,j}^{(i)} r_{k,j}^{(i)}}{\sum_{i=I_0+1}^{I} q_{k,j}^{(i)}}, & \text{if } \hat{q}_{k,j} = 1 \\ 0, & \text{otherwise} \end{cases}$$

$\mathbf{t}_{j-1}^{/(1)}$	$\mathbf{t}_{j-1}^{/(2)}$	$\mathbf{t}_{j-1}^{/(3)}$	$\mathbf{t}_{j-1}^{/(4)}$	$\mathbf{t}_{j-1}^{/(5)}$
0	0	0	0	0
1	0	1	1	1
0	0	0	0	0
1	1	1	0	1
0	0	0	0	0

$\Rightarrow$

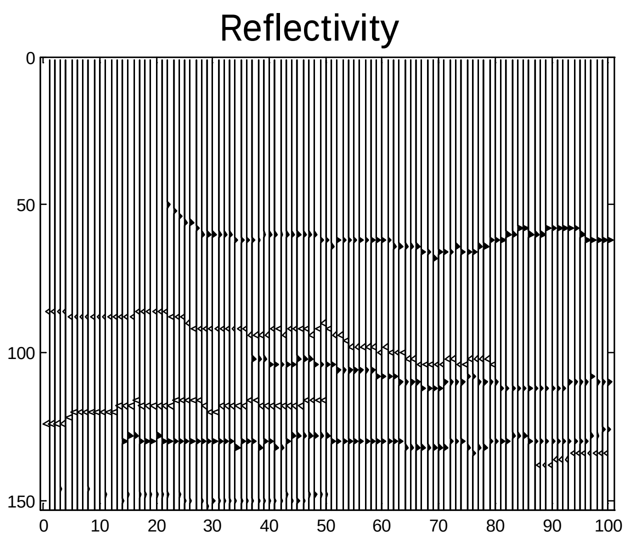
$\hat{\mathbf{t}}_{j-1}^{/}$
0
1
0
1
0

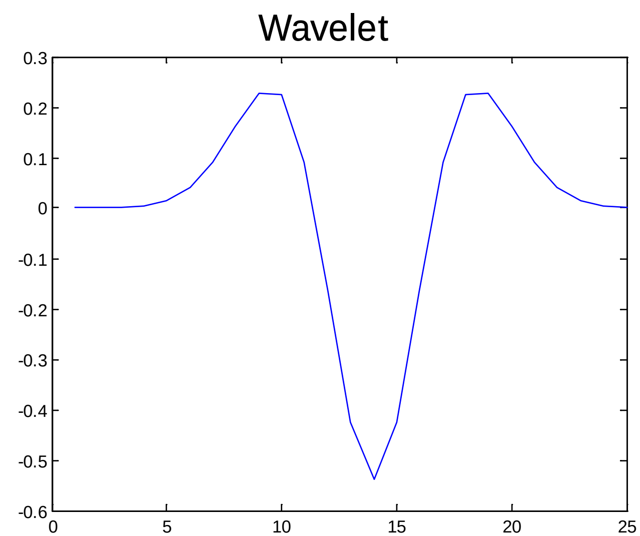
$\mathbf{r}_j^{(1)}$	$\mathbf{r}_j^{(2)}$	$\mathbf{r}_j^{(3)}$	$\mathbf{r}_j^{(4)}$	$\mathbf{r}_j^{(5)}$
0	0	0	0	0
4	0	4.5	4	3.5
0	0	0	0	0
-2	-2.5	-2	0	-1.5
0	0	0	0	0

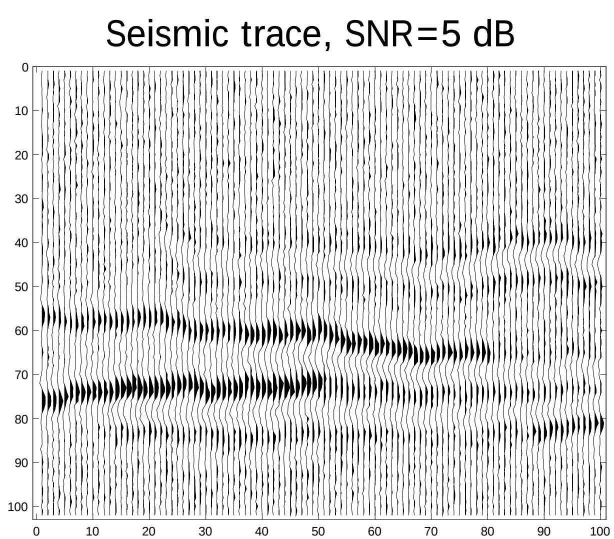
$\Rightarrow$

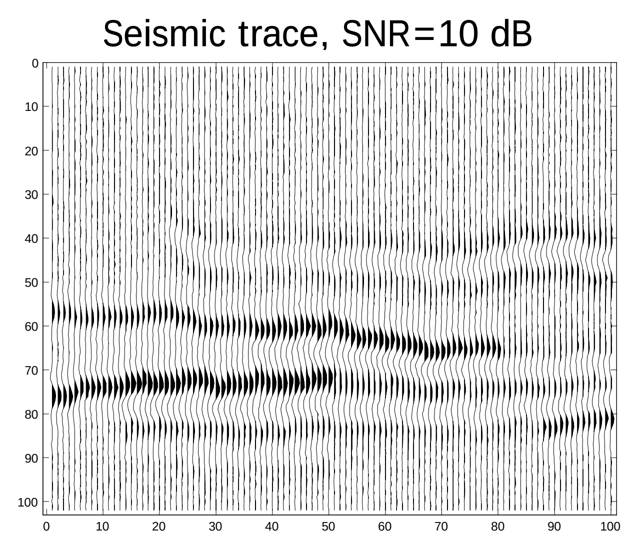
$\hat{\mathbf{r}}_j$
0
4
0
-2
0

Synthetic Data Results

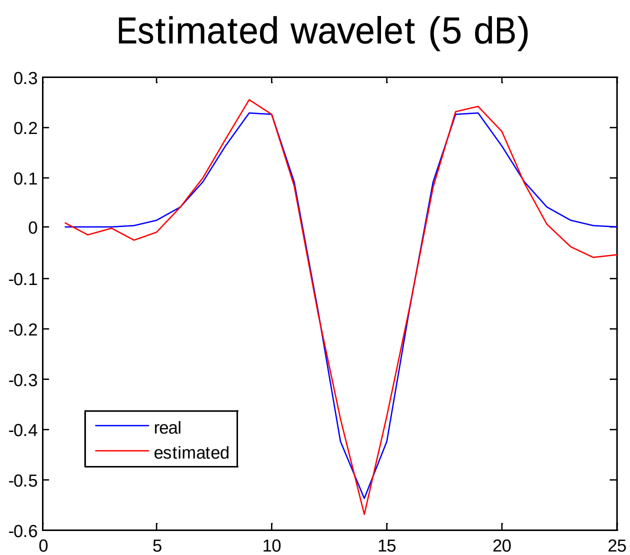


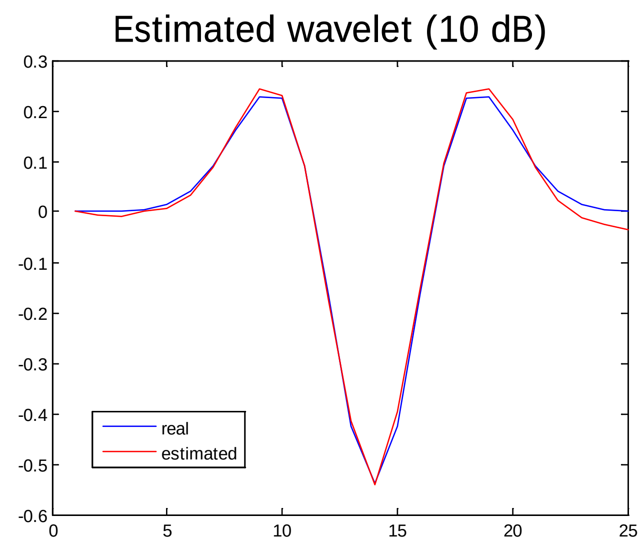






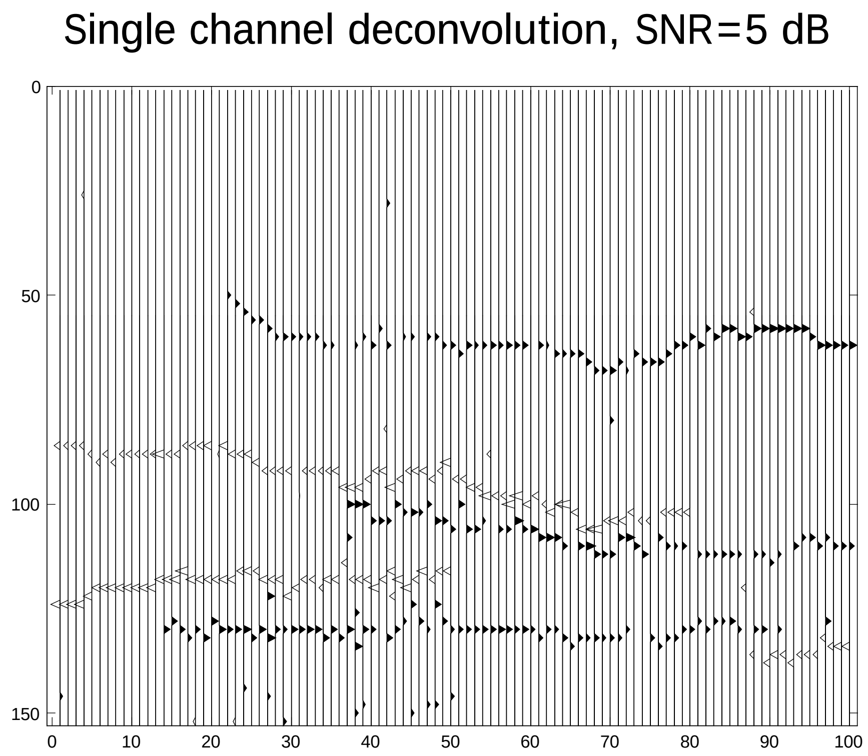
Synthetic Results – Estimated Parameters

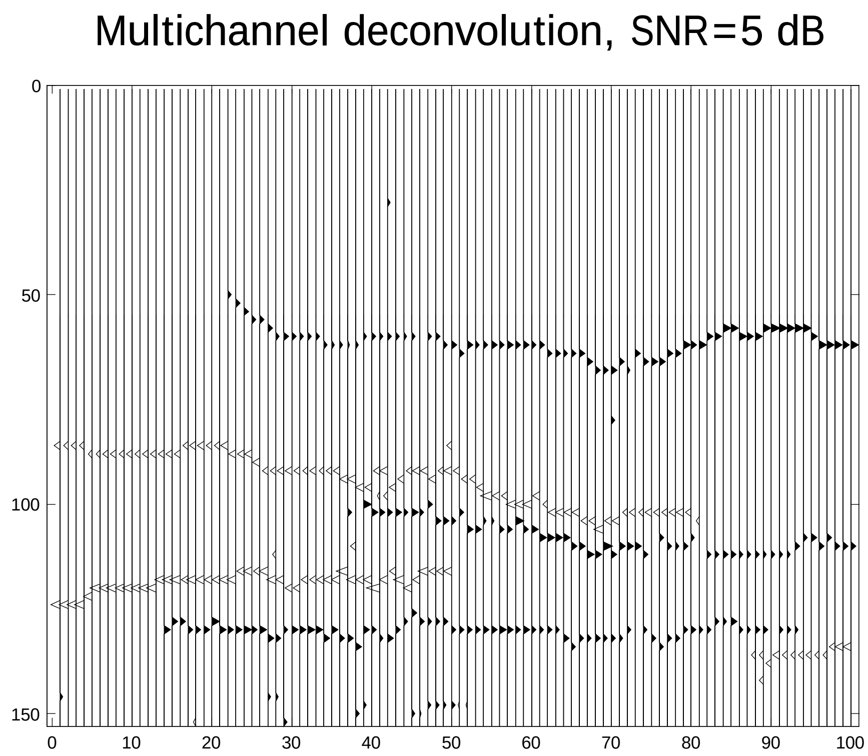




	λ	σ_1	σ_w	a	$\mu^{/}$	μ^{-}	$\mu^{\backslash}$	ε
True	0.0551	1	0.132	0.999	0.0066	0.0399	0.0081	0.0012
Estimated (5 dB)	0.0533	1.2230	0.1094	0.8133	0.0080	0.0222	0.0096	0.0146
Estimatd (10 dB)	0.0541	1.0925	0.0657	0.8177	0.0082	0.0308	0.0090	0.0069

Synthetic Data Results, SNR=5 dB

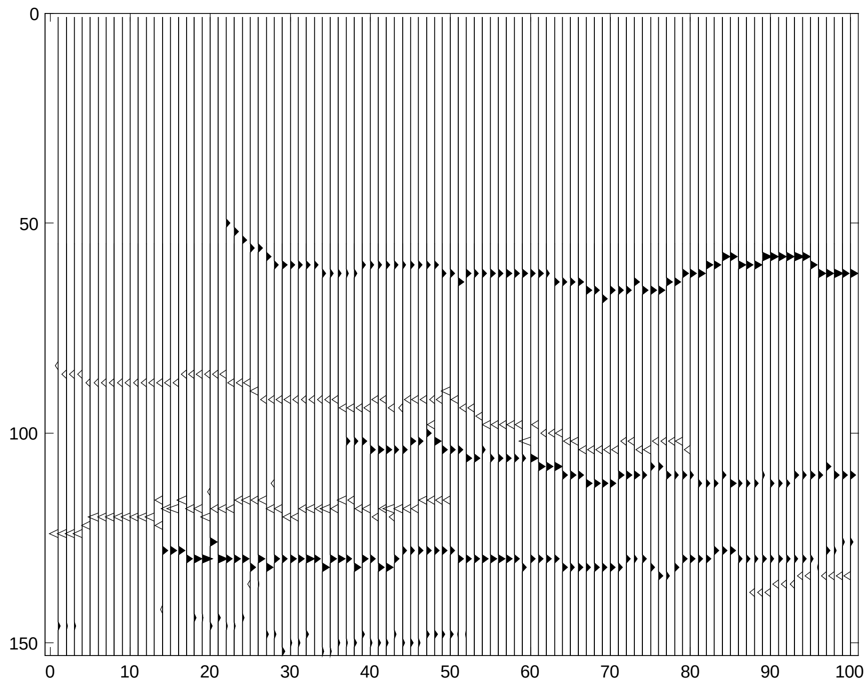




Synthetic Data Results, SNR=10 dB

Single channel deconvolution, SNR=10 dB

Multichannel deconvolution, SNR=10 dB



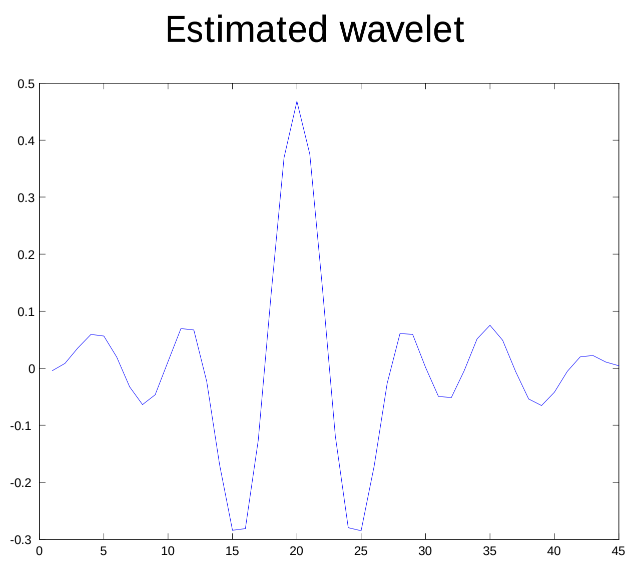
Synthetic Results – Performance Quality Measures

The following loss functions were used to quantify the performance of the algorithms:

1. $L^{miss+false} = \left\|\hat{\mathbf{r}} - \mathbf{r}\right\|_1 + N^{miss} + N^{false}$
2. $L^{miss} = \left\|\hat{\mathbf{r}} - \mathbf{r}\right\|_1 + N^{miss}$
3. $L^{false} = \left\|\hat{\mathbf{r}} - \mathbf{r}\right\|_1 + N^{false}$
4. $L^{SSQ} = \left\|\hat{\mathbf{r}} - \mathbf{r}\right\|_2^2$
5. $L_2^{miss+false}, L_2^{miss}, L_2^{false}$ give partial credit to reflectors that are close to their true positions.

SNR (5dB)		$L^{miss+flase}$	L^{miss}	L^{false}	L^{SSQ}	$L_2^{miss+flase}$	L_2^{miss}	L_2^{false}
5	SC	512.3	414.3	365.3	276.1	293.7	235.2	186.2
	MC	372.6	300.6	265.6	172.9	216.1	173.6	138.6
10	SC	243.9	192.9	175.9	116.8	140.5	109.5	92.5
	MC	196.3	149.3	145.3	75.4	123.6	92.6	89.6

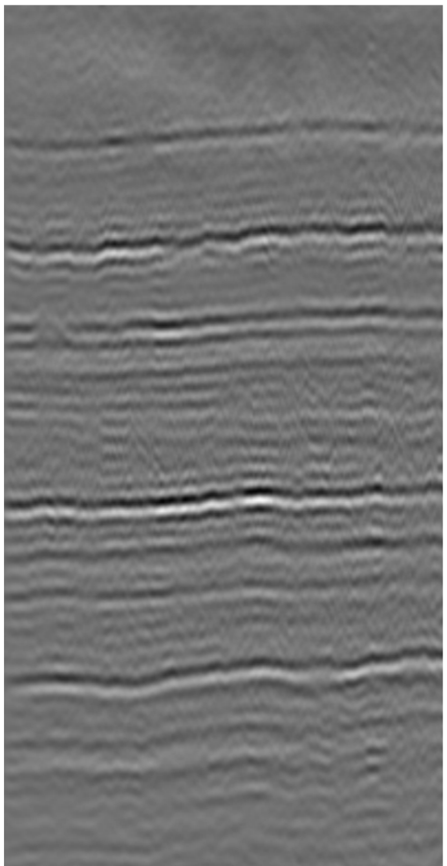
Real Data – Estimated Parameters



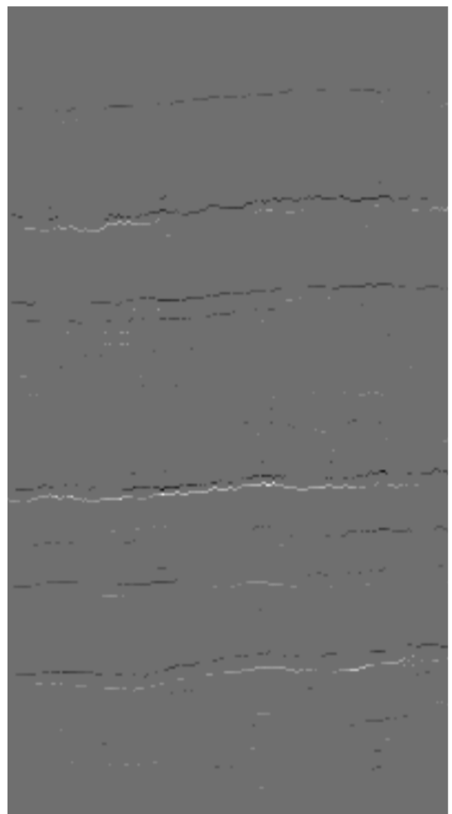
λ	σ_1	σ_w	a	$\mu^{/}$	μ^{-}	$\mu^{\backslash}$	ε
0.0382	3.9488	0.7024	0.8989	0.0015	0.0112	0.0008	0.0252

Real Data Results

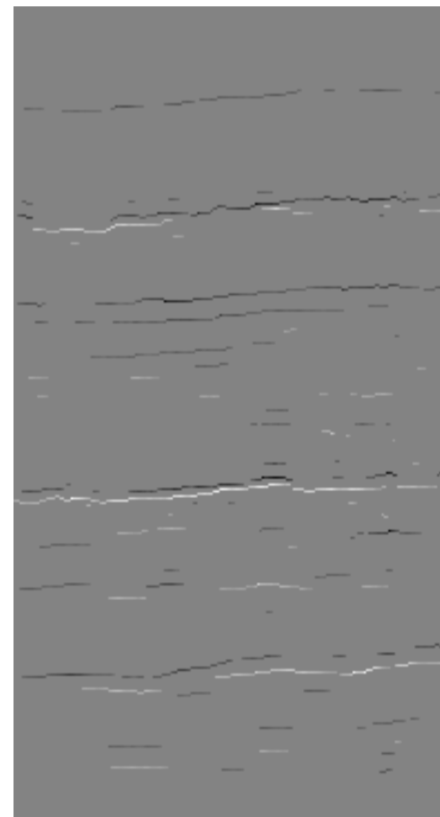
Real data



Single channel deconvolution



Multichannel deconvolution



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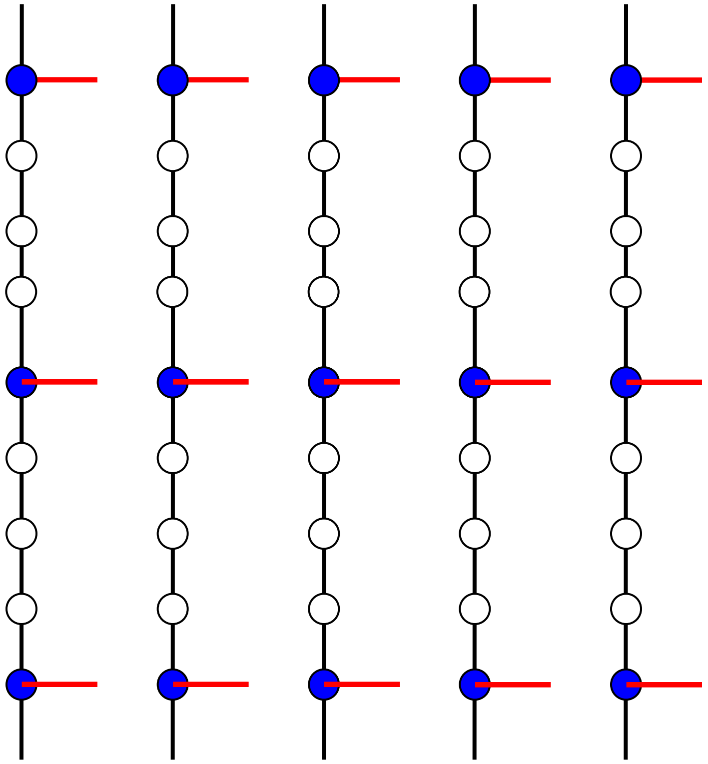
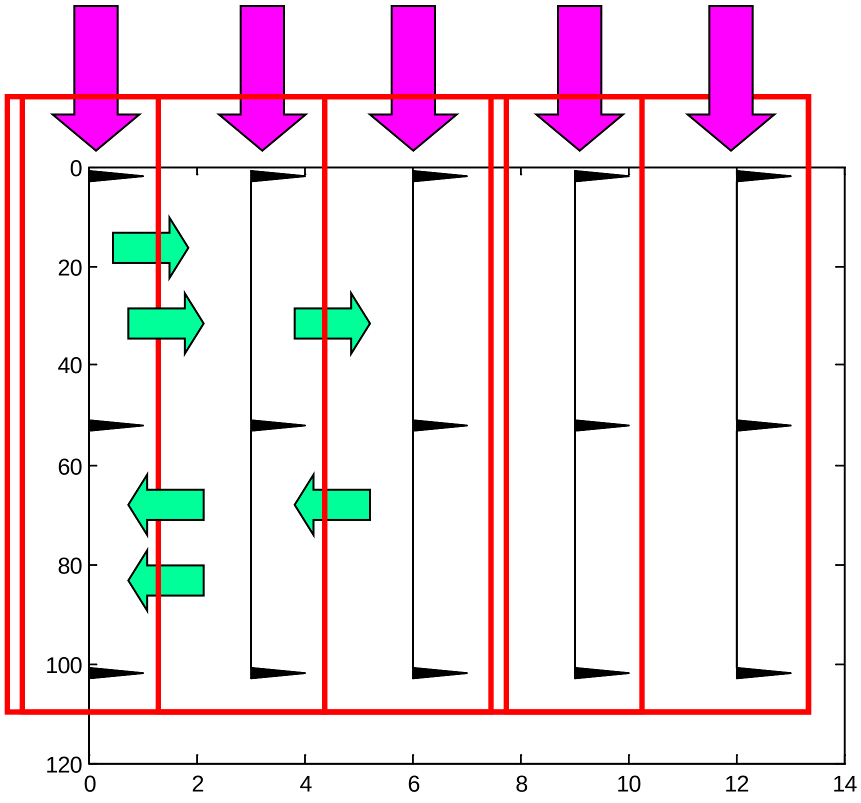
Deconvolution By Smoothing

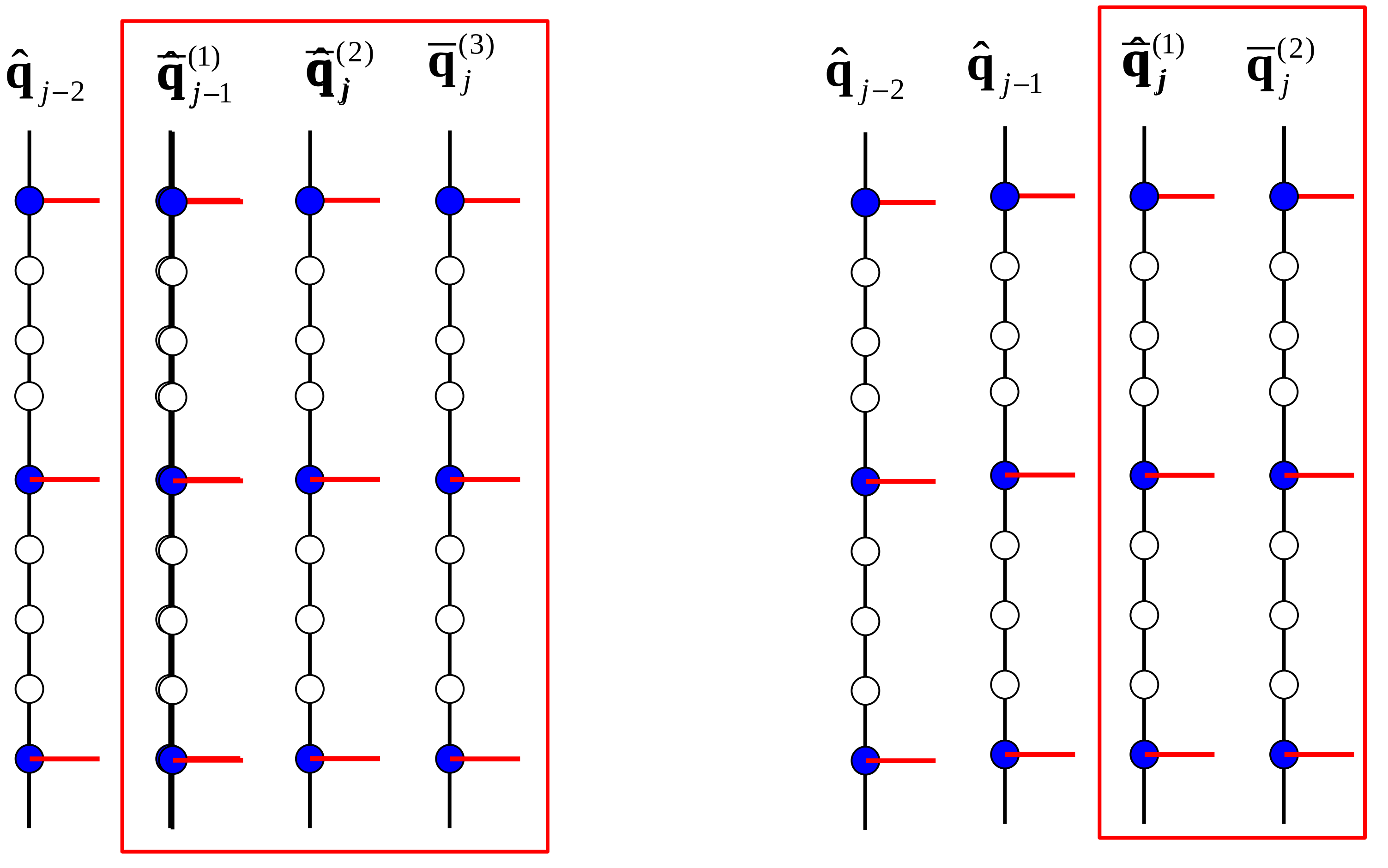
- Uses the MBG-I reflectivity prior model.
- Uses the same parameter estimation method as the first proposed algorithm.
- Uses a column recursive deconvolution scheme, similar to the one of the previous algorithm.
- Accounts for observation columns subsequent to $\mathbf{y}_j$ in $\mathbf{r}_j$'s estimation process.

Smoothing Scheme





Smoothing Windows



Smoothing Scheme (cont.)

q_j	q_{j+1}
1	1
0	0
1	1
0	0
1	1

r_j	r_{j+1}
1	1
0	0
1	1
0	0
1	1

t_{j-1}^{-}	t_j^{-}
1	1
0	0
1	1
0	0
1	1

	$\bar{t}_{j-1}^{-}$		$\bar{q}_j$		$\bar{r}_j$
t_{j-1}^{-}	1	q_j	1	r_j	1
	0		0		0
	1		1		1
	0		0		0
	1		1		1
t_j^{-}	1	q_{j+1}	1	r_{j+1}	1
	0		0		0
	1		1		1
	0		0		0
	1		1		1

Deconvolution Scheme

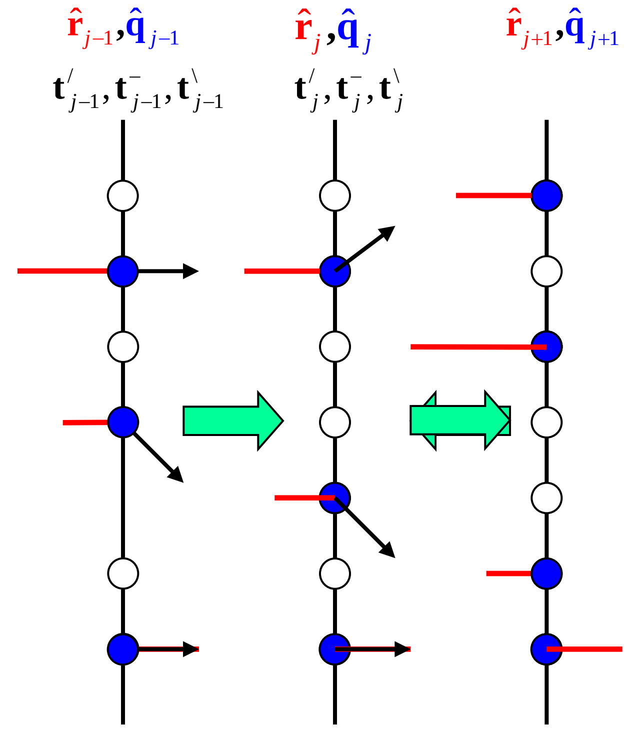
- Uses the following estimation procedure:
  - First column: $(\hat{\bar{\mathbf{r}}}_1, \hat{\bar{\mathbf{q}}}_1) = \underset{\bar{\mathbf{r}}_1, \bar{\mathbf{q}}_1, \bar{\mathbf{t}}_0^{/}, \bar{\mathbf{t}}_0^{-}, \bar{\mathbf{t}}_0^{\backslash}}{\operatorname{argmax}}\ p(\bar{\mathbf{r}}_1, \bar{\mathbf{q}}_1, \bar{\mathbf{t}}_0^{/}, \bar{\mathbf{t}}_0^{-}, \bar{\mathbf{t}}_0^{\backslash} \mid \bar{\mathbf{y}}_1)$
  - Middle column: $(\hat{\bar{\mathbf{r}}}_j, \hat{\bar{\mathbf{q}}}_j, \hat{\bar{\mathbf{t}}}_{j\text{-}1}^{/}, \hat{\bar{\mathbf{t}}}_{j\text{-}1}^{-}, \hat{\bar{\mathbf{t}}}_{j\text{-}1}^{\backslash})$
    $= \underset{\bar{\mathbf{r}}_j, \bar{\mathbf{q}}_j, \bar{\mathbf{t}}_{j\text{-}1}^{/}, \bar{\mathbf{t}}_{j\text{-}1}^{-}, \bar{\mathbf{t}}_{j\text{-}1}^{\backslash}}{\operatorname{argmax}}\ p(\bar{\mathbf{r}}_j, \bar{\mathbf{q}}_j, \bar{\mathbf{t}}_{j\text{-}1}^{/}, \bar{\mathbf{t}}_{j\text{-}1}^{-}, \bar{\mathbf{t}}_{j\text{-}1}^{\backslash} \mid \bar{\mathbf{y}}_j, \hat{\mathbf{q}}_{j-1}, \hat{\mathbf{r}}_{j-1})$
  - Last column: $(\hat{\mathbf{r}}_J, \hat{\mathbf{q}}_J, \hat{\mathbf{t}}_{J-1}^{/}, \hat{\mathbf{t}}_{J-1}^{-}, \hat{\mathbf{t}}_{J-1}^{\backslash})$
    $= \underset{\mathbf{r}_J, \mathbf{q}_J, \mathbf{t}_{J-1}^{/}, \mathbf{t}_{J-1}^{-}, \mathbf{t}_{J-1}^{\backslash}}{\operatorname{argmax}}\ p(\mathbf{r}_J, \mathbf{q}_J, \mathbf{t}_{J-1}^{/}, \mathbf{t}_{J-1}^{-}, \mathbf{t}_{J-1}^{\backslash} \mid \mathbf{y}_J, \hat{\mathbf{q}}_{J-1}, \hat{\mathbf{r}}_{J-1})$
- Uses a further extended version of the MPM algorithm which maximizes: $p(\bar{r}_{k,j}, \bar{q}_{k,j}, \bar{t}_{k,j-1}^{/}, \bar{t}_{k,j-1}^{-}, \bar{t}_{k,j-1}^{\backslash} \mid \bar{\mathbf{y}}_j, \hat{\mathbf{q}}_{j-1}, \hat{\mathbf{r}}_{j-1})$

Smoothing's Gibbs Sampler

- Used by the smoothing's MPM algorithm.
- Simulates observations of $\overline{\mathbf{r}}_j, \overline{\mathbf{q}}_j, \overline{\mathbf{t}}^{/}_{j-1}, \overline{\mathbf{t}}^{-}_{j-1}, \overline{\mathbf{t}}^{\backslash}_{j-1}$ from $p(\overline{\mathbf{r}}_j, \overline{\mathbf{q}}_j, \overline{\mathbf{t}}^{/}_{j-1}, \overline{\mathbf{t}}^{-}_{j-1}, \overline{\mathbf{t}}^{\backslash}_{j-1} \mid \overline{\mathbf{y}}_j, \hat{\mathbf{r}}_{j-1}, \hat{\mathbf{q}}_{j-1})$.
- The algorithm samples from:
  - $p(r_{k,j}, q_{k,j} \mid \mathbf{y}_j, \mathbf{r}_{-k,j}, \mathbf{q}_{-k,j}, \hat{\mathbf{r}}_{j-1}, \mathbf{r}_{j+1}, \hat{\mathbf{q}}_{j-1}, \mathbf{q}_{j+1}, \mathbf{t}^{/}_{j-1}, \mathbf{t}^{-}_{j-1}, \mathbf{t}^{\backslash}_{j-1}, \mathbf{t}^{/}_{j}, \mathbf{t}^{-}_{j}, \mathbf{t}^{\backslash}_{j})$
    $\sim Bi(\lambda^{m}_{k,j}) \cdot N\left(m_m, V_m\right)$
  - The multichannel Gibbs sampler's conditional distributions

Smoothing's Gibbs Sampler Algorithm

1. Initialization: choice of $\overline{\mathbf{r}}_j^{(0)}, \overline{\mathbf{q}}_j^{(0)}, \overline{\mathbf{t}}_{j-1}^{/\,(0)}, \overline{\mathbf{t}}_{j-1}^{-\,(0)}, \overline{\mathbf{t}}_{j-1}^{\backslash\,(0)}$
2. For $i \leq I$
   1. for $k=1,\ldots N_r$
      - detection step:
        - compute $\lambda_{k,j}^m, \mu^/, \mu^-, \mu^\backslash$
        - simulate $\overline{t}_{k,j}^{/\,(i)} \sim Bi(\mu^/), \overline{t}_{k,j}^{-\,(i)} \sim Bi(\mu^-),$
          $\overline{t}_{k,j}^{\backslash\,(i)} \sim Bi(\mu^\backslash), \overline{q}_{k,j}^{(i)} \sim Bi(\lambda_{k,j}^m)$
      - estimation step:
        - simulation of $\overline{r}_{k,j}^{(i)}$ where $\overline{r}_{k,j}^{(i)} \sim N\left(m_b, V_b\right)$
          if $\overline{q}_{k,j}^{(i)} = 1$ and $\overline{r}_{k,j}^{(i)} = 0$ if $\overline{q}_{k,j}^{(i)} = 0$
   2. for $k+1=2N_r$ follow the multichannel Gibbs sampler procedure



Smoothing's MPM algorithm

The smoothing's MPM algorithm follows the steps:

1. For $i=1,\ldots,I$ simulate $\bar{\mathbf{r}}_j^{(i)}, \bar{\mathbf{q}}_j^{(i)}, \bar{\mathbf{t}}_{j-1}^{/\,(i)}, \bar{\mathbf{t}}_{j-1}^{-\,(i)}, \bar{\mathbf{t}}_{j-1}^{\backslash\,(i)}$ using the Gibbs sampler
2. For $i=1,\ldots,N_r$
   - detection step:

$$\hat{\bar{t}}^{/}_{k,j-1} = \begin{cases} 1 & \text{if } \frac{1}{I-I_0}\sum_{i=I_0+1}^{I} \bar{t}_{k,j-1}^{/\,(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}, \quad \hat{\bar{t}}^{-}_{k,j-1} = \begin{cases} 1 & \text{if } \frac{1}{I-I_0}\sum_{i=I_0+1}^{I} \bar{t}_{k,j-1}^{-\,(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{\bar{t}}^{\backslash}_{k,j-1} = \begin{cases} 1 & \text{if } \frac{1}{I-I_0}\sum_{i=I_0+1}^{I} \bar{t}_{k,j-1}^{\backslash\,(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}, \quad \hat{\bar{q}}_{k,j} = \begin{cases} 1 & \text{if } \frac{1}{I-I_0}\sum_{i=I_0+1}^{I} \bar{q}_{k,j}^{(i)} > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Smoothing's MPM algorithm (cont.)

- estimation step:

$$\hat{\bar{r}}_{k,j} = \begin{cases} \dfrac{\sum_{i=I_0+1}^{I} \bar{q}_{k,j}^{(i)} \bar{r}_{k,j}^{(i)}}{\sum_{i=I_0+1}^{I} \bar{q}_{k,j}^{(i)}}, & \text{if } \hat{\bar{q}}_{k,j} = 1 \\ 0, & \text{otherwise} \end{cases}$$

Smoothing's MPM algorithm (cont.)

$\bar{\mathbf{q}}_j^{(1)}$	$\bar{\mathbf{q}}_j^{(2)}$	$\bar{\mathbf{q}}_j^{(3)}$	$\bar{\mathbf{q}}_j^{(4)}$	$\bar{\mathbf{q}}_j^{(5)}$	$\Rightarrow$	$\hat{\mathbf{q}}_j$
0	0	0	0	0		0
1	0	1	1	1		1
0	0	0	0	0		0
1	1	1	0	1		1
0	0	0	0	0		0
0	0	0	0	0		0
1	0	1	1	1		1
0	0	0	0	0		0
1	1	1	0	1		1
0	0	0	0	0		0

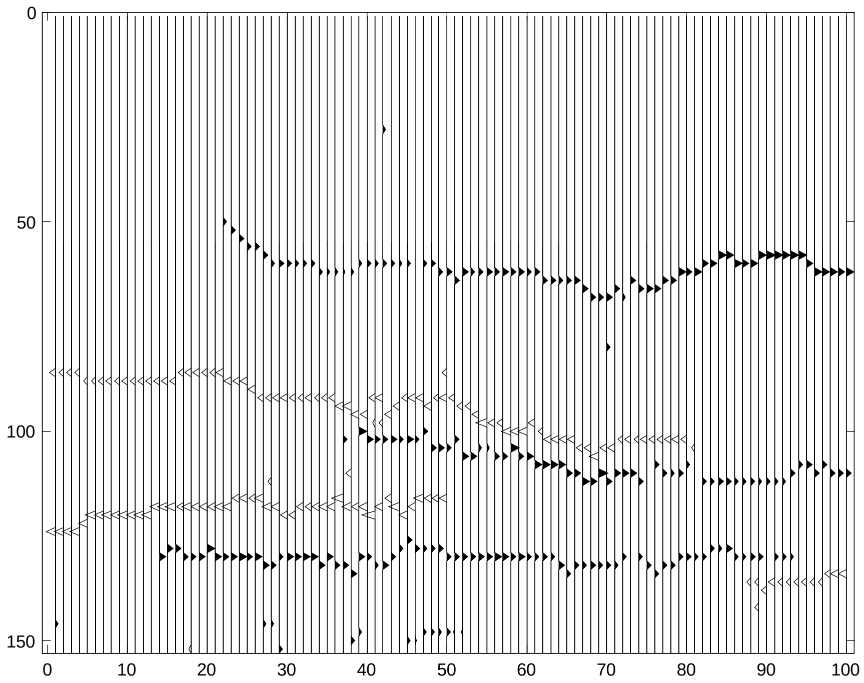
(A brace around the first five entries of this result column is labelled $\hat{\mathbf{q}}_j$.)

$\bar{\mathbf{r}}_j^{(1)}$	$\bar{\mathbf{r}}_j^{(2)}$	$\bar{\mathbf{r}}_j^{(3)}$	$\bar{\mathbf{r}}_j^{(4)}$	$\bar{\mathbf{r}}_j^{(5)}$	$\Rightarrow$	$\hat{\mathbf{r}}_j$
0	0	0	0	0		0
4	0	4.5	4	3.5		4
0	0	0	0	0		0
-2	-2.5	-2	0	-1.5		-2
0	0	0	0	0		0
0	0	0	0	0		0
4	0	4.5	4	3.5		4
0	0	0	0	0		0
-2	-2.5	-2	0	-1.5		-2
0	0	0	0	0		0

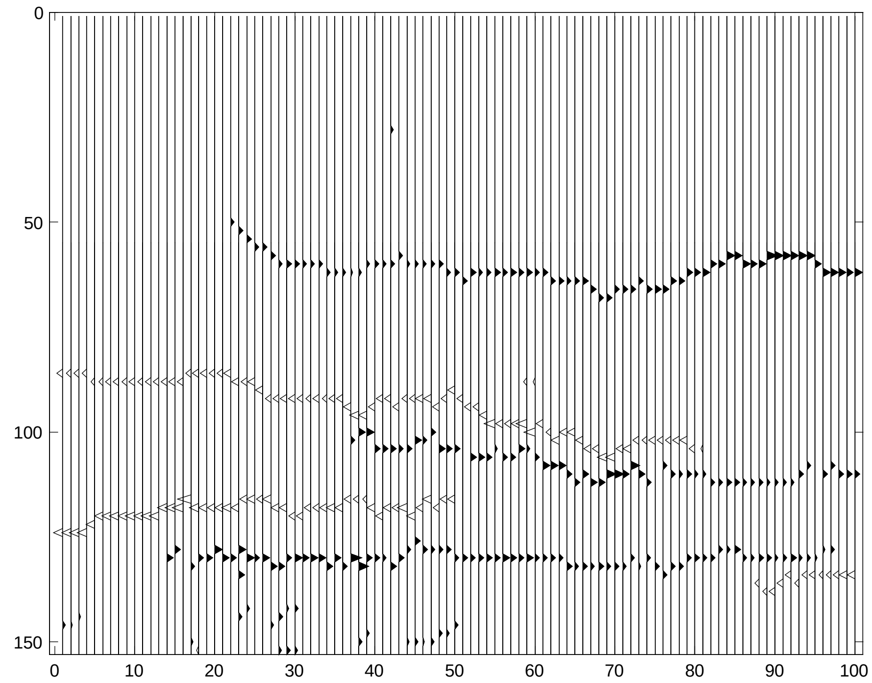
(A brace around the first five entries of this result column is labelled $\hat{\mathbf{r}}_j$.)

Synthetic Data Results, SNR=5 dB

Multichannel deconvolution, SNR=5 dB



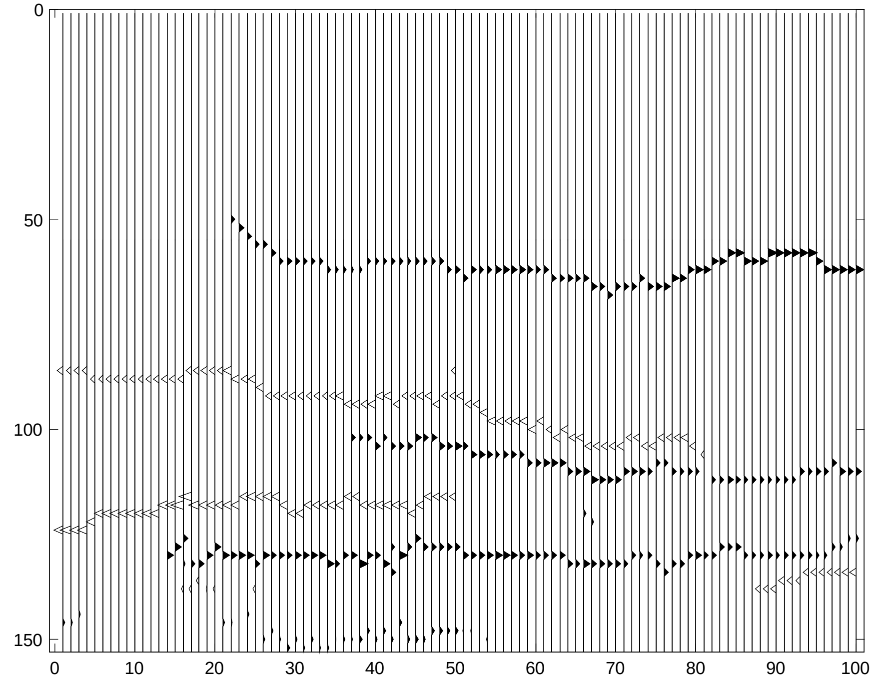
Deconvolution by smoothing, SNR=5 dB



Synthetic Data Results, SNR=10 dB

Multichannel deconvolution, SNR=10 dB

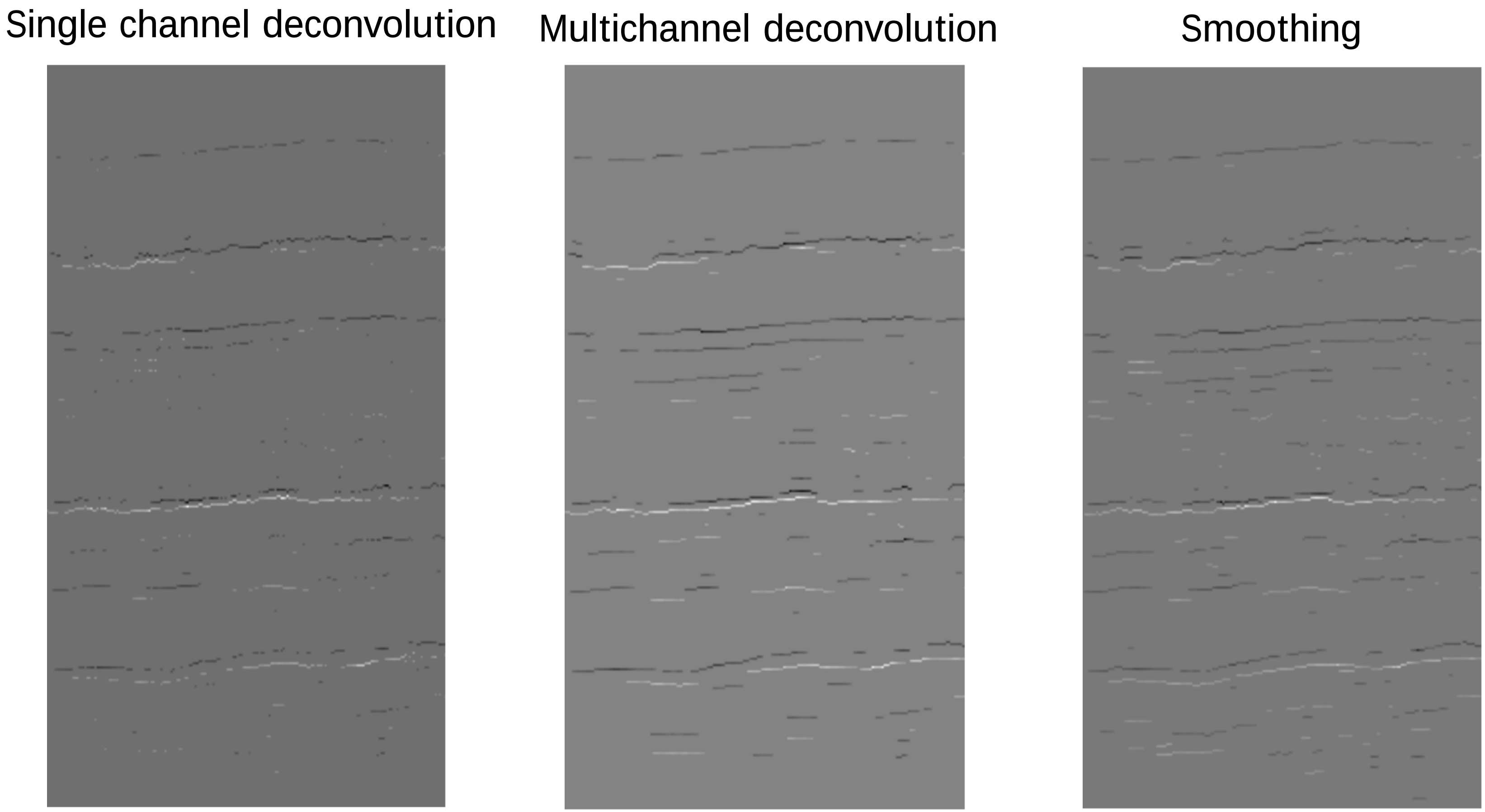
Deconvolution by smoothing, SNR=10 dB



Synthetic Results – Performance Quality Measures

SNR (5dB)		$L^{miss+flase}$	L^{miss}	L^{false}	L^{SSQ}	$L_2^{miss+flase}$	L_2^{miss}	L_2^{false}
5	SC	512.3	414.3	365.3	276.1	293.7	235.2	186.2
	MC	372.6	300.6	265.6	172.9	216.1	173.6	138.6
	Smoothing	315.4	252.4	227.4	136.4	187.7	150.2	125.2
10	SC	243.9	192.9	175.9	116.8	140.5	109.5	92.5
	MC	196.3	149.3	145.3	75.4	123.6	92.6	89.6
	Smoothing	153.8	114.8	114.8	41.1	109.5	81.5	81.5

Real Data Results



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Conclusions

- We presented two stochastic blind multichannel deconvolution algorithms.
- The proposed parameter estimation method successfully recovers the MBG I model's parameters.
- Both proposed algorithms produce better deconvoltuion results than the single channel blind deconvolution method.
- The smoothing algorithm produces better deconvoltuion results than the proposed multichannel algorithm.
- The performance of both blind deconvoltuion schemes improves as the SNR increases.

Future Research

- Replacing the MBG I model by the MBG II model.
- Analyzing the smoothing algorithm's performance for larger smoothing windows
- Using wavelet estimation methods which can estimate both wavelet's length and maximum position.