EIGENVALUE DECOMPOSITION BASED ESTIMATORS OF CARRIER FREQUENCY OFFSET IN MULTICARRIER UNDERWATER ACOUSTIC COMMUNICATION

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ABSTRACT

We propose computationally efficient carrier frequency offset estimators for multicarrier underwater acoustic communication using identical pilot tones equi-spaced in the frequency domain. The first estimator uses the phase of the maximal eigenvector of a channel-dependent correlation matrix. Next, the phase of the minimal eigenvector of a channel-independent correlation matrix is combined with the first estimation using a weighted linear least squares principle. The third estimator solves a generalized eigenvalue decomposition problem by jointly considering the two correlation matrices, and then performs a similar second step as the previous estimator. Simulations and pool trials show that the proposed estimators achieve similar performance as common estimation techniques while surpassing them in severe environments.

Index Terms— multicarrier communication, carrier frequency offset, underwater acoustic communication

1. INTRODUCTION

Carrier frequency offset (CFO) in orthogonal frequency division multiplexing (OFDM) communication systems may cause inter-carrier interference and degrades the performance of the OFDM decoder [1, 2, 6, 7, 8, 9]. Contrary to the radio communication channel, the time variations of the underwater acoustic communication (UAC) are non negligible and result in non uniform Doppler shifts [3]. There are two common CFO estimators in UAC OFDM using block-by-block processing. Li et al. introduced [3] a nonlinear least-squares (LS) CFO estimate using equi-spaced pilot symbols in each block, which are also used to estimate the channel impulse response (CIR). The authors also employed the LS principle with null symbols in the frequency domain. Both methods require a grid search in the frequency domain. A block-to-block processing approach for UAC OFDM systems was suggested by Carracosa and Stojanovic [4], where the non-uniform phase offset is tracked from one block to the subsequent block.

Recently, low complexity CFO estimators for OFDM in UAC channel have been suggested [5] that replace the need for exhaustive grid search. Using identical pilot tones equi-spaced in the frequency domain results in a periodic time-domain block signal with a period equal to the number of pilot tones (the issue of the peak to average power ratio of this type of signaling is thoroughly discussed in [5]). After retaining the channel-independent part of each periodic segment, a small-sized correlation matrix between these parts is constructed. One of the estimators proposed in [5] is based on determining the eigenvector associated with the minimal eigenvalue of this channel-independent correlation matrix, and then the CFO is obtained using the LS estimator given the phases of this eigenvector.

Herein, we propose CFO estimators which not only use a channel-independent correlation matrix [5], but also a channel-dependent correlation matrix constructed from correlating the channel-dependent parts of the segments of the periodic time domain block signal. The first estimator is based solely on this channel-dependent correlation matrix. We first determine the eigenvector associated with the maximal eigenvalue of this matrix, and then the CFO is estimated using a linear LS estimator given the phases of this eigenvector. The second estimator is a weighted linear LS estimator given the phases of the minimal eigenvector of the channel independent correlation matrix and the maximal eigenvector of the channel dependent correlation matrix. Finally, the third estimator solves a generalized eigenvalue decomposition (GEVD) problem by considering jointly the two correlation matrices, and then performing a similar second step as the previous estimators. Numerical simulations indicate that the root mean square error (RMSE) performance of the GEVD estimator is superior to that of the other estimators for low signal to noise ratio (SNR) and in various underwater channels. Water tank experiments further show that the proposed estimators outperform previously suggested CFO estimators.

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2. SIGNAL MODEL

Consider a zero-padded OFDM block with time duration T and K carriers, where the kth carrier frequency is $f(k) = f_0 + k\Delta f$, $k = 0, \ldots, K-1$ and $\Delta f = 1/T$ is the carrier spacing. The $K \times 1$ vector of symbols is denoted by $\mathbf{s}$ with its kth element defined by $s(k) = e^{j\phi(k)}$, meaning that a phase modulation is being used. Hereon we use the QPSK constellation. We use Q pilot symbols, equi-spaced in frequency with spacing $G = K/Q$, i.e., the frequency of the qth pilot carrier is $f(qG)$, $q = 0, \ldots, Q-1$. The $P \times 1$ zero-padded discrete-time transmitted signal, where $P = K + L$, is $\mathbf{r} = \mathbf{T}_{zp}\mathbf{F}_K^H\mathbf{s}$, where $\mathbf{F}_K$ is a $K \times K$ Fourier matrix with the (m, n)th element given by $\frac{1}{\sqrt{K}}e^{-j2\pi/K \cdot mn}$, $\mathbf{T}_{zp} = [\mathbf{I}_K, \mathbf{0}_K\mathbf{0}_L^T]^T$ is a $P \times K$ zero-padding matrix, $\mathbf{I}_n$ is the $n \times n$ identity matrix, $\mathbf{0}_n$ is a $n \times 1$ vector of zero elements, and L is the length of the zero-padding. The unknown discrete-time baseband CIR representing a multipath channel is described by the $L \times 1$ vector $\mathbf{h} = [h(0), \ldots, h(L-1)]^T$. After coarse Doppler shift compensation, the $P \times 1$ received vector $\mathbf{y}$ that still consists of a frequency independent residual Doppler shift component is [3, 12, 5]

$$\mathbf{y} = \mathbf{\Gamma}_K(\epsilon_0)\mathbf{H}\mathbf{T}_{zp}\mathbf{F}_K^H\mathbf{s} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{H}$ is a $P \times P$ Toeplitz matrix with first column and first row given by $[\mathbf{h}^T, \mathbf{0}_{P-L}^T]^T$ and $[h(0), \mathbf{0}_{P-1}^T]^T$, respectively, $\mathbf{n}$ is a $P \times 1$ noise vector modeled as a zero-mean circular complex white Gaussian with covariance matrix $\sigma_n^2\mathbf{I}_P$ and

$$\mathbf{\Gamma}_K(\epsilon_0) = \text{diag}(1, e^{j\frac{\epsilon_0}{K}}, \ldots, e^{j\frac{\epsilon_0}{K}(P-1)} \tag{2}$$

where ϵ_0 is the carrier frequency offset normalized by Δf. We note that when estimating the CIR using identical pilots, the last L elements of $\mathbf{y}$ hold irrelevant data and thus we simply discard them instead of using the overlap-and-add operation usually used in zero padded OFDM receivers.

The problem we discuss is briefly expressed as follows: Given the received signal, $\mathbf{y}$, estimate the normalized CFO.

3. CHANNEL-DEPENDENT CFO ESTIMATOR

By using identical pilot tones, i.e. $s(qG) = u$, where u is one of the possible symbols, the time domain signal is [5]

$$\begin{aligned}\tilde{s}(n) &= \sum_{q=0}^{Q-1}\frac{1}{\sqrt{K}}\underbrace{s(qG)}_{=u}e^{j\frac{2\pi}{Q}nq} + \underbrace{\sum_{k\in\mathcal{S}_D}\frac{1}{\sqrt{K}}s(k)e^{j\frac{2\pi}{K}nk}}_{\triangleq\eta(n)} \\ &= \frac{Qu}{\sqrt{K}}\delta[n \bmod Q] + \eta(n)\,, n = 0, \ldots, K-1 \end{aligned}\tag{3}$$

where $\mathcal{S}_D$ is the set of indices of the data symbols and $\delta[\cdot]$ is the delta function. The signal contains G peaks with absolute values equal to $Q/\sqrt{K}$ at times $n = 0, Q, \ldots, K-Q$ in the presence of a noise-like term $\eta(n)$. For $K \gg Q$ this noise term is distributed as a zero mean Gaussian random variable with variance equals to $(K-Q)/K$. By assuming that Q is large enough so that $\eta(n)$ is negligible compared to $Qu/\sqrt{K}$, and by neglecting the additive noise term contribution in (1), we get, after substituting (3) into (1), that the vector $\mathbf{y}$ can be segmented into G segments where each contains Q samples, where the gth segment $\mathbf{y}_g = \mathbf{y}(1 + gQ : (g+1)Q)$, $g = 0, \ldots, G-1$, is given as

$$\mathbf{y}_g \cong \sqrt{Q}u\mathbf{\Gamma}_Q(\epsilon_0)\mathbf{h}_Q\alpha_g(\epsilon_0)\,, g = 0, \ldots, G-1 \tag{4}$$

where $\mathbf{\Gamma}_Q$ is obtained by taking the $Q \times Q$ top left block of $\mathbf{\Gamma}_K$, $\alpha_g(\epsilon_0) = 1/\sqrt{G}e^{-j\frac{2\pi}{G}g\epsilon_0}$ and $\mathbf{h}_Q = [\mathbf{h}, 0, \ldots, 0]^T$ is the $Q \times 1$ vector containing the CIR $\mathbf{h}$ in its first L samples. The last $Q-L$ samples of $\mathbf{y}_g$ do not contribute to the estimate, therefore we replace it by the $L \times 1$ vector,

$$\tilde{\mathbf{y}}_g \cong \sqrt{Q}u\mathbf{\Gamma}_L(\epsilon_0)\mathbf{h}\alpha_g(\epsilon_0) \tag{5}$$

where $\mathbf{\Gamma}_L(\epsilon_0)$ obtained by taking the $L \times L$ top left block of $\mathbf{\Gamma}_K$. By collecting all data vectors in (5) we arrive to the $GL \times 1$ measurement vector $\tilde{\mathbf{y}} = [\tilde{\mathbf{y}}_1^T, \ldots, \tilde{\mathbf{y}}_{G-1}^T]^T$ given as,

$$\tilde{\mathbf{y}} \cong \sqrt{Q}u(\boldsymbol{\alpha}(\epsilon_0) \otimes \mathbf{\Gamma}_L)\mathbf{h} \tag{6}$$

where $\otimes$ is the Kronecker product, and the $G \times 1$ unit-norm vector $\boldsymbol{\alpha}(\epsilon_0)$ is defined as

$$\boldsymbol{\alpha}(\epsilon_0) = \frac{1}{\sqrt{G}}[1, \ldots, e^{-j\frac{2\pi}{G}(G-1)\epsilon_0}]^T \tag{7}$$

At this point we can formulate an LS model. Given the measurement $\tilde{\mathbf{y}}$, we want to find the CIR, $\mathbf{h}$, and the CFO, ϵ, that minimize the cost function

$$L(\epsilon, \mathbf{h}) = \|\tilde{\mathbf{y}} - \sqrt{Q}u(\boldsymbol{\alpha}(\epsilon) \otimes \mathbf{\Gamma}_L)\mathbf{h}\|^2 \tag{8}$$

The CIR estimate is therefore $\hat{\mathbf{h}} = G(\boldsymbol{\alpha}(\epsilon_0) \otimes \mathbf{\Gamma}_L)^H\tilde{\mathbf{y}}$. Substituting it back into (8), after a few simple steps we find that the estimated CFO is the one that maximizes

$$\ell(\epsilon) = \boldsymbol{\alpha}^H(\epsilon)\hat{\mathbf{R}}_{CD}\boldsymbol{\alpha}(\epsilon) \tag{9}$$

where the $G \times G$ channel-dependent cross correlation matrix is

$$\hat{\mathbf{R}}_{CD} = \mathbf{Y}^H\mathbf{I}(1:L,:)^T\mathbf{I}(1:L,:)\mathbf{Y} \tag{10}$$

where $\mathbf{I}(1:L,:)$ is a $L \times Q$ matrix obtained by taking the L first rows of the $Q \times Q$ identity matrix, and the $Q \times G$ matrix $\mathbf{Y}$ containing all the G segments of the block signal is

$$\mathbf{Y} = [\mathbf{y}_0, \ldots, \mathbf{y}_{G-1}] \tag{11}$$

Instead of estimating the CFO by performing an exhaustive search over (9) we suggest to perform a two step estimation approach. First, we estimate the vector $\boldsymbol{\alpha}$ which maximizes (9). This vector is the eigenvector $\mathbf{u}_{\max}$ associated with the maximal eigenvalue of $\hat{\mathbf{R}}_{CD}$. Then, given the phases of this principal eigenvector, we can estimate the CFO using the following LS model

$$\arg(\mathbf{u}_{\max}(\hat{\mathbf{R}}_{CD})) \cong -\frac{2\pi}{G}\mathbf{g}\epsilon \tag{12}$$

where $\mathbf{g} = [0, 1, \ldots, G-1]^T$. The channel-dependent CFO estimate is

$$\hat{\epsilon} = -\frac{G}{2\pi\|\mathbf{g}\|^2}\arg(\mathbf{u}_{\max}^T(\hat{\mathbf{R}}_{CD}))\mathbf{g} \tag{13}$$

The computational complexity of this process is $\mathcal{O}(G^2L)$. Notice that the first L taps are selected due to their high pilot signal to noise ratio. For most fading channels, a smaller number than L will be sufficient.

4. LINEAR COMBINATION OF CFO ESTIMATES

In [5] it is shown that the CFO can be estimated in a complementary way to the previous estimate by looking for the vector $\boldsymbol{\alpha}$ that minimizes [5]

$$\ell^{'}(\boldsymbol{\alpha}) = \boldsymbol{\alpha}^H\hat{\mathbf{R}}_{CI}\boldsymbol{\alpha} \tag{14}$$

where the $G \times G$ channel-independent cross correlation matrix is

$$\hat{\mathbf{R}}_{CI} = \mathbf{Y}^H\mathbf{I}(L+1:Q,:)^T\mathbf{I}(L+1:Q,:)\mathbf{Y} \tag{15}$$

where $\mathbf{I}(L+1:Q,:)$ is defined similar to $\mathbf{I}(1:L,:)$. The vector $\boldsymbol{\alpha}$ which minimizes (14) is the eigenvector $\mathbf{u}_{\min}$ associated with the minimal eigenvalue of $\hat{\mathbf{R}}_{CI}$. The estimated CFO can then be solved using the phases of this eigenvector similarly to (13).

We suggest to use the phases of both $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$ to determine the CFO using a joint LS model given as

$$\begin{bmatrix} \arg[\mathbf{u}_{\max}(\hat{\mathbf{R}}_{CD})] \\ \arg[\mathbf{u}_{\min}(\hat{\mathbf{R}}_{CI})] \end{bmatrix} \cong \begin{bmatrix} -\frac{2\pi}{G}\mathbf{g} \\ -\frac{2\pi}{G}\mathbf{g} \end{bmatrix}\epsilon \tag{16}$$

The CFO is determined as the one that minimizes the following weighted LS optimization problem

$$\begin{aligned}\hat{\epsilon} \;\; = \;\; & \underset{\epsilon}{\operatorname{argmin}}\,\{\beta \cdot \|\arg[\mathbf{u}_{\min}(\hat{\mathbf{R}}_{CI})] + \frac{2\pi}{G}\mathbf{g}\epsilon\|^2 \\ & +(1-\beta)\cdot\|\arg[\mathbf{u}_{\max}(\hat{\mathbf{R}}_{CD})] + \frac{2\pi}{G}\mathbf{g}\epsilon\|^2\}\end{aligned} \tag{17}$$

Taking the derivative w.r.t. ϵ and equating the result to zero yields that the combined CFO estimate is

$$\begin{aligned}\hat{\epsilon} \;\; = \;\; & -\frac{G}{2\pi\|\mathbf{g}\|^2}(\beta\arg(\mathbf{u}_{\min}(\hat{\mathbf{R}}_{CI})) \\ & +(1-\beta)\arg(\mathbf{u}_{\max}(\hat{\mathbf{R}}_{CD})))^T\mathbf{g}\end{aligned} \tag{18}$$

where $0 \le \beta \le 1$. As might be expected, the combined estimator is an arithmetic average between both estimations. For $\beta = 0.5$ we get the LS solution. A natural selection of weights is $\beta = L/Q$, which accounts for the relative number of samples used for each estimate.

5. GENERALIZED EVD-BASED ESTIMATOR

The channel-dependent cost function attempts to find a vector $\boldsymbol{\alpha}$ that maximizes $\hat{\mathbf{R}}_{CD}$, whereas the channel-independent cost function attempts to find the same vector $\boldsymbol{\alpha}$ that minimizes the complementary matrix $\hat{\mathbf{R}}_{CI}$. Instead of solving two independent eigenvalue decomposition problems, we suggest to solve a single eigenvalue decomposition problem by considering jointly the channel-dependent data and the channel-independent data. In other words, a combined model can be proposed to maximize these quantities ratios. i.e. the optimization problem can be rephrased as a Rayleigh quotient

$$\ell^{''}(\boldsymbol{\alpha}) = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}}\,\frac{\boldsymbol{\alpha}^H\hat{\mathbf{R}}_{CD}\boldsymbol{\alpha}}{\boldsymbol{\alpha}^H\hat{\mathbf{R}}_{CI}\boldsymbol{\alpha}} \tag{19}$$

We thus encounter a well-known generalized eigenvalue problem, which is equivalent to the simple problem of finding the eigenvalue $\mathbf{u}_{\max}$ associated with the maximal eigenvalue of the matrix $\hat{\mathbf{R}}_{CI}^{-1}\hat{\mathbf{R}}_{CD}$. Solving an LS optimization problem as with the previous methods but now given the phases of $\mathbf{u}_{\max}(\hat{\mathbf{R}}_{CI}^{-1}\hat{\mathbf{R}}_{CD})$ yields that the generalized eigenvalue decomposition (GEVD) estimate of the CFO is

$$\hat{\epsilon} = -\frac{G}{2\pi\|\mathbf{g}\|^2}\arg(\mathbf{u}_{\max}(\hat{\mathbf{R}}_{CI}^{-1}\hat{\mathbf{R}}_{CD})^T)\mathbf{g} \tag{20}$$

which involves similar computation complexity as the previous estimates.

6. SIMULATIONS AND REAL DATA RESULTS

We evaluate the performance of the proposed estimates using numerical simulations and by examining the results of experiments in a water tank.

In the simulations we used OFDM blocks of $K = 2048$ QPSK symbols with $G = 8$ and bandwidth of $W = K\Delta f =$ 12.5 KHz. The normalized CFO was fixed to $\epsilon_0 = 0.2$. First, we considered a channel with $L = 100$ taps, corresponding to a delay spread of 8 ms with 15 paths of descending amplitudes. We evaluate the RMSE of the CFO using 5 estimates: 1) Channel dependent ($\beta = 0$); 2) Channel independent from [5] ($\beta = 1$); 3) Weighted LS with equal weights ($\beta = 0.5$); 4) Weighted LS ($\beta = L/Q$), and 5) GEVD-based. Fig. 1 shows the RMSE performance (the simulated CIR presented at the bottom is fixed for all trials) versus the SNR in terms of E_b/N_0. Each value represents statistics of 10,000 realizations of noise and data symbols. Clearly, the GEVD based estimator achieves the best results, outperforming the estimate proposed in [5] in low SNR. As expected, when applying the channel dependent estimate ($\beta = 0$), a noise floor caused by the data tones is reached for high SNR.

We further investigate the effect of the CIR delay spread on the estimate. Fig. 2 shows the RMSE of the CFO estimates as a function of the channel delay-spread in terms of L/Q for a fixed SNR of 3 dB. As expected, the performance degrades for long channels. It seems that the GEVD estimate illustrates the best performance for all scenarios with price of minor degradation in computational complexity, caused by the $G \times G$ matrix inversion needed (since G is typically small).

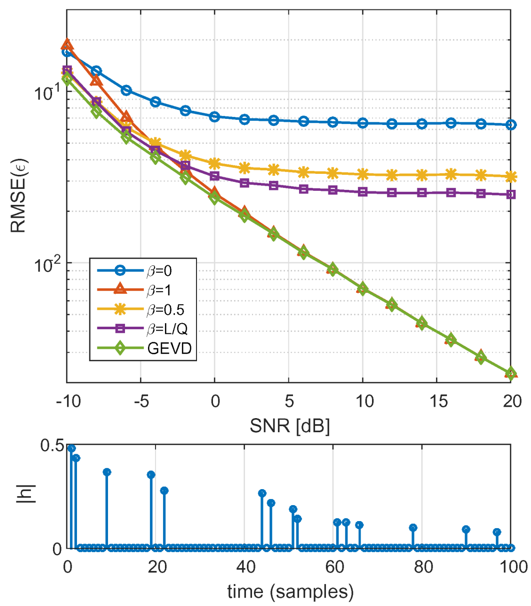

Fig. 1. CFO RMSE versus the SNR.

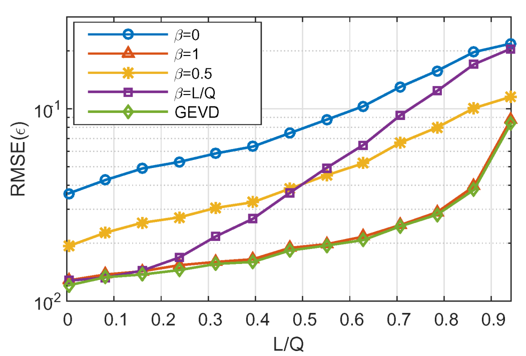

Fig. 2. CFO RMSE versus the CIR delay spread.

Pool trials were held in a 10m $\times$ 20m $\times$ 10m water tank. The receiving and transmitting transducers were placed at the center of the pool 2m apart at 3m depth. Since the typical CIR in the tank is fairly long, we used $K = 2048$, $G = 4$ and $L = 250$, i.e. $L/Q = 0.49$. A total of 1400 OFDM blocks were transmitted, including both static and motion scenarios. We compared the proposed methods with the state of the art null carriers CFO estimation [3], where 114 equi-spaced null carriers were used. Fig. 3 shows the CFO estimates of two packets of motion scenarios. In each set a different displacement was performed. In the top plot, the displacement was large, as can be seen all methods follow the harmonic oscillation of the transducer in the water. The bottom plot illustrates the results of much subtle displacements. Here we show the differences between the best performing methods (as expected, the channel-dependent estimator in Section 3 produced poor results due to the long delay spread).

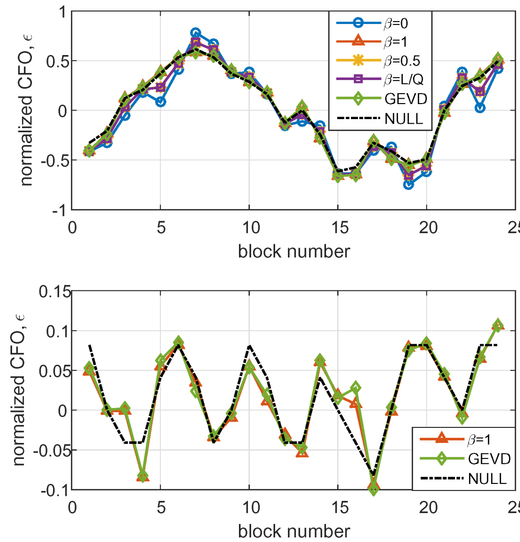

Fig. 3. CFO estimates of dynamic scenarios in a water tank.

7. CONCLUSION

We have proposed EVD-based CFO estimators for OFDM in UAC channels. The first uses the phase of the maximal eigenvector of the channel-dependent correlation matrix. The second solves a joint LS problem given the first estimator and the phase of the minimal eigenvector of the channel-independent correlation matrix. The third employs the phases of the maximal eigenvector by solving a generalized eigenvalue decomposition problem given the channel-independent and channel-dependent correlation matrices. Simulations, as well as pool trials, show that the second estimator outperforms the other suggested estimator and the previously published estimators. Further analysis, which explains the improved performance in low SNR together with relating the methods formulation to optimizations problems, are in the focus of future research.

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