

A TUNABLE BEAMFORMER FOR ROBUST SUPERDIRECTIONAL BEAMFORMING

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ABSTRACT

Conventional superdirective beamforming is a well-known multi-microphone enhancement method with superior directivity factor (DF). However, it suffers from an inferior white noise gain (WNG), which is expressed by high sensitivity to uncorrelated noise and array inaccuracies. In this work, a beamformer with tunable superdirective gain is introduced. The proposed approach plays a role of a regularized superdirective beamformer, where instead of constraining the WNG, we minimize both white noise and diffuse noise energy in the optimization problem. In addition, by using a tunable regularization parameter, we control the amount of the beamformer DF and WNG. This single boresight solution is then extended to a multiple linear constraint beamformer, with any user-determined spatial or frequency constraints. The beamformer gain response simulations exhibit a robust and controllable solution with an efficient DF/WNG tradeoff.

Index Terms— Linear microphone arrays, delay-and-sum beamformer, superdirective beamformer, robust beamforming, white noise gain, directivity factor.

1. INTRODUCTION

In many acoustic communication systems, the input speech signals are corrupted by reverberation and noise. As a result, multi-microphone processing for signal enhancement became an essential task in speech-controlled applications. Superdirective fixed beamforming is an effective enhancement method for array processing, which provides high directive gain for closely-spaced elements [1]. However, the high sensitivity of this beamformer to uncorrelated noise, sensor noise, spatial white noise and position errors, significantly degrades its use in practice [1, 2, 3]. Dealing with this type of errors can be assessed efficiently by the beamformer white noise gain (WNG), a reliable robustness measure.

Many algorithms for robust superdirective beamforming were developed, mostly by an explicit maximization of the array gain for a given acceptable white noise amplification (or similar robustness measure) constraint [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Gilbert and Morgan [4] were among the first who addressed the constrained array optimization problem. Cox et al. [1] introduced an optimal beamformer for maximum array gain with a WNG constraint. Other methods include various forms of the optimization problem, such as an adaptive beamformer with an iterative implementation algorithm [5], handling different mismatch errors [6, 7], diagonal loading of the sample covariance matrix [19],

etc. Later studies exploited the probability density functions of the array characteristics errors for the optimization, instead of controlling the WNG level [8, 9, 10, 11]. Some methods suggested nonlinear or worst-case performance optimization [11, 12], and considered broadband superdirective beamforming [10, 11, 12].

In this paper, we address the problem of designing a robust superdirective distortionless beamformer, which maximizes a weighted sum of both the directivity factor (DF) and WNG. Although many solutions for similar problems exist, most of them lack closed-form expressions for the constrained optimization problem, and usually involve iterative steps [5] or reformulation in convex optimization forms (such as second-order cone programming [6, 13]). Moreover, in many diagonal loading methods, for example, the process of setting an optimal value for the regularization factor is rather ad hoc [20] or requires prior knowledge of the signal and interference [21, 22, 23]. We propose an innovative approach, in which instead of constraining it, we maximize the WNG as well, and obtain a new form of a regularized superdirective beamformer. In practice, we propose an equivalent procedure in which we aim to minimize the noise energy at the beamformer output, by controlling the angular integral on the relevant noise fields. We obtain an alternative regularized solution, with an intuitive closed-form expression for the regularization parameter. This parameter is controlled by a tuning angle, which sets the weight of the WNG and the DF blend, in the optimization problem. The proposed beamformer achieves an effective compromise of high DF together with robustness against white noise input, with full user-control of the desired properties.

This paper is organized as follows. In Section 2, we define the problem and the signal model. In Section 3, several useful performance measures are introduced. Next, in Section 4, we describe three key conventional fixed beamformers, solutions of various gain optimization problems. In Section 5, we present the tunable beamformer, our proposed distortionless solution for control of the WNG and the DF. Additionally, a generalized-multiple linear constraints beamformer is introduced. Simulation results are presented in Section 6. Finally, conclusions are given in Section 7.

2. SIGNAL MODEL AND PROBLEM FORMULATION

We consider a plane wave, in the far field, that propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on a uniform linear sensor array consisting of M omnidirectional microphones, where the distance between two successive sensors equals to δ . The direction of the source signal to the array is parameterized by the azimuth angle θ . In this context, the steering

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vector (of length M) is given by

$$\mathbf{d}(\omega, \theta) = \begin{bmatrix} 1 & e^{-j\omega\tau_0 \cos \theta} & \dots & e^{-j(M-1)\omega\tau_0 \cos \theta} \end{bmatrix}^T, \quad (1)$$

where the superscript T is the transpose operator, $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, and $\tau_0 = \delta/c$ is the delay between two successive sensors at the angle $\theta = 0^\circ$.

We are interested in fixed beamformers with small values of δ , like in superdirective [1], [5] or differential beamforming [2], [14], where the main lobe is at the angle $\theta = 0^\circ$ (endfire direction) and the desired signal propagates from the same angle. Then, our objective is to design linear array beamformers, which are able to achieve supergains at the endfire with a better control of white noise amplification.

With this signal model, the observation signal vector (of length M) is

$$\begin{aligned} \mathbf{y}(\omega) &= [Y_1(\omega) \ Y_2(\omega) \ \dots \ Y_M(\omega)]^T \\ &= \mathbf{x}(\omega) + \mathbf{v}(\omega) \\ &= \mathbf{d}(\omega)X(\omega) + \mathbf{v}(\omega), \end{aligned} \quad (2)$$

where $Y_m(\omega)$ is the m th microphone signal, $\mathbf{x}(\omega) = \mathbf{d}(\omega)X(\omega)$, $X(\omega)$ is the desired signal, $\mathbf{d}(\omega) = \mathbf{d}(\omega, 0^\circ)$ is the steering vector at $\theta = 0^\circ$ (direction of the source), and $\mathbf{v}(\omega)$ is the additive noise signal vector.

By applying a complex-valued linear filter (of length M), $\mathbf{h}(\omega)$, to the observation signal vector, $\mathbf{y}(\omega)$, we obtain the beamformer output [15]:

$$\begin{aligned} Z(\omega) &= \mathbf{h}^H(\omega)\mathbf{y}(\omega) \\ &= \mathbf{h}^H(\omega)\mathbf{d}(\omega)X(\omega) + \mathbf{h}^H(\omega)\mathbf{v}(\omega), \end{aligned} \quad (3)$$

where $Z(\omega)$ is an estimate of the desired signal, $X(\omega)$, and the superscript H is the conjugate-transpose operator. In our context, the distortionless constraint is desired, i.e.,

$$\mathbf{h}^H(\omega)\mathbf{d}(\omega) = 1. \quad (4)$$

3. PERFORMANCE MEASURES

In this section, we present some useful performance measures. The first important measure is the beampattern or directivity pattern, which describes the sensitivity of the beamformer to a plane wave impinging on the array from a direction θ . It is given by

$$\begin{aligned} \mathcal{B}[\mathbf{h}(\omega), \theta] &= \mathbf{d}^H(\omega, \theta)\mathbf{h}(\omega) \\ &= \sum_{m=1}^M H_m(\omega) e^{j(m-1)\omega\tau_0 \cos \theta}. \end{aligned} \quad (5)$$

In [16], we define the gain in the signal-to-noise ratio (SNR), as the ratio between the input and the output SNR:

$$\mathcal{G}[\mathbf{h}(\omega)] = \frac{\text{oSNR}[\mathbf{h}(\omega)]}{\text{iSNR}(\omega)} = \frac{|\mathbf{h}^H(\omega)\mathbf{d}(\omega)|^2}{\mathbf{h}^H(\omega)\mathbf{\Gamma}_v(\omega)\mathbf{h}(\omega)}, \quad (6)$$

where $\mathbf{\Gamma}_v(\omega)$ is the pseudo-coherence matrix of $\mathbf{v}(\omega)$ [16].

The most convenient way to evaluate the sensitivity of the array to some of its imperfections is via the so-called WNG, which measures the array gain to a white-noise input. It is defined by taking $\mathbf{\Gamma}_v(\omega) = \mathbf{I}_M$ in (6), where \mathbf{I}_M is the $M \times M$ identity matrix, i.e.,

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{|\mathbf{h}^H(\omega)\mathbf{d}(\omega)|^2}{\mathbf{h}^H(\omega)\mathbf{h}(\omega)}. \quad (7)$$

We can easily find that the maximum WNG is

$$\mathcal{W}_{\max} = M, \quad (8)$$

which is frequency independent. The white noise amplification is the most serious problem with superdirective beamformers, which restricts their deployment in practice.

Another important measure, which measures diffuse noise reduction and quantifies how the microphone array performs in the presence of reverberation is the DF (i.e., the SNR-gain for a diffuse-noise input). Considering a spherically isotropic (diffuse) noise field, the DF is defined as

$$\begin{aligned} \mathcal{D}[\mathbf{h}(\omega)] &= \frac{|\mathcal{B}[\mathbf{h}(\omega), 0^\circ]|^2}{\frac{1}{2} \int_0^\pi |\mathcal{B}[\mathbf{h}(\omega), \theta]|^2 \sin \theta d\theta} \\ &= \frac{|\mathbf{h}^H(\omega)\mathbf{d}(\omega)|^2}{\mathbf{h}^H(\omega)\mathbf{\Gamma}_{0,\pi}(\omega)\mathbf{h}(\omega)}, \end{aligned} \quad (9)$$

where

$$\mathbf{\Gamma}_{0,\pi}(\omega) = \frac{1}{2} \int_0^\pi \mathbf{d}(\omega, \theta)\mathbf{d}^H(\omega, \theta) \sin \theta d\theta. \quad (10)$$

The elements of the $M \times M$ matrix $\mathbf{\Gamma}_{0,\pi}(\omega)$ are

$$\begin{aligned} [\mathbf{\Gamma}_{0,\pi}(\omega)]_{ij} &= \frac{\sin[\omega(j-i)\tau_0]}{\omega(j-i)\tau_0} \\ &= \text{sinc}[\omega(j-i)\tau_0], \end{aligned} \quad (11)$$

with $[\mathbf{\Gamma}_{0,\pi}(\omega)]_{mm} = 1$, $m = 1, 2, \dots, M$. It is easy to verify that the maximum DF is

$$\mathcal{D}_{\max}(\omega) = \mathbf{d}^H(\omega)\mathbf{\Gamma}_{0,\pi}^{-1}(\omega)\mathbf{d}(\omega), \quad (12)$$

which is frequency dependent. The maximum DF is referred as supergain when it is close to M^2 [18]. This gain can be achieved but at the expense of white noise amplification.

Then, one of the foremost issues in practice is how to compromise between $\mathcal{W}[\mathbf{h}(\omega)]$ and $\mathcal{D}[\mathbf{h}(\omega)]$. Ideally, we would like $\mathcal{D}[\mathbf{h}(\omega)]$ to be as large as possible with $\mathcal{W}[\mathbf{h}(\omega)] \geq 1$.

4. CONVENTIONAL FIXED BEAMFORMERS

In this section, we briefly discuss three important conventional fixed beamformers: delay-and-sum, superdirective, and robust superdirective.

The simplest and the most well-known beamformer is the delay-and-sum (DS), which is derived by maximizing the WNG [eq. (7)] subject to the distortionless constraint (4). We easily get

$$\mathbf{h}_{\text{DS}}(\omega) = \frac{\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\mathbf{d}(\omega)} = \frac{\mathbf{d}(\omega)}{M}. \quad (13)$$

Therefore, the WNG and DF are, respectively,

$$\mathcal{W}[\mathbf{h}_{\text{DS}}(\omega)] = M = \mathcal{W}_{\max} \quad (14)$$

and

$$\mathcal{D}[\mathbf{h}_{\text{DS}}(\omega)] = \frac{M^2}{\mathbf{d}^H(\omega)\mathbf{\Gamma}_{0,\pi}(\omega)\mathbf{d}(\omega)} \geq 1. \quad (15)$$

Clearly, the DS beamformer maximizes the WNG and never amplifies the diffuse noise since $\mathcal{D}[\mathbf{h}_{\text{DS}}(\omega)] \geq 1$. However, in reverberant and noisy environments, it is essential to have high DF for good speech enhancement (i.e., dereverberation and noise reduction). But, unfortunately, this does not happen, in general, with the DS beamformer, which is known to perform very poorly when the reverberation time of the room is high, even with a large number of microphones.

The second important beamformer is obtained by maximizing the DF [eq. (9)] subject to the distortionless constraint (4). We get the well-known superdirective beamformer [1]:

$$\mathbf{h}_{\text{SD}}(\omega) = \frac{\mathbf{\Gamma}_{0,\pi}^{-1}(\omega)\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\mathbf{\Gamma}_{0,\pi}^{-1}(\omega)\mathbf{d}(\omega)}. \quad (16)$$

This filter is a particular form of the celebrated minimum variance distortionless response (MVDR) beamformer [24], [25]. Also, (16) corresponds to the directivity pattern of the hypercardioid of order $M - 1$ [14]. We deduce that the WNG and the DF are, respectively,

$$\mathcal{W}[\mathbf{h}_{\text{SD}}(\omega)] = \frac{[\mathbf{d}^H(\omega)\mathbf{\Gamma}_{0,\pi}^{-1}(\omega)\mathbf{d}(\omega)]^2}{\mathbf{d}^H(\omega)\mathbf{\Gamma}_{0,\pi}^{-2}(\omega)\mathbf{d}(\omega)} \quad (17)$$

and

$$\mathcal{D}[\mathbf{h}_{\text{SD}}(\omega)] = \mathbf{d}^H(\omega)\mathbf{\Gamma}_{0,\pi}^{-1}(\omega)\mathbf{d}(\omega) = \mathcal{D}_{\max}(\omega). \quad (18)$$

Finally, the last conventional beamformer of interest is obtained by maximizing the DF subject to a constraint on the WNG. Using the distortionless constraint, we find the robust superdirective beamformer [1], [5]:

$$\mathbf{h}_{\text{R},\epsilon}(\omega) = \frac{[\epsilon\mathbf{I}_M + \mathbf{\Gamma}_{0,\pi}(\omega)]^{-1}\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)[\epsilon\mathbf{I}_M + \mathbf{\Gamma}_{0,\pi}(\omega)]^{-1}\mathbf{d}(\omega)}, \quad (19)$$

where $\epsilon \geq 0$ is a Lagrange multiplier. It is clear that (19) is a regularized (or robust) version of (16), where ϵ serves as the regularization parameter. This parameter tries to find a good compromise between a supergain and white noise amplification. A small ϵ leads to a large DF and a low WNG, while a large ϵ gives a low DF and a large WNG. Two interesting cases are $\mathbf{h}_{\text{R},0}(\omega) = \mathbf{h}_{\text{SD}}(\omega)$ and $\mathbf{h}_{\text{R},\infty}(\omega) = \mathbf{h}_{\text{DS}}(\omega)$. While $\mathbf{h}_{\text{R},\epsilon}(\omega)$ has some control on white noise amplification, it is certainly not easy to find an intuitive meaning or a closed-form expression for ϵ given a desired value of the WNG.

5. TUNABLE BEAMFORMER

Since we want to compromise between the WNG and the DF, each representing a contradictory physical need, we suggest to address the optimization problem from a different point of view. Instead of targeting to maximize the beamformer DF with a constraint on the WNG, we would like to minimize the amplification of noise that passes through the system. Accordingly, we propose to reduce the weighted combination of the relevant noise energies, meaning, at the beamformer output, we should minimize some white noise plus

some diffuse noise energy subject to the distortionless constraint, i.e.,

$$\begin{aligned} & \min_{\mathbf{h}(\omega)} \mathbf{h}^H(\omega) [\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)] \mathbf{h}(\omega) \\ & \text{subject to } \mathbf{h}^H(\omega)\mathbf{d}(\omega) = 1, \end{aligned} \quad (20)$$

where

$$\mathbf{\Gamma}_{\psi,\pi}(\omega) = \frac{1}{2} \int_{\psi}^{\pi} \mathbf{d}(\omega, \theta) \mathbf{d}^H(\omega, \theta) \sin \theta d\theta, \quad (21)$$

and $0 \leq \psi \leq \pi$. We note that the first addend in (20) is inversely proportional to the WNG, whereas the second addend, for $\psi = 0$, is inversely proportional to the DF. It can be shown that the elements of the $M \times M$ matrix $\mathbf{\Gamma}_{\psi,\pi}(\omega)$ are

$$[\mathbf{\Gamma}_{\psi,\pi}(\omega)]_{ij} = \frac{e^{j\omega(j-i)\tau_0} \cos \psi - e^{-j\omega(j-i)\tau_0}}{2j\omega(j-i)\tau_0}, \quad (22)$$

with

$$[\mathbf{\Gamma}_{\psi,\pi}(\omega)]_{mm} = \frac{1 + \cos \psi}{2}, \quad m = 1, 2, \dots, M. \quad (23)$$

In (20), with the matrix $\mathbf{\Gamma}_{\psi,\pi}(\omega)$, we minimize the diffuse noise from the angle ψ to π , while with ϵ_ψ , we control the amount of white noise we wish to minimize. In order to derive an applicable solution of the minimization problem, we add a normalization constraint. One way to normalize the weighted combination of the relevant noise energies is to restrict $\text{tr}[\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)]$ to M , with $\text{tr}(\cdot)$ denoting the trace of a square matrix. Therefore, with (23), we get

$$M \cdot \left(\epsilon_\psi + \frac{1 + \cos \psi}{2} \right) = M, \quad (24)$$

hence

$$\epsilon_\psi = \frac{1 - \cos \psi}{2}. \quad (25)$$

Similarly to the conventional beamformers (13, 16) derivation, the minimization of (20) leads to the tunable beamformer:

$$\mathbf{h}_{\text{T},\psi}(\omega) = \frac{[\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)]^{-1}\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)[\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)]^{-1}\mathbf{d}(\omega)}. \quad (26)$$

We can see that $\mathbf{h}_{\text{T},0}(\omega) = \mathbf{h}_{\text{SD}}(\omega)$ and $\mathbf{h}_{\text{T},\pi}(\omega) = \mathbf{h}_{\text{DS}}(\omega)$. This approach of angular integration over the noise suggests an interesting and useful alternative for the conventional optimization methods.

This idea can be generalized by adding more constraints to the optimization problem. Suppose that we want a null in the direction π , the additional constraint is $\mathbf{h}^H(\omega)\mathbf{d}(\omega, \pi) = 0$. Therefore, the criterion to optimize is

$$\begin{aligned} & \min_{\mathbf{h}(\omega)} \mathbf{h}^H(\omega) [\epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega)] \mathbf{h}(\omega) \\ & \text{subject to } \mathbf{h}^H(\omega)\mathbf{C}(\omega) = \mathbf{i}^T, \end{aligned} \quad (27)$$

where

$$\mathbf{C}(\omega) = \begin{bmatrix} \mathbf{d}^T(\omega) \\ \mathbf{d}^T(\omega, \pi) \end{bmatrix}^T, \quad (28)$$

$$\mathbf{i} = [1 \quad 0]^T. \quad (29)$$

We find some kind of cardioid of order $M - 1$:

$$\mathbf{h}_{\text{C},\psi}(\omega) = \mathbf{\Upsilon}_\psi^{-1}(\omega)\mathbf{C}(\omega)[\mathbf{C}^H(\omega)\mathbf{\Upsilon}_\psi^{-1}(\omega)\mathbf{C}(\omega)]^{-1}\mathbf{i}, \quad (30)$$

where

$$\mathbf{\Upsilon}_\psi(\omega) = \epsilon_\psi \mathbf{I}_M + \mathbf{\Gamma}_{\psi,\pi}(\omega). \quad (31)$$

6. SIMULATIONS

One of the key factors that controls the frequency response is the array physical structure. The number of microphones M , and the spacing distance δ vastly impact the beamformer WNG and DF [1, 16]. Increasing the number of microphones M leads to a higher maximal WNG and DF, whereas higher δ changes the WNG/DF tradeoff [1]. In the proposed approach, we control the compromise between the WNG and the DF, or alternately between the white noise and the diffuse noise output energies, by setting an appropriate value for the parameter ψ . Fine tuning of ψ determines the exact WNG/DF tradeoff, hence allowing full control of the beamformer response. Next, we demonstrate the proposed beamformer with few representative values of ψ , and compare it to the regularized superdirective beamformer.

First, we simulated the tunable beamformer (26). In Fig. 1(a)-(b), we show an example of its WNG and DF, with $\psi = 10^\circ$. This value satisfies both acceptable white noise amplification, also in low frequencies, and relatively high DF. When compared to the regularized superdirective beamformer (19) (with $\epsilon = 1 \cdot 10^{-4}$), this value provides a better WNG, but a bit lower DF.

Next, we demonstrate the response of the multiple linear constraints beamformer (30). An example of its WNG and DF is illustrated in Fig. 1(c)-(d). Here we chose $\psi = 0.5^\circ$ for higher DF (at the expense of a lower yet still tolerable WNG). In addition, the obtained frequency response here is much more similar to the regularized superdirective beamformer (19).

Choosing a higher value for ψ would enlarge the WNG but conversely decrease the DF [reducing the integral boundary range in (21)]. This type of behavior is examined in Fig. 2, where we analyze the WNG and DF responses of (26) versus the angle ψ , for few representative frequencies. Evidently, a larger ψ gives more weight to $\epsilon_\psi \mathbf{I}_M$ coefficient in (20), providing a higher WNG and lower DF.

Examining Figs. 1-2, especially with respect to the regularized superdirective beamformer [5, 16, 17], we note that the tunable beamformer and the constrained superdirective beamformer share a lot in common. In fact, for very small values of ψ and ϵ , the tunable beamformer approximates (19), since for $\psi \ll 1$ [rad] we can denote $\Gamma_{\psi,\pi}(\omega) \approx \Gamma_{0,\pi}(\omega)$ and demand $\epsilon_\psi \stackrel{\text{def}}{=} \epsilon$.

Actually, for $\psi \ll 1$ we can obtain a direct and simple relation between the two. Using first-order Taylor expansion of (25), we get

$$\epsilon_\psi \approx \frac{\psi^2}{4}. \quad (32)$$

i.e., with $\psi \stackrel{\text{def}}{=} 2\sqrt{\epsilon}$ and $\psi \ll 1$, we get $\mathbf{h}_{T,\psi}(\omega) \approx \mathbf{h}_{R,\epsilon}(\omega)$.

7. CONCLUSIONS

We have proposed a new approach for robust superdirective beamforming, by introducing the tunable beamformer. This solution comprises a useful alternative for the constrained superdirective beamformer. It allows us to control the amount of the white noise and the diffuse noise energy we minimize, by tuning the parameter ψ . We derived a closed-form expression for the regularization parameter, which varies only from 0 to 1, as opposed to most of the common approaches, in which the parameter varies up to infinity with no intuitive physical meaning for its value. In addition, this solution was generalized to a multiple linear constraints beamformer, where any

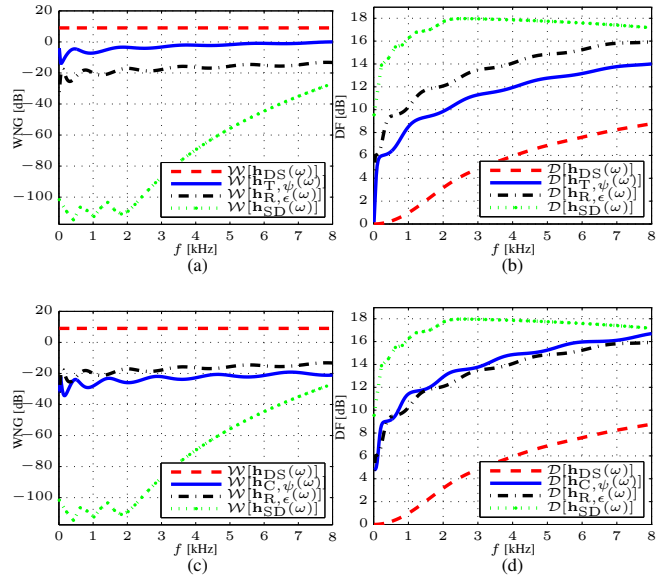


Fig. 1: Array gains of the proposed beamformers versus frequency, with $M = 8$ microphones and $\delta = 1$ cm. The left figures illustrate the beamformer WNG (solid line) versus frequency. As a reference, $\mathcal{W}[\mathbf{h}_{DS}(\omega)] = \mathcal{W}_{\max}$ (dashed line), $\mathcal{W}[\mathbf{h}_{SD}(\omega)]$ (dotted line), and $\mathcal{W}[\mathbf{h}_{R,\epsilon}(\omega)]$ (dot-dash-dot line, $\epsilon = 10^{-4}$) are plotted. The right figures illustrate the proposed beamformer DF (solid line). As a reference, $\mathcal{D}[\mathbf{h}_{SD}(\omega)] = \mathcal{D}_{\max}(\omega)$ (dashed line), $\mathcal{D}[\mathbf{h}_{DS}(\omega)]$ (dotted line), and $\mathcal{D}[\mathbf{h}_{R,\epsilon}(\omega)]$ (dot-dash-dot line) are plotted. (a)-(b) WNG and DF of $\mathbf{h}_{T,\psi}(\omega)$ (26), with $\psi = 10^\circ$. (c)-(d) WNG and DF of $\mathbf{h}_{C,\psi}(\omega)$ (28), with $\psi = 0.5^\circ$ and null in π direction.

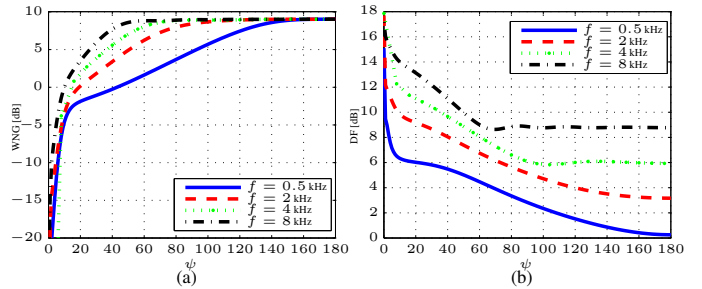


Fig. 2: (a)-(b) WNG and DF curves of $\mathbf{h}_{T,\psi}(\omega)$ (26) versus ψ , with $M = 8$ microphones and $\delta = 1$ cm, at $f = 0.5$ kHz (solid line), $f = 2$ kHz (dashed line), $f = 4$ kHz (dotted line), and $f = 8$ kHz (dot-dash-dot line).

type of spatial or frequency linear constraint can be satisfied. We demonstrated design examples, both for the WNG and DF measurements, and examined the influence of the angle ψ on the WNG-DF tradeoff.

The proposed angular approach with tunable regularization parameter offers a new perspective for angular noise field analysis and regularized robust beamforming, and may be analyzed for more uses and expansions in the future.

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