A Reduced Bandwidth Binaural MVDR Beamformer

Shmulik Markovich Golan and Sharon Gannot
School of Engineering
Bar-Ilan University, 56000 Ramat-Gan, Israel
shmulik.markovich@gmail.com; gannot@eng.biu.ac.il

Israel Cohen
Department of Electrical Engineering
Technion, Technion City, Haifa 32000, Israel
icohen@ee.technion.ac.il

***Abstract*—In this contribution a novel reduced-bandwidth iterative binaural MVDR beamformer is proposed. The proposed method reduces the bandwidth requirement between hearing aids to a single channel, regardless of the number of microphones. The algorithm is proven to converge to the optimal binaural MVDR in the case of a rank-1 desired source correlation matrix. Comprehensive simulations of narrow-band and speech signals demonstrate the convergence and the optimality of the algorithm.**

I. Introduction

Beamforming [1] is known to outperform single channel algorithms in noise reduction tasks. Two of the most common beamformers are the multichannel Wiener filter (MWF), which minimizes the mean squared error (MSE), and the minimum variance distortionless response (MVDR), which minimizes the noise variance under the constraint of no distortion of the desired speech.

A distributed sensor network [2] is comprised of multiple sub-arrays, each consisting of several sensors, a signal processor and a wireless communication module. The spatial diversity improves the signal perception and allows for further improvement compared with a single array of sensors. Sub-arrays are battery operated in many cases, and energy resources are the main limitation of the sensor network structure. Wireless communication is the most energy consuming operation. Straightforward optimal algorithms which are based on sharing all sensors data among the sub-arrays lead to a shorter system lifetime and in many scenarios are unacceptable.

The binaural hearing aid is an example for a sensor network comprised of two sub-arrays (left and right), each consisting of one or more microphones. Doclo et al. [3] showed the advantage of binaural algorithms over monaural algorithms, using data of both laterals rather than using only local sensors. They proposed the iterative distributed binaural speech distortion weighted multichannel Wiener filter (SDW-MWF) which requires transmitting a single audio channel between laterals. A proof of the algorithm convergence to the optimal MWF beamformer was given for the scenario of a desired source correlation matrix with rank-1. Bertrand and Moonen [4] proposed the distributed adaptive node-specific minimum mean squared error (MMSE) signal estimation (DANSE) algorithm which required k transmission channels and extended the iterative distributed SDW-MWF (DB-MWF) to multiple nodes (sub-arrays) for desired source correlation matrix with rank-k.

In this contribution, a novel reduced bandwidth iterative algorithm for a distributed MVDR beamformer with application to binaural hearing aids is presented. The proposed algorithm requires a single channel transmission between laterals. It is well known that the MVDR beamformer is a special case of the SDW-MWF. Here, a distributed version of the MVDR beamformer is derived directly. The convergence of the iterative procedure to the binaural MVDR is proved for a desired source correlation matrix with rank-1. The proposed method is shown to outperform the monaural MVDR, where the output is generated by filtering only local sensors, without communication with the other side.

Maintaining spatial cues is a desired property of any binaural beamformer. The binaural MVDR maintains the spatial cues of the desired source as can be deduces from its distortionless response. However, spatial cues of the interfering sources are not maintained.

The paper is organized as follows. In Sec. II, the binaural hearing aids problem is formulated. In Sec. III, a closed-form solution for the binaural MVDR based on all sensors data is derived. In Sec.IV, the novel reduced bandwidth iterative MVDR algorithm is proposed. The proof of convergence in the rank-1 scenario appears in the Sec. V. Finally, the algorithm is evaluated for narrow-band stationary signals, and for speech signals in reverberant environments in Sec. VI.

Keeping spatial cues is a desired property of any binaural beamformer. The binaural MVDR keeps the spatial cues of the desired source directly from its definition. However, spatial cues of interfering sources are not kept in general.

II. Problem Formulation

The problem is formulated in the short-time Fourier transform (STFT) domain. Consider a desired speech signal $s(\ell, k)$ impinging on two microphone arrays in the left and right hearing aid apparatuses placed in a reverberant environment. The received signals are contaminated by a stationary noise $\boldsymbol{v}(\ell, k)$. From here on we omit the time and frequency indexes for brevity. The signals received by the left and right arrays are given by $\boldsymbol{z}_l = \boldsymbol{h}_l s + \boldsymbol{v}_l$ and $\boldsymbol{z}_r = \boldsymbol{h}_r s + \boldsymbol{v}_r$, respectively, where $\boldsymbol{h}_l, \boldsymbol{h}_r$ are the acoustic transfer function (ATF)s relating the desired source and the left and right arrays. Define the vectors comprised of a concatenation of the left and right signals $\boldsymbol{z} = \begin{bmatrix} \boldsymbol{z}_l{}^T & \boldsymbol{z}_r{}^T \end{bmatrix}^T = \boldsymbol{h}s + \boldsymbol{v}$, where $\boldsymbol{v} = \begin{bmatrix} \boldsymbol{v}_l{}^T & \boldsymbol{v}_r{}^T \end{bmatrix}^T$ and $\boldsymbol{h} = \begin{bmatrix} \boldsymbol{h}_l{}^T & \boldsymbol{h}_r{}^T \end{bmatrix}^T$. Denote the covariance matrix of the received signals:

$$\boldsymbol{\Phi}_{zz} = \sigma^2 \boldsymbol{h}^\dagger \boldsymbol{h} + \boldsymbol{\Phi}_{vv} = \begin{bmatrix} \boldsymbol{\Phi}_{ll} & \boldsymbol{\Phi}_{lr} \\ \boldsymbol{\Phi}_{rl} & \boldsymbol{\Phi}_{rr} \end{bmatrix} \tag{1}$$

where $\boldsymbol{\Phi}_{vv} = \mathrm{E}\left[\boldsymbol{v}\boldsymbol{v}^\dagger\right]$ is the covariance matrix of the stationary noise. The goal of the standard binaural MVDR beamformer is to reduce the noise power at two reference microphones at the left and right apparatuses, by using all microphone data, and while keeping the desired speech components undistorted. In this contribution a distributed version of the binaural MVDR problem is addressed. The solution should limit communication bandwidth between laterals without sacrificing the performance.

III. Closed-form Binaural MVDR

The binaural MVDR beamformer consists of two beamformers designed for reproducing the desired signal components as received by reference microphones in each lateral, while minimizing the overall noise power. The output signals of the closed-form beamformer are

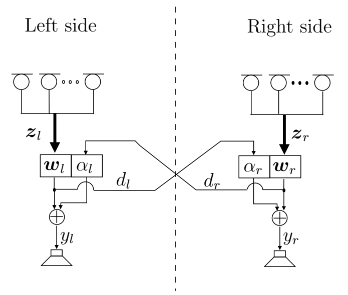


Fig. 1. Diagram of the distributed binaural MVDR

given by $y_l^o = (\boldsymbol{w}^{ol})^\dagger \boldsymbol{z}$ and $y_r^o = (\boldsymbol{w}^{or})^\dagger \boldsymbol{z}$. The closed-form solution is given by:

$$\boldsymbol{w}^{ol} = \left((\tilde{\boldsymbol{h}}^l)^\dagger \boldsymbol{\Phi}_{zz}{}^{-1} \tilde{\boldsymbol{h}}^l \right)^{-1} \boldsymbol{\Phi}_{zz}{}^{-1} \tilde{\boldsymbol{h}}^l \tag{2a}$$

$$\boldsymbol{w}^{or} = \left((\tilde{\boldsymbol{h}}^r)^\dagger \boldsymbol{\Phi}_{zz}{}^{-1} \tilde{\boldsymbol{h}}^r \right)^{-1} \boldsymbol{\Phi}_{zz}{}^{-1} \tilde{\boldsymbol{h}}^r \tag{2b}$$

where the left and right relative transfer function (RTF)s are defined as:

$$\tilde{\boldsymbol{h}}^l = (h_{l,1})^{-1} \boldsymbol{h} = \left[\begin{array}{cc} (\tilde{\boldsymbol{h}}_l^l)^T & (\tilde{\boldsymbol{h}}_r^l)^T \end{array} \right]^T$$

$$\tilde{\boldsymbol{h}}^r = (h_{r,1})^{-1} \boldsymbol{h} = \left[\begin{array}{cc} (\tilde{\boldsymbol{h}}_l^r)^T & (\tilde{\boldsymbol{h}}_r^r)^T \end{array} \right]^T$$

The first microphones are arbitrarily chosen as the reference microphones. Throughout this contribution the subscript notations $(\cdot)_l$ and $(\cdot)_r$ are used for denoting the vector components corresponding to the left and right laterals, respectively. The superscript notations $(\cdot)^l$ and $(\cdot)^r$ are used to denote variables which are used for calculating the outputs of the left and right apparatuses, respectively. Note that the left and right MVDR beamformers are parallel in the rank-1 case.

IV. Proposed method

In this section, we introduce a batch iterative algorithm which converges to the closed-form solution introduced in the previous section. We assume that the second moments of the observed data are available or can be estimated without errors. At each iteration, each side calculates the MVDR beamformer based on its local microphones and the channel received from the lateral side. Its contribution to the binaural beamformer is transmitted to the lateral side which in turn also updates its coefficients in a similar manner. Each iteration is therefore comprised of updating both left and right beamformers. A diagram of the algorithm is depicted in Fig. 1. Consider the ith iteration of the algorithm. Without loss of generality, we assume that the left side is the first to update its beamformer. The data available to the left side is its own microphones and the received channel from the right side in the previous iteration

$$d_r^{i-1} = (\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{z}_r. \tag{3}$$

The MVDR equation at the ith iteration at the left side is given by

$$\left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \end{array} \right] = \underset{\boldsymbol{w}_l^i, \alpha_l^i}{\operatorname{argmin}} \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \end{array} \right]^\dagger \mathrm{E} \left[\left[\begin{array}{c} \boldsymbol{z}_l \\ d_r^{i-1} \end{array} \right] \left[\begin{array}{c} \boldsymbol{z}_l \\ d_r^{i-1} \end{array} \right]^\dagger \right] \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \end{array} \right] \tag{4}$$

$$\text{s.t.} \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \end{array} \right]^\dagger \left[\begin{array}{c} \tilde{\boldsymbol{h}}_l^l \\ (\boldsymbol{w}_r^{i-1})^\dagger \tilde{\boldsymbol{h}}_r^l \end{array} \right] = 1.$$

The last minimization can be reformulated as a constrained minimization of the binaural MVDR, where the right coefficients are constant up to a scaling factor $\alpha_l^i \boldsymbol{w}_r^{i-1}$:

$$\left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \end{array} \right] = \underset{\boldsymbol{w}_l^i, \alpha_l^i}{\operatorname{argmin}} \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right]^\dagger \boldsymbol{\Phi}_{zz} \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right] \tag{5}$$

$$\text{s.t.} \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right]^\dagger \tilde{\boldsymbol{h}}^l = 1.$$

The minimization is performed by using Lagrange multipliers. Define the Lagrangian of the left side:

$$\begin{aligned} \mathcal{L}_l(\boldsymbol{w}_l^i, \alpha_l^i, \lambda_l^i) = & \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right]^\dagger \boldsymbol{\Phi}_{zz} \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right] \\ & + \lambda_l^i \left(\left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right]^\dagger \tilde{\boldsymbol{h}}^l - 1 \right) \\ & + (\lambda_l^i)^* \left((\tilde{\boldsymbol{h}}^l)^\dagger \left[\begin{array}{c} \boldsymbol{w}_l^i \\ \alpha_l^i \boldsymbol{w}_r^{i-1} \end{array} \right] - 1 \right). \end{aligned} \tag{6}$$

Minimizing (6) is performed by solving the partial derivatives $\frac{\partial \mathcal{L}_l}{\partial (\boldsymbol{w}_l^i)^\dagger} = \boldsymbol{0}$, $\frac{\partial \mathcal{L}_l}{\partial (\alpha_l^i)^*} = 0$, $\frac{\partial \mathcal{L}_l}{\partial (\lambda_l^i)^*} = 0$ and is given by:

$$\lambda_l^i = \left(-\|\bar{\boldsymbol{h}}_l^{l,i}\|^2_{(\bar{\boldsymbol{\Phi}}_{ll}^i)^{-1}} - \frac{\|(\tilde{\boldsymbol{h}}_r^l)^\dagger \boldsymbol{w}_r^{i-1}\|^2}{(\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}_r^{i-1}} \right)^{-1} \tag{7a}$$

$$\boldsymbol{w}_l^i = \lambda_l^i (\bar{\boldsymbol{\Phi}}_{ll}^i)^{-1} \bar{\boldsymbol{h}}_l^{l,i} \tag{7b}$$

$$\alpha_l^i = -\frac{(\boldsymbol{w}_r^{i-1}) \boldsymbol{\Phi}_{rl} \boldsymbol{w}_l^i + \lambda_l^i (\boldsymbol{w}_r^{i-1})^\dagger \tilde{\boldsymbol{h}}_r^l}{(\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}_r^{i-1}} \tag{7c}$$

where

$$\bar{\boldsymbol{\Phi}}_{ll}^i = \boldsymbol{\Phi}_{ll} - \frac{\boldsymbol{\Phi}_{lr} \boldsymbol{w}_r^{i-1} (\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{\Phi}_{rl}}{(\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}_r^{i-1}} \tag{8a}$$

$$\bar{\boldsymbol{h}}_l^{l,i} = \frac{(\boldsymbol{w}_r^{i-1})^\dagger \tilde{\boldsymbol{h}}_r^l \boldsymbol{\Phi}_{lr} \boldsymbol{w}_r^{i-1}}{(\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}_r^{i-1}} - \tilde{\boldsymbol{h}}_l^l \tag{8b}$$

and the matrix norm with respect to the matrix $\boldsymbol{A}$ is denoted by $\|\cdot\|_{\boldsymbol{A}}$. Note that the second moments required for the calculation are available since

$$\boldsymbol{\Phi}_{lr} \boldsymbol{w}_r^{i-1} = \mathrm{E}\left[\boldsymbol{z}_l (d_r^{i-1})^* \right]$$

$$(\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}_r^{i-1} = \mathrm{E}\left[\|d_r^{i-1}\|^2 \right].$$

The expression $(\boldsymbol{w}_r^{i-1})^\dagger \tilde{\boldsymbol{h}}_r^l = \frac{(\boldsymbol{w}_r^{i-1})^\dagger \boldsymbol{h}_r}{h_{l,1}}$ equals the RTF between the desired source components at d_r^{i-1} and $z_{l,1}$. It can be estimated by exploiting the non-stationarity of speech or by the generalized eigenvalue decomposition (GEVD). The minimization of the right Lagrangian is solved in a similar manner. The algorithm is summarized in Alg. 1, where we also define

$$\bar{\boldsymbol{\Phi}}_{rr}^i = \boldsymbol{\Phi}_{rr} - \frac{\boldsymbol{\Phi}_{lr} \boldsymbol{w}_l^i (\boldsymbol{w}_l^i)^\dagger \boldsymbol{\Phi}_{lr}}{(\boldsymbol{w}_l^i)^\dagger \boldsymbol{\Phi}_{ll} \boldsymbol{w}_l^i} \tag{9}$$

$$\bar{\boldsymbol{h}}_r^{r,i} = \frac{(\boldsymbol{w}_l^i)^\dagger \tilde{\boldsymbol{h}}_l^r \boldsymbol{\Phi}_{rl} \boldsymbol{w}_l^i}{(\boldsymbol{w}_l^i)^\dagger \boldsymbol{\Phi}_{ll} \boldsymbol{w}_l^i} - \tilde{\boldsymbol{h}}_r^r. \tag{10}$$

The algorithm is initialized with the monaural MVDR on the left side. In the following section the proposed algorithm is proved to converge to the binaural MVDR beamformer in the rank-1 case.

V. Convergence of the distributed MVDR to the binaural MVDR

The variance of the MVDR output is denoted by $\mathcal{J}(\boldsymbol{w}) = \boldsymbol{w}^\dagger \boldsymbol{\Phi}_{zz} \boldsymbol{w}$. Consider the ith iteration at the left side $\mathcal{J}\left(\boldsymbol{w}^{l,i}\right)$, where $\boldsymbol{w}^{l,i} = \left[\begin{array}{cc} (\boldsymbol{w}_l^i)^T & \alpha i (\boldsymbol{w}_r^{i-1})^T \end{array} \right]^T$. By manipulation of the left constraint $(\boldsymbol{w}^{l,i})^\dagger \tilde{\boldsymbol{h}}^l = 1$ we obtain that $\left((\tilde{h}^r_{l,1})^* \boldsymbol{w}^{l,i}\right)^\dagger \tilde{\boldsymbol{h}}^r = 1$. Since $(\tilde{h}^r_{l,1})^* \boldsymbol{w}^{l,i}$ belongs to the minimization range of the right side, the MVDR variance after updating the right weight coefficients (at the ith iteration) is upper bounded by $\mathcal{J}\left(\boldsymbol{w}^{r,i}\right) \leq \mathcal{J}\left((\tilde{h}^r_{l,1})^* \boldsymbol{w}^{l,i}\right) = |(\tilde{h}^r_{l,1})|^2 \mathcal{J}\left(\boldsymbol{w}^{l,i}\right)$, where $\boldsymbol{w}^{r,i} = \begin{bmatrix} ari(\boldsymbol{w}^i_l)^T & (\boldsymbol{w}^i_r)^T \end{bmatrix}^T$. By manipulation of the right constraint $(\boldsymbol{w}^{r,i})^\dagger \tilde{\boldsymbol{h}}^r = 1$ we obtain that $\left((\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,i}\right)^\dagger \tilde{\boldsymbol{h}}^l = 1$. Therefore the MVDR variance after updating the left weight coefficients (at iteration $i+1$) is upper bounded by $\mathcal{J}\left(\boldsymbol{w}^{l,i+1}\right) \leq \mathcal{J}\left((\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,i}\right) = |(\tilde{h}^l_{r,1})|^2 \mathcal{J}\left(\boldsymbol{w}^{r,i}\right)$. We therefore obtain the following inequality $\mathcal{J}\left(\boldsymbol{w}^{l,i+1}\right) \leq |(\tilde{h}^l_{r,1})|^2 |(\tilde{h}^r_{l,1})|^2 \mathcal{J}\left(\boldsymbol{w}^{l,i+1}\right)$, and since $\tilde{h}^l_{r,1} = (\tilde{h}^r_{l,1})^{-1}$ we conclude that the variance of the left MVDR is monotonically non-increasing $\mathcal{J}\left(\boldsymbol{w}^{l,i+1}\right) \leq \mathcal{J}\left(\boldsymbol{w}^{l,i}\right)$. In a similar way, we conclude that the variance of the right MVDR is also monotonically non-increasing $\mathcal{J}\left(\boldsymbol{w}^{r,i+1}\right) \leq \mathcal{J}\left(\boldsymbol{w}^{r,i}\right)$. $\mathcal{J}(\boldsymbol{w})$ is trivially lower bounded by 0 and therefore $\mathcal{J}(\boldsymbol{w}^{l,\infty})$, $\mathcal{J}(\boldsymbol{w}^{r,\infty})$ must converge. Consider the above inequalities for $i \to \infty$:

$$\mathcal{J}\left(\boldsymbol{w}^{l,\infty}\right) \leq \mathcal{J}\left((\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,\infty}\right) \tag{11a}$$

$$\mathcal{J}\left(\boldsymbol{w}^{r,\infty}\right) \leq \mathcal{J}\left((\tilde{h}^r_{l,1})^* \boldsymbol{w}^{l,\infty}\right). \tag{11b}$$

Dividing Eq. (11b) by $|\tilde{h}^r_{l,1}|^2$, noting that $\tilde{h}^r_{l,1}\tilde{h}^l_{r,1} = 1$ and combining both inequalities we have that $\mathcal{J}\left((\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,\infty}\right) \leq \mathcal{J}\left(\boldsymbol{w}^{l,\infty}\right) \leq \mathcal{J}\left((\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,\infty}\right)$ and due to the Squeeze theorem an equality holds

$$\mathcal{J}\left(\boldsymbol{w}^{l,\infty}\right) = \mathcal{J}\left((\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,\infty}\right). \tag{12}$$

Since $\mathcal{L}_l(\boldsymbol{w}^\infty_l, \alpha^\infty_l, \lambda^\infty_l)$ is the only local minimum as shown in Sec. IV, and since $(\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,\infty}$ belongs to the minimization range they coincide $\boldsymbol{w}^{l,\infty} = (\tilde{h}^l_{r,1})^* \boldsymbol{w}^{r,\infty}$, and the spatial filters after convergence are parallel. Notice that $\alpha^\infty_l = (\tilde{h}^l_{r,1})^*$ and $\alpha^\infty_r = (\tilde{h}^r_{l,1})^*$.

In order to prove that $\boldsymbol{w}^{ol} = \boldsymbol{w}^{l,\infty}$ we define the projection matrix to the desired signal subspace $\boldsymbol{P}^\parallel = \dfrac{\boldsymbol{h}\boldsymbol{h}^\dagger}{\boldsymbol{h}^\dagger\boldsymbol{h}}$ and to its null subspace $\boldsymbol{P}^\perp = \boldsymbol{I} - \dfrac{\boldsymbol{h}\boldsymbol{h}^\dagger}{\boldsymbol{h}^\dagger\boldsymbol{h}}$, where $\boldsymbol{I}$ is the identity matrix. Notice that the left and right constraints guarantee that at any iteration $i > 1$ the parallel components remain constant $\boldsymbol{P}^\parallel \boldsymbol{w}^{l,i} = \boldsymbol{w}^{l\parallel} = \boldsymbol{w}^{ol\parallel}$, $\boldsymbol{P}^\parallel \boldsymbol{w}^{r,i} = \boldsymbol{w}^{r\parallel} = \boldsymbol{w}^{or\parallel}$. Substitute $\boldsymbol{w}^{l,i} = (\boldsymbol{P}^\parallel + \boldsymbol{P}^\perp)\boldsymbol{w}^{l,i} = \boldsymbol{w}^{l\parallel} + \boldsymbol{w}^{l\perp,i}$ in the left Lagrangian (6)

$$\begin{aligned}\mathcal{L}_l(\boldsymbol{w}^i_l, \alpha^i_l, \lambda^i_l) &= (\boldsymbol{w}^{l\parallel} + \boldsymbol{w}^{l\perp,i})^\dagger \boldsymbol{\Phi}_{zz} (\boldsymbol{w}^{l\parallel} + \boldsymbol{w}^{l\perp,i}) \\ &+ \lambda^i_l \left((\boldsymbol{w}^{l\parallel})^\dagger \tilde{\boldsymbol{h}}^l_l + (\boldsymbol{w}^{r\parallel})^\dagger \tilde{\boldsymbol{h}}^l_r - 1\right) \\ &+ (\lambda^i_l)^* \left((\tilde{\boldsymbol{h}}^l_l)^\dagger \boldsymbol{w}^{l\parallel}_l + (\tilde{\boldsymbol{h}}^l_r)^\dagger \boldsymbol{w}^{l\parallel}_r - 1\right).\end{aligned} \tag{13}$$

The partial derivative $\frac{\partial \mathcal{L}_l}{\partial \boldsymbol{w}^{l\perp,i}_l}$ equals:

$$\begin{aligned}\frac{\partial \mathcal{L}_l}{\partial \boldsymbol{w}^{l\perp,i}_l} &= \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\parallel\right)_{ll} \boldsymbol{w}^{l\parallel}_l + \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\perp\right)_{ll} \boldsymbol{w}^{l\perp,i}_l \\ &+ \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\parallel\right)_{lr} \boldsymbol{w}^{l\parallel}_r + \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\perp\right)_{ll} \boldsymbol{w}^{l\perp,i}_r.\end{aligned} \tag{14}$$

In a similar manner we reformulate $\mathcal{L}_r(\boldsymbol{w}^i_r, \alpha^i_r, \lambda^i_r)$ and evaluate its partial derivative $\frac{\partial \mathcal{L}_r}{\partial \boldsymbol{w}^{r\perp,i}_l}$

$$\begin{aligned}\frac{\partial \mathcal{L}_r}{\partial \boldsymbol{w}^{r\perp,i}_l} &= \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\parallel\right)_{ll} \boldsymbol{w}^{r\parallel}_l + \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\perp\right)_{ll} \boldsymbol{w}^{r\perp,i}_l \\ &+ \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\parallel\right)_{lr} \boldsymbol{w}^{r\parallel}_r + \left(\boldsymbol{P}^\perp \boldsymbol{\Phi}_{zz} \boldsymbol{P}^\perp\right)_{ll} \boldsymbol{w}^{r\perp,i}_r.\end{aligned} \tag{15}$$

Hence, the partial derivatives with respect to the left part of the orthogonal weights simultaneously equal zero $\frac{\partial \mathcal{L}_r}{\partial \boldsymbol{w}^{r\perp,i}_l}|_{\boldsymbol{w}^{r,\infty}} = \alpha^\infty_r \frac{\partial \mathcal{L}_l}{\partial \boldsymbol{w}^{l\perp,i}_l}|_{\boldsymbol{w}^{l,\infty}} = 0$ when $i \to \infty$. The right partial derivative simultaneously equal zero in a similar manner. Finally the global minimum is reached, since the parallel part of the weights equals its optimum, and since all the partial derivatives according to the left and right orthogonal parts of the weights equal zero. Furthermore, since the global Lagrangian under minimization has a single minimum, it has been reached. Hence, it is concluded that

$$\boldsymbol{w}^{l,\infty} = \boldsymbol{w}^{ol} \tag{16a}$$

$$\boldsymbol{w}^{r,\infty} = \boldsymbol{w}^{or} \blacksquare \tag{16b}$$

begin
  for $i = 1, 2, ..$ **do**
    if $i = 1$ **then**
      $\boldsymbol{w}^1_l = (\boldsymbol{\Phi}_{ll})^{-1} \tilde{\boldsymbol{h}}^l_l \left((\tilde{\boldsymbol{h}}^l_l)^\dagger (\boldsymbol{\Phi}_{ll})^{-1} \tilde{\boldsymbol{h}}^l_l\right)^{-1}$
      $\alpha^1_l = 0$
      $d^1_r = \boldsymbol{0}$
    else
      $\lambda^i_l = \left(-\|\bar{\boldsymbol{h}}^{l,i}_l\|^2_{(\bar{\boldsymbol{\Phi}}^i_{ll})^{-1}} - \frac{\|(\tilde{\boldsymbol{h}}^l_r)^\dagger \boldsymbol{w}^{i-1}_r\|^2}{(\boldsymbol{w}^{i-1}_r)^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}^{i-1}_r}\right)^{-1}$
      $\boldsymbol{w}^i_l = \lambda^i_l (\bar{\boldsymbol{\Phi}}^i_{ll})^{-1} \bar{\boldsymbol{h}}^{l,i}_l$
      $\alpha^i_l = -\frac{(\boldsymbol{w}^{i-1}_r)\boldsymbol{\Phi}_{rl}\boldsymbol{w}^i_l + \lambda^i_l (\boldsymbol{w}^{i-1}_r)^\dagger \tilde{\boldsymbol{h}}^l_r}{(\boldsymbol{w}^{i-1}_r)^\dagger \boldsymbol{\Phi}_{rr} \boldsymbol{w}^{i-1}_r}$
      $\boldsymbol{w}^i_l = \frac{\boldsymbol{w}^i_l}{(\tilde{\boldsymbol{h}}^l_l)^\dagger \boldsymbol{w}^i_l}$
    end
    $d^i_l = (\boldsymbol{w}^i_l)^\dagger \boldsymbol{z}_l$
    $y^i_l = (\boldsymbol{w}^i_l)^\dagger \boldsymbol{z}_l + (\alpha^i_l)^* d^{i-1}_r$
    Transmit d^i_l to the right side
    $\lambda^i_r = \left(-\|\bar{\boldsymbol{h}}^{r,i}_r\|^2_{(\bar{\boldsymbol{\Phi}}^i_{rr})^{-1}} - \frac{\|(\tilde{\boldsymbol{h}}^r_l)^\dagger \boldsymbol{w}^i_l\|^2}{(\boldsymbol{w}^i_l)^\dagger \boldsymbol{\Phi}_{ll} \boldsymbol{w}^i_l}\right)^{-1}$
    $\boldsymbol{w}^i_r = \lambda^i_r (\bar{\boldsymbol{\Phi}}^i_{rr})^{-1} \bar{\boldsymbol{h}}^{r,i}_r$
    $\alpha^i_r = -\frac{(\boldsymbol{w}^i_l)\boldsymbol{\Phi}_{lr}\boldsymbol{w}^i_r + \lambda^i_r (\boldsymbol{w}^i_l)^\dagger \tilde{\boldsymbol{h}}^r_l}{(\boldsymbol{w}^i_l)^\dagger \boldsymbol{\Phi}_{ll} \boldsymbol{w}^i_l}$
    $\boldsymbol{w}^i_r = \frac{\boldsymbol{w}^i_r}{(\tilde{\boldsymbol{h}}^r_r)^\dagger \boldsymbol{w}^i_r}$
    $d^i_r = (\boldsymbol{w}^i_r)^\dagger \boldsymbol{z}_r$
    $y^i_r = (\boldsymbol{w}^i_r)^\dagger \boldsymbol{z}_r + (\alpha^i_r)^* d^i_l$
    Transmit d^i_r to the left side
  end
end

Algorithm 1: Distributed binaural MVDR

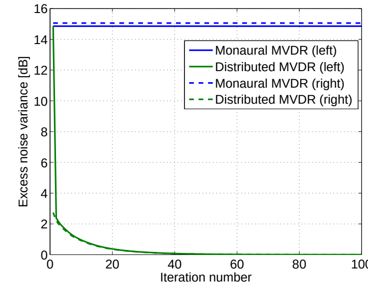
VI. Experimental Study

The proposed algorithm was evaluated using both narrow-band stationary signals and speech signals in a reverberant room. The proposed method was compared with the closed-form binaural MVDR, and the closed-form monaural MVDR. The narrow-band scenario is comprised of 1 desired source and 2 interfering sources received by 2 sub-arrays each comprised of 2 sensors with random transfer function (TF)s. The sources are generated directly at the STFT domain as complex Gaussian random variables uncorrelated between time frames. A spatially white sensor noise is added to the received signals. The signal to interference ratio (SIR) and signal to noise ratio (SNR) were set to 0dB and 20dB, respectively. The various correlation matrices were assumed to be known. In Fig. 2(a) a comparison between the noise variance of the proposed distributed MVDR and the monaural MVDR normalized by the noise variance of the binaural MVDR is shown for the left and right lateral. 50 Monte-carlo trials were used to produce the graphs. The noise variance is plotted vs. the iteration number. It is clear from the figure that the proposed algorithm converges to the binaural MVDR.

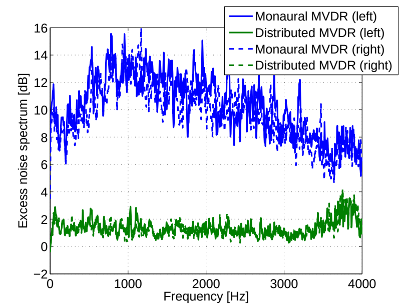
The wide-band speech scenario is comprised of 1 desired speaker and 2 interference signals received by 2 sub-arrays comprised of 2 microphones in a simulated $4 \times 4 \times 3\text{m}^3$ room environment, with a reverberation time set to 150ms. The SIR and SNR were set to 15dB and 60dB respectively. The 2 sub-arrays were oriented in parallel. The inter sub-array distance was set to 17cm. The distance between the 2 microphones at each sub-array was set to 5cm. The signals were sampled at 8KHz and were transformed into the STFT domain with 2048 points and 75% overlap. The various algorithms operated in each frequency bin independently. The signals were filtered in the time-domain[1]. The noise and speech correlation matrices were estimated in noise-only and in speech and noise segments, respectively, using a simple energy threshold voice activity detector (VAD). The desired signal RTF was estimated by selecting the major eigenvector of the GEVD of the received signals correlation matrix and the noise-only correlation matrix. The average noise PSD at the outputs of the proposed distributed algorithm and the monaural MVDR algorithms normalized by the noise PSD at the output of the binaural MVDR is depicted in Fig. 2(b). The graphs were obtained by averaging 20 Monte-Carlo experiments. The number of iterations of the distributed MVDR algorithm was set to 10. Due to estimation errors and filter windowing the noise PSD of the proposed distributed MVDR is 1.5dB higher than the noise PSD of the binaural MVDR. In Fig. 3 the distributed MVDR is compared with the binaural and monaural MVDR by subjective assessment of speech sonograms. It is clearly seen that the proposed distributed binaural MVDR outperforms the monaural MVDR, and that its performance is equivalent to the binaural MVDR.

VII. Conclusions

A novel distributed MVDR beamformer has been introduced. The proposed method reduces the energy consumption by requiring a single transmission channel between two sub-arrays, regardless of the number of microphones. The algorithm is proved to converge to the optimal binaural MVDR when the desired source correlation matrix has a rank-1. The algorithm is applied to the binaural hearing aid problem. The experimental study demonstrates the superior performance of the proposed algorithm in comparison with the monaural MVDR. The convergence to the binaural MVDR is analytically proven for the narrow-band case. A time recursive version of the algorithm can be obtained by using a recursive estimation of the involved correlation matrices.

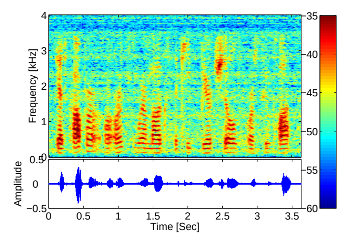


(a) Noise variance normalized by the noise variance at the output of the binaural MVDR

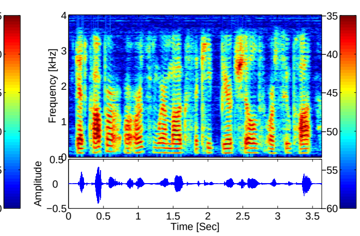


(b) Noise PSD normalized by the noise PSD at the output of the binaural MVDR

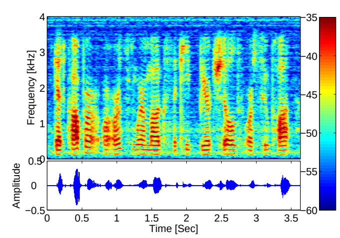
Fig. 2. Excess noise variance in narrow-band scenario and excess noise PSD in wide-band scenario



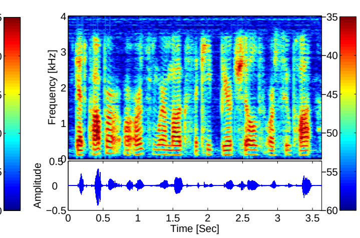
(a) Microphone number 1



(b) Binaural MVDR



(c) Monaural MVDR



(d) Distributed MVDR

Fig. 3. Sonograms of left signals in an example scenario

References

[1] S. Gannot and I. Cohen, "Adaptive Beamforming and Postfiltering," in Jacob Benesty, M. Mohan Sondhi and Yiteng (Arden) Huang (Eds.), *Springer Handbook of Speech Processing*, Springer 2007, Part H, Ch. 47, pp. 945-978.

[1]The noise power spectral density (PSD) and the ATFs were assumed to be time-invariant, hence the MVDR beamformers were also time-invariant.

[2] D. Culler, D. Estrin, and M. Srivastava, "Overview of sensor networks," *Computer*, vol. 37, no. 8, pp. 41–49, Aug. 2004.

[3] S. Doclo, M. Moonen, T. Van den Bogaert, and J. Wouters, "Reduced-bandwidth and distributed MWF-based noise reduction algorithms for binaural hearing aids," *IEEE Trans. Audio, Speech and Language Processing*, vol. 17, no. 1, pp. 38–51, Jan. 2009.

[4] A. Bertrand and M. Moonen, "Distributed adaptive node-specific signal estimation in fully connected sensor networks - Part I: sequential node updating ," accepted for publication in *IEEE Trans. Signal Processing*.