BAYESIAN FOCUSING TRANSFORMATION FOR COHERENT WIDEBAND ARRAY PROCESSING

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ABSTRACT

In this paper, we develop a Bayesian Focusing Transformation (BFT) for wideband array processing which utilizes a weighted extension of the Wavefield Interpolated Narrowband Generated Subspace (WINGS) focusing approach, also derived here. The BFT minimizes the mean-square error of the transformation, thus, achieving improved focusing accuracy over the entire bandwidth. We also propose a time progressing Bayesian focused beamformer which incorporates a direction finding stage. Experimental results demonstrate the performance of the BFT compared to other focusing schemes and show some of the benefits achieved by the BFT in adaptive beamforming applications.

***Index Terms*—** MVDR, Bayesian approach, adaptive beamforming, direction finding

1. INTRODUCTION

Wideband adaptive beamforming techniques can be classified into two main categories. The first category is the non-coherent category which employs either time domain or frequency domain techniques. The non-coherent time domain techniques utilize spatial adaptive filters whose coefficients are adjusted to suppress the interferences while recovering the desired signal [2]. The non-coherent frequency domain techniques implement a narrowband beamformer in each frequency bin. All the methods associated with the non-coherent category are computationally expensive, have slow convergence rate due to a large number of adaptive coefficients, and are prone to signal cancellation problem in coherent source scenarios. The second category is the coherent category for wideband adaptive beamforming which incorporates a focusing procedure for signal subspace alignment [5]. This procedure involves a pre-processor implemented as a linear transformation matrix which focuses the signal subspaces at different frequencies to a single frequency, followed by a time domain narrowband beamformer. The main benefits of the coherent category are low computational complexity, the ability to combat the signal cancellation problem and improved convergence properties.

There are two basic approaches to design focusing matrices. The first approach is based on minimizing the transformation error at the Directions of Arrival (DOAs). This method requires preliminary estimates of DOAs [5], which is a drawback. The second approach is based on spatial interpolation schemes, and does not require initial DOA estimates [6]. However it requires the array to satisfy a spatial interpolation rule.

Recently, Doron et al. [3], presented a new focusing transformation based on the wavefield modeling theory [4] which does not require any preliminary DOA estimates and may be applied to any array with a known arbitrary geometry. Using the new focusing matrix, a virtual Wavefield Interpolated Narrowband Generated Subspace (WINGS) array can be constructed. The WINGS data has a narrowband array manifold while preserving the wideband spectral content of the wideband signals, allowing the use of a narrowband beamformer.

In this paper, we propose a third approach, namely a Bayesian approach for focusing transformation design. In this approach we bring into account the uncertainty of the DOAs by modeling them as random variables with prior statistics.We derive a Bayesian Focusing Transformation (BFT) minimizing the mean-square error (MSE) of the transformation, thus achieving improved focusing accuracy over the entire bandwidth. The BFT utilizes a weighted extension of the WINGS, to be also developed here. The proposed focusing transformation is a compromise between the first focusing approach, which requires preliminary DOAs estimates, and the spatial interpolation based second approach, which does not require any DOAs estimates.

The paper is organized as follows. In Section 2, we formulate the problem of interest. In Section 3, the BFT is derived. Section 4 presents a short overview of the WINGS and a weighted extension is suggested which helps in the derivation of the BFT. In Section 5, the focused MVDR beamformer is presented. In Section 6, a time progressing algorithm is proposed which incorporates a Direction Finding (DF) stage operating on the focused date followed by the Bayesian focused beamformer. In section 7, we consider a simple case of a linear array and demonstrate some of the benefits achieved by the BFT. Section 8 concludes our work.

2. PROBLEM FORMULATION

Consider an arbitrary array of N sensors sampling a wavefield generated by P statistically independent wideband sources, in the presence of additive noise. For simplicity, we confine our discussion to the free and far field model. The signal measured at the output of the *n*th sensor can be written as

$$x_n(t) = \sum_{p=1}^{P} s_p(t - \tau_{np}) + n_n(t),\ n = 1, .., N, \quad (1)$$

where $\{s_p(t)\}_{p=1}^{P}$ and $\{n_n(t)\}_{n=1}^{N}$ denote the radiated wideband signals and the additive noise processes, respectively. The parameters $\{\tau_{np}\}$ are the delays associated with the signal propagation time from the *p*th source to the *n*th sensor. Let $\{\gamma_i\}_{i=1}^{P}$ be the DOAs of the sources. $\gamma \equiv \theta$ in 2-D and $\gamma \equiv (\theta, \varphi)$ in 3-D where θ in the azimuth angle and φ is the elevation angle. For simplicity, we deal with the 2-D case. Each T seconds of the received data are transformed to frequency domain and divided into K snapshots yielding the following matrix equation

$$\begin{aligned} \mathbf{x}_k(w_j) &= \mathbf{A}_{\boldsymbol{\theta}}(w_j)\mathbf{s}_k(w_j) + \mathbf{n}_k(w_j), \\ j &= 1, 2, ...J,\ k = 1, 2, ...K \end{aligned} \quad (2)$$

where $\mathbf{x}_k(w_j)$, $\mathbf{s}_k(w_j)$ and $\mathbf{n}_k(w_j)$ denote vectors whose elements are the discrete Fourier coefficients of the measurements, of the unknown sources signals and of the noise, respectively at the *k*th subinterval and frequency w_j. $\mathbf{A}_{\boldsymbol{\theta}}(w_j)$ is the $N \times P$ direction matrix

$$\mathbf{A}_{\boldsymbol{\theta}}(w_j) \equiv [\mathbf{a}_{\theta_1}(w_j), \mathbf{a}_{\theta_2}(w_j), ..., \mathbf{a}_{\theta_P}(w_j)]. \quad (3)$$

The vector $\mathbf{a}_\theta(w)$, referred to as the *array manifold* vector, is the response of the array to an incident plane wave at frequency w and DOA θ. We assume that the noise vectors $\mathbf{n}_k(w_j)$ are independent samples of stationary, zero mean Gaussian random process, with unknown covariance $\sigma_j^2\mathbf{I}$. The signal vectors $\mathbf{s}_k(w_j)$ are independent samples of stationary , zero mean Gaussian random process which were fed into auto regressive filter.The noise process is assumed uncorrelated with the signal process. The Bayesian approach employs statistical model where the DOAs, $\{\theta_i\}_{i=1}^{P}$ are statistically independent random variables.

3. BAYESIAN FOCUSING TRANSFORMATION

Our goal is to derive a linear focusing transformation $T(w_j)$ optimal in the MMSE sense, i.e. find $T(w_j)$ which minimizes the following expectation

$$\begin{aligned} T_{MMSE}(w_j) &= \underset{\mathbf{T}(w_j)}{\arg\min} \Big\{ E_{\boldsymbol{\theta}|\mathbf{Y}} \Big\{ \\ &\quad \|\mathbf{A}_{\boldsymbol{\theta}}(w_0) - \mathbf{T}(w_j)\mathbf{A}_{\boldsymbol{\theta}}(w_j)\|_F^2 \mid \mathbf{Y} \Big\}\Big\}, \end{aligned} \quad (4)$$

where w_0 is the focusing frequency, $\mathbf{Y}$ is the observed data and $\|\cdot\|_F$ denotes the Frobenious norm. It can be shown that

$$\begin{aligned} & E_{\boldsymbol{\theta}|\mathbf{Y}} \Big\{ \|\mathbf{A}_{\boldsymbol{\theta}}(w_0) - \mathbf{T}(w_j)\mathbf{A}_{\boldsymbol{\theta}}(w_j)\|_F^2 \mid \mathbf{Y} \Big\} \quad (5) \\ = & \int_{\Gamma = -\pi}^{\pi} d\theta \, \|\mathbf{a}_\theta(w_0) - \mathbf{T}(w_j)\mathbf{a}_\theta(w_j)\|^2 \sum_{i=1}^{P} f_{\theta_i|\mathbf{Y}}(\theta|\, \mathbf{Y}) . \end{aligned}$$

$\|\cdot\|$ is the Euclidian norm and $f_{\theta_i|\mathbf{Y}}(\theta|\, \mathbf{Y})$ are the conditional PDFs of the DOAs. Defining

$$\rho^2(\theta) \triangleq \sum_{i=1}^{P} f_{\theta_i|\mathbf{Y}}(\theta|\, \mathbf{Y}), \quad (6)$$

and substituting (6) into right-hand side of (5) yields the following integral to be minimized

$$\begin{aligned} T_{MMSE}(w_j) &= \underset{\mathbf{T}(w_j)}{\arg\min} \int_{\Gamma} \quad (7) \\ &\quad d\theta \, \|\rho(\theta)(\mathbf{a}_\theta(w_0) - \mathbf{T}(w_j)\mathbf{a}_\theta(w_j))\|^2 . \end{aligned}$$

In the following section, we derive a closed form solution of (7). First, a brief overview of the WINGS is presented and then a weighted extension is derived. The formalism to be used during this development will help to solve (7).

4. WINGS FOCUSING APPROACH

In this section, we review the main points of the WINGS, and develop a weighted extension which incorporates an angular weighting function.

4.1. WINGS

The WINGS focusing approach [3] is based on the wavefield modeling theory [4] according to which, the output of almost any array **x(w)** of arbitrary geometry can be written as a product of array geometry dependent part and wavefield dependent part, i.e. $\mathbf{x}(w) = \mathbf{G}(w)\boldsymbol{\psi}(w)$ where $\mathbf{G}(w)$ is a sampling matrix which is independent of the wavefield and the coefficient vector $\boldsymbol{\psi}(w)$ is independent of the array. By the wavefield modeling formalism, the steering vector can be expressed by terms of orthogonal decomposition

$$\mathbf{a}_\theta(w) = \sum_n \mathbf{g}_n(w) h_n^*(\theta), \quad (8)$$

where $\mathbf{g}_n(w)$ are the columns of the sampling matrix $\mathbf{G}(w)$ and $\{h_n(\theta)\}$ is an orthogonal basis set in $\mathbf{L}_2(\Gamma)$. In 2-D, we use the Fourier basis, i.e. $h_n(\theta) = \frac{1}{\sqrt{2\pi}} e^{-in\theta}$. In the WINGS approach it is desired to find $\mathbf{T}(w_j)$ which satisfies the following equation

$$\mathbf{a}_\theta(w_0) = \mathbf{T}(w_j)\mathbf{a}_\theta(w_j) + \mathbf{e}_\theta(w_j), \ \ \forall\theta \quad (9)$$

Using (8), the error term can be expressed as

$$\mathbf{e}_\theta(w_j) = [\mathbf{G}(w_0) - \mathbf{T}(w_j)\mathbf{G}(w_j)]\,\mathbf{b}_\theta, \tag{10}$$

where the vector $\mathbf{b}_\theta$ contains the basis functions $\{h_n(\theta)\}$ as its elements. Let ε_j be defined as the L_2 norm of the error

$$\varepsilon_j^2 \triangleq \frac{1}{N}\int_\Gamma d\theta\,\|\mathbf{e}_\theta(w_j)\|^2. \tag{11}$$

The elements of $\mathbf{b}_\theta$ comprise a complete and orthogonal basis set over $\mathbf{L}_2(\Gamma)$, thus, one may consider (10) to be the orthogonal decomposition of the error vector $\mathbf{e}_\theta(w_j)$. We can use Parseval's identity and derive the following Least-Square(LS) minimization problem

$$\varepsilon_j^2 = \frac{1}{N}\|\mathbf{G}(w_0) - \mathbf{T}(w_j)\mathbf{G}(w_j)\|_F^2. \tag{12}$$

(12) has a well known solution

$$\mathbf{T}(w_j) = \mathbf{G}(w_0)\mathbf{G}^\dagger(w_j), \tag{13}$$

where $\mathbf{G}^\dagger(w_j)$ denotes the pseudo-inverse of $\mathbf{G}(w_j)$. For more additional details see [3] and [4].

4.2. Weighted WINGS

We now extend the WINGS transformation (13) to incorporate an angular weighting function $\rho(\theta)$, which may be used to enhance the LS fit of the array manifold within a pre-selected angular region

$$\begin{aligned}\min_{\mathbf{T}(w_j)} \varepsilon_j^2 &= \frac{1}{N}\int_{\boldsymbol{\Gamma}} d\theta\,\|\rho(\theta)\mathbf{e}_\theta(w_j)\|_F^2 \qquad (14)\\ &= \frac{1}{N}\int_{\boldsymbol{\Gamma}} d\theta\,\|\rho(\theta)(\mathbf{a}_\theta(w_0) - \mathbf{T}(w_j)\mathbf{a}_{\boldsymbol{\theta}}(w_j))\|^2.\end{aligned}$$

Note that this generalized form includes many focusing matrices as private cases. It reduces to the previously described WINGS by taking $\rho(\theta) \equiv 1$. Taking $\rho(\theta) = \sum_i \delta(\theta - \hat{\theta}_i)$ yields the focusing matrices originally proposed for wideband DOA estimation in the pioneering work of [5] which focuses at the preliminary estimates of the DOAs $\left\{\hat{\theta}_i\right\}$.

In order to solve (14) let us find $\mathbf{C}(w)$, the orthogonal decomposition of the product $\rho(\theta)\mathbf{a}_\theta(w)$

$$[\mathbf{C}(w)]_{mn} \equiv \int_\Gamma d\theta\rho(\theta)\,[(\mathbf{a}_\theta(w)]_m\,h_n(\theta). \tag{15}$$

Let $\rho(\theta) = \sum_n \rho_n h_n(\theta)$ be the orthogonal decomposition of the angular weighting function $\rho(\theta)$, then inserting (8) we may write

$$\begin{aligned}[\mathbf{C}(w)]_{mn} &= \int_\Gamma d\theta \sum_p \rho_p h_p(\theta) \sum_l \mathbf{G}_{ml}(w) h_l^*(\theta) h_n(\theta)\\ &= \sum_{p,l} \rho_p \mathbf{G}_{ml}(w) \int_\Gamma d\theta h_n(\theta) h_p(\theta) h_l^*(\theta). \qquad (16)\end{aligned}$$

In 2-D

$$\int_\Gamma d\theta h_n(\theta) h_p(\theta) h_l^*(\theta) = \frac{1}{\sqrt{2\pi}}\delta_{n+p-1}, \tag{17}$$

which yields, for the 2-D case

$$[\mathbf{C}(w)]_{mn} = \frac{1}{\sqrt{2\pi}}\sum_p \rho_p \mathbf{G}_{m,n+p}(w). \tag{18}$$

We now insert into (14) the orthogonal decomposition $\rho(\theta)\mathbf{a}_\theta(w) = \mathbf{C}(w)\mathbf{b}_\theta$ and get the following minimization integral

$$\varepsilon_j^2 = \frac{1}{N}\int_\Gamma d\theta\,\|[\mathbf{C}(w_0) - \mathbf{T}(w_j)\mathbf{C}(w_j)]\mathbf{b}_{\boldsymbol{\theta}}\|^2. \tag{19}$$

Now we can use Parseval's identity

$$\varepsilon_j^2 = \frac{1}{N}\|[\mathbf{C}(w_0) - \mathbf{T}(w_j)\mathbf{C}(w_j)]\|_F^2. \tag{20}$$

(20) is a LS problem with the following solution

$$\mathbf{T}(w_j) = \mathbf{C}(w_0)\mathbf{C}^\dagger(w_j). \tag{21}$$

From the above derivation one can easily see that (14) has exactly the same form like (7). Now, we can easily derive the closed form expression for the MMSE Bayesian focusing transformation:

$$\mathbf{T}_{MMSE}(w_j) = \mathbf{C}(w_0, \rho(\theta))\mathbf{C}^\dagger(w_j, \rho(\theta)) \tag{22}$$

where $\rho(\theta)$ is given by (6).

5. MVDR FOCUSED BEAMFORMER

In this section, we describe a framework example of the BFT for the MVDR adaptive beamformer using the SMI implementation. The MVDR - SMI method is implemented for the wideband case in the frequency domain, i.e. it is implemented as a narrowband beamformer at each frequency bin (see e.g. [1]). The DFT implementation of the frequency domain MVDR-SMI beamformer is based on estimation the narrowband sample covariance matrix at each frequency bin

$$\hat{\mathbf{R}}(w_j) = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k(w_j)\mathbf{x}_k^H(w_j). \tag{23}$$

The narrowband MVDR - SMI adaptive weight vector is computed at each frequency bin as

$$\hat{\mathbf{w}}_\theta(w_j) = \frac{\hat{\mathbf{R}}^{-1}(w_j)\mathbf{a}_\theta(w_j)}{\mathbf{a}_\theta^H(w_j)\hat{\mathbf{R}}^{-1}(w_j)\mathbf{a}_\theta(w_j)}. \tag{24}$$

The MVDR-SMI focused adaptive beamformer may be simply implemented as a narrowband adaptive beamformer operating on the temporal focused data vector

$$\begin{aligned}\mathbf{y}_k(n) &= \sum_{j=1}^{J}\mathbf{T}(w_j)\mathbf{x}_k(w_j)e^{iw_jnT_s} \\ &\cong \mathbf{A}_\gamma(w_0)\mathbf{s}(n)+\mathbf{n}_{trans}(n), \qquad (25)\end{aligned}$$

where $\mathbf{s}(n)$ is the temporal vector of wideband unknown source signals within the focused frequency band $[w_1 : w_J]$, T_s is the sampling time interval and $\mathbf{n}_{trans}(n)$ is the transformed noise. The sample covariance matrix of the focused vector is estimated by

$$\hat{\mathbf{R}}_{\text{focused}} = \frac{1}{KJ}\sum_{k,n}\mathbf{y}_k(n)\mathbf{y}_k^H(n). \qquad (26)$$

Now, the focused coherent adaptive beamformer MVDR weight vector is simply computed in the time domain by

$$\hat{\mathbf{w}}_{\theta,wings} = \frac{\hat{\mathbf{R}}_{\text{focused}}^{-1}\mathbf{a}_\theta(w_0)}{\mathbf{a}_\theta^H(w_0)\hat{\mathbf{R}}_{\text{focused}}^{-1}\mathbf{a}_\theta(w_0)}, \qquad (27)$$

where w_0 is the focusing frequency. For more details, see [1] and [3].

6. TIME PROGRESSING ALGORITHM

In this section we present a time progressing algorithm which is based on the proposed Bayesian focusing approach. The BFT assumes that the aposteriori PDFs of the DOAs are available, however, in practice we need to estimate these PDFs. Let us assume a Gaussian model for the DOAs, so, we have to estimate the first two moments of each DOA. The conditional mean of θ_i is approximated by $\hat{\theta}_{i\_DF}$ which is the estimate of the DF algorithm. The standard deviation is taken to be half the beam width of the array. A block diagram of the proposed algorithm is given in figure 1. Each T seconds of data is divided into K snapshots, on which, a Bayesian focusing transformation is applied and yields the focused vector. The design of the focusing transformation uses the estimated aposteriori PDFs from the previous T seconds, while in the first T seconds, the algorithm uses $\rho(\theta) \equiv 1$. The focused temporal vectors $\{\mathbf{y}_k\}_{k=1}^{K}$are used as inputs to the focused MVDR beamformer and for updating the estimation of the conditional PDFs.

7. EXPERIMENTAL RESULTS

In this section, we present a simulation example comparing the performance of the BFT to that of other methods. We consider P= 6 statistically independent wideband sources propagating towards a linear array of N=10 sensors. The spacing between two successive elements is $d = \frac{\lambda_{\min}}{2}$, where $\lambda_{\min}$ corresponds to the highest frequency of the bandwidth. The true DOA vector is $\boldsymbol{\theta} = \{50°, 70°, 93°, 100°, 119°, 135°\}$ where $90°$ is the broadside direction. The bandwidth of the sources is $60Hz$ taken around f_c=$100Hz$. We used the MUSIC algorithm [7] for DOA estimation. The desired signal is assumed to be the signal from $100°$. The Signal to Interference Ratio (SIR) is $9dB$, and the Signal to Noise Ratio(SNR) is $0dB$.

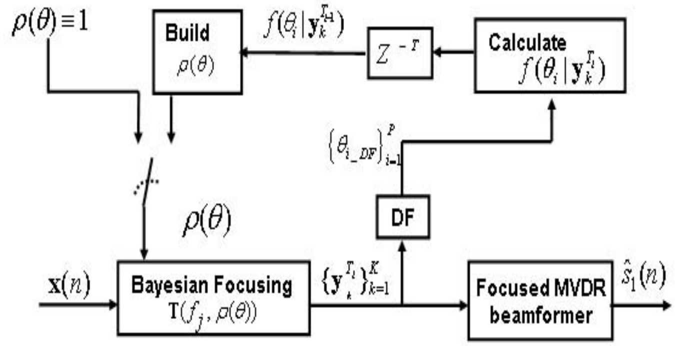

Fig. 1. Block diagram of the Bayesian focused MVDR beamformer time progressing algorithm.

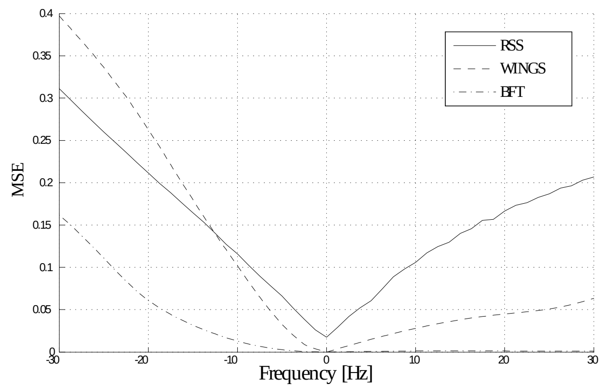

Fig. 2. MSE transformation curves as a function of frequency in base band.

Each $T = 10$ seconds of the received data vector were converted to baseband, divided into $K = 16$ snapshots and transformed to the frequency domain. We compare 4 different methods implementing wideband adaptive beamforming, the RSS [5] - which requires preliminary DOAs estimates, WINGS, and the BFT - are all coherent methods. The forth method is the non coherent adaptive beamformer. The results were obtained by averaging over 500 independent runs.

Figure 2 shows the frequency dependent MSE of the focusing transformation for the various coherent methods. The MSE is a summation of the errors over the true DOAs. It can be clearly seen that the Bayesian focusing transformation

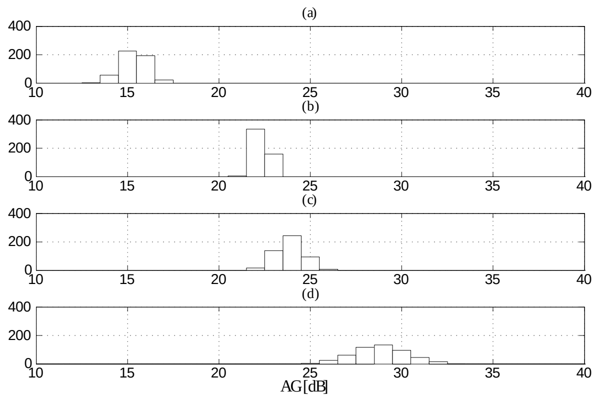

Fig. 3. Histogram of the array gain acheived by the various approaches. (a) Non coherent processing. (b) RSS. (c) WINGS. (d) BFT

achieves the lowest error along the entire bandwidth.

Figure 3 shows a histogram of the array gain (AG) values for the various methods. The AG is defined by

$$AG = \frac{SIR_{out}}{SIR_{in}}, \tag{28}$$

where SIR_{out} and SIR_{in} are the signal to interference ratios at the beamformer output and input, respectively. The lowest AG is that of the non-coherent processing, since it has a large number of beamformer coefficients to adjust. The AG achieved by the BFT is roughly $29dB$ while that of the WINGS and RSS is only about $23dB$. The superior performance of the BFT is expected due to the fact that the BFT has the lowest error across the entire bandwidth, yeilding more accurate focused data.

8. CONCLUSIONS

The proposed Bayesian approach takes into account the uncertainty in the DOAs during the focusing process. We have developed closed form expression for the BFT. To this end we have derived a weighted extension to the WINGS focusing transformation. The BFT assumes that the PDFs of the DOAs are known and yields an optimized MSE focusing transformation. We also proposed a time progressing algorithm which employs a DF algorithm and our Bayesian focused beamformer. We demonstrated that the Bayesian approach achieves lower transformation error over the entire bandwidth, thus yielding a high quality focused vector relative to other checked focusing approaches. The AG achieved by each of the considered methods illustrates the benefits of the BFT relative to over focusing methods and also demonstrates the advantage of the coherent methods over non-coherent processing.

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