Electronics
Computers
Communications

Andrew and Erna Viterbi Faculty
of Electrical Engineering

Technion
Israel Institute of Technology

Time Varying Carrier Frequency Offset Estimation in Multicarrier Underwater Acoustic Communication

Gilad Avrashi

Supervised by **Prof. Israel Cohen** and **Dr. Alon Amar**

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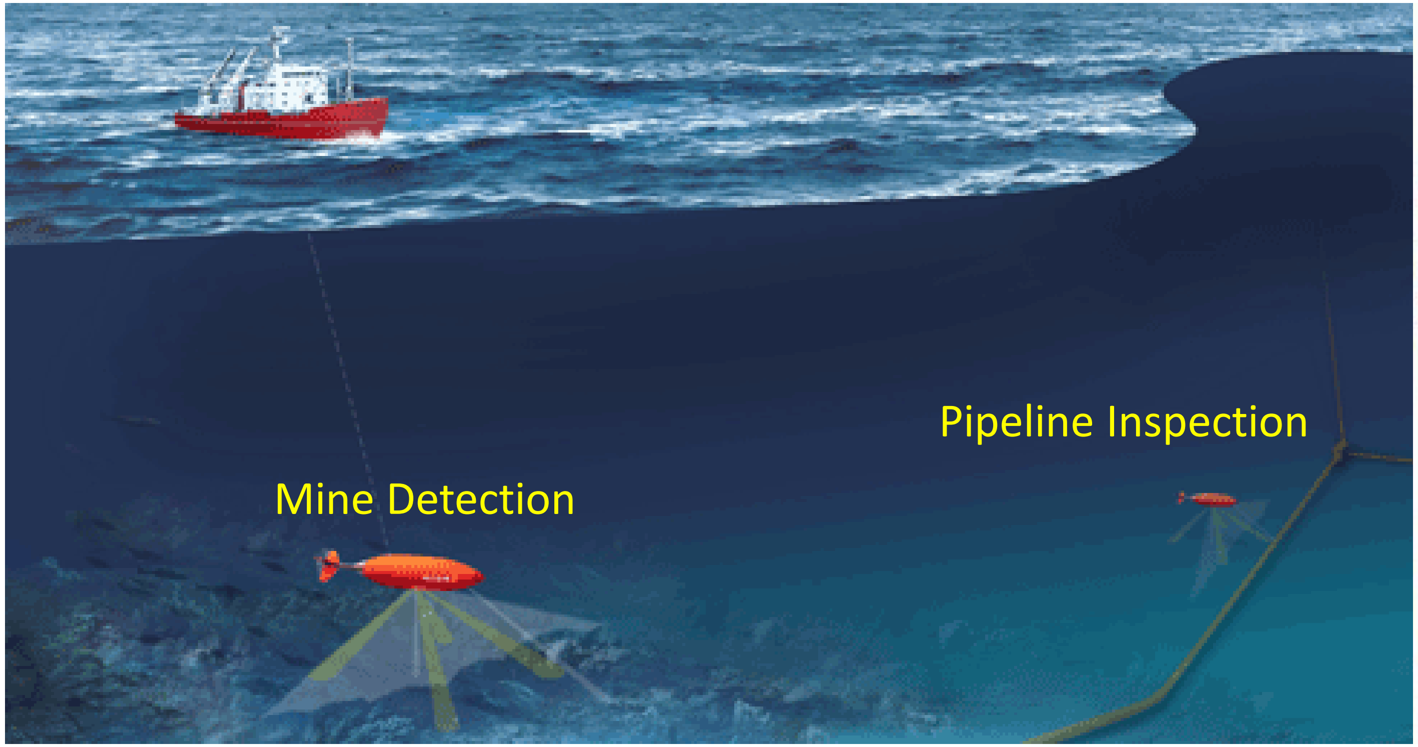
Introduction

Why underwater communications?

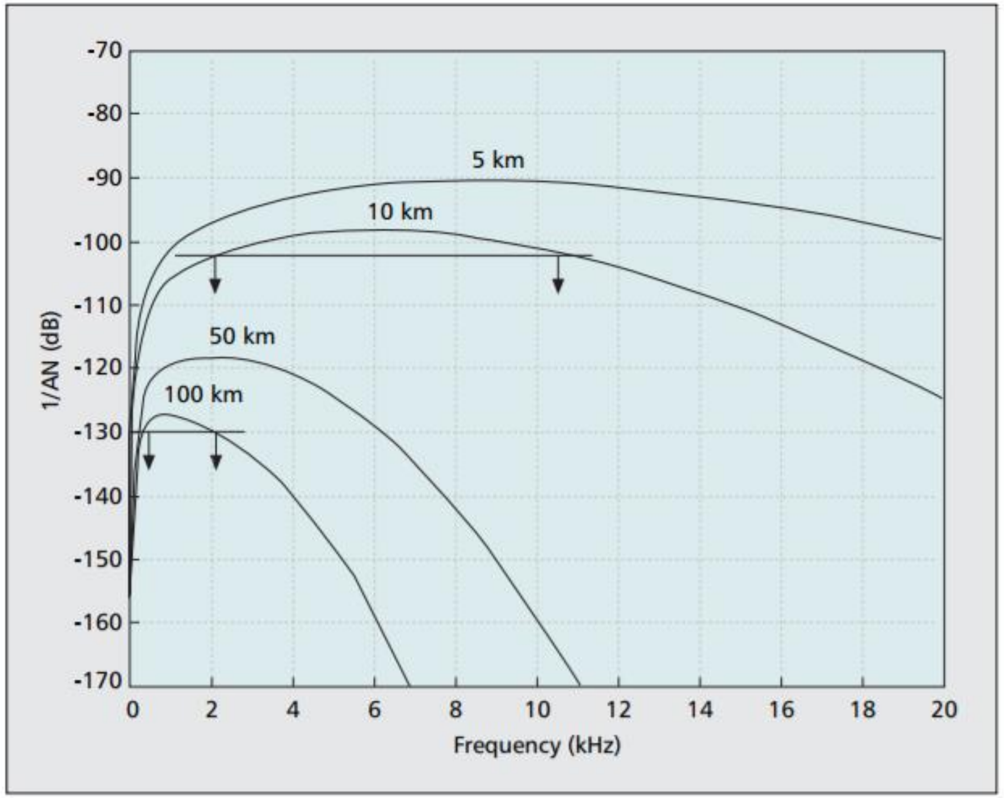


Autonomous Underwater Vehicles

Manned Vehicles

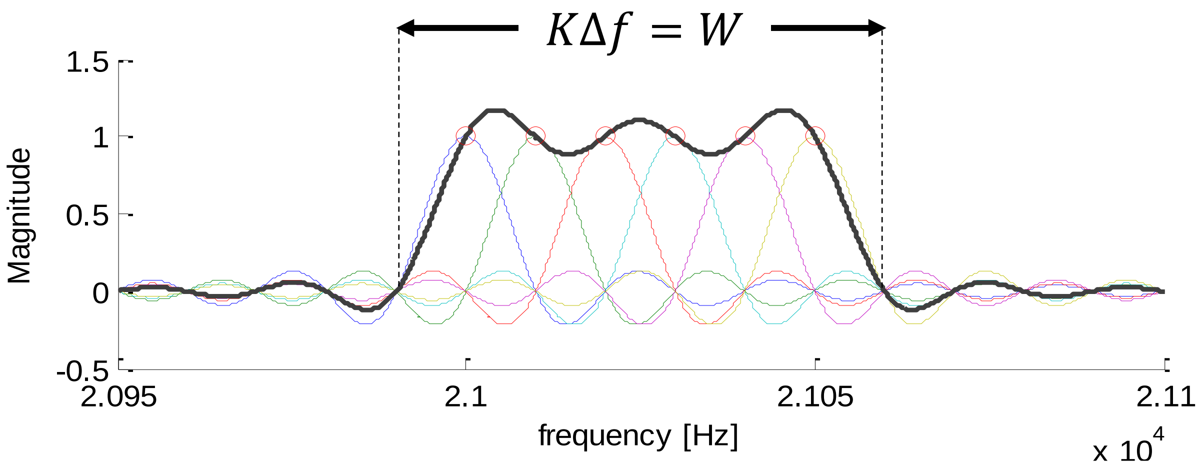
Challenges of Underwater Communications

- EM signals are attenuated quickly in the UW medium → pressure waves (sound) have been chosen for long range communications
- Sound waves characteristics:
  - Propagation speed: 1500 m/s (times 200,000 slower than EM waves!)
  - Frequency dependent losses
  - Frequency related ambient noise



Orthogonal Frequency Division Multiplexing

- The comm. bandwidth is divided into sub-carriers
- Each subcarrier is modulated to carry a digital communication symbol
- Pros:
  - Easy to implement using FFT operations
  - Robustness to frequency selective channels
  - Simple channel equalizer
- Cons:
  - Very sensitive to frequency shifts
  - High peak-to-average power ratio (PAPR)

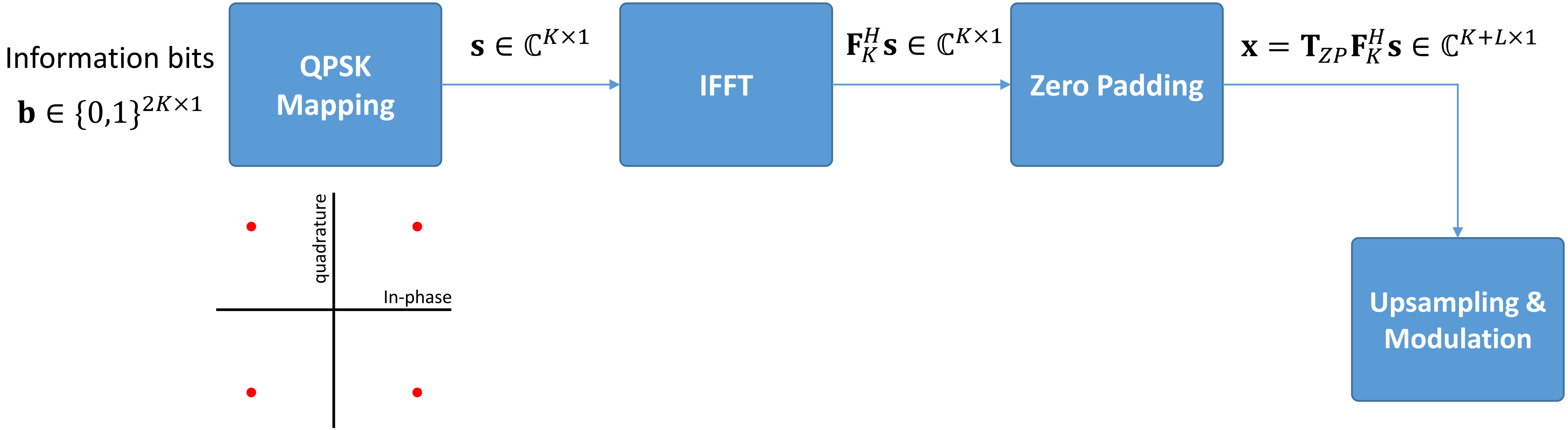


Orthogonality is achieved by $\Delta f = \frac{1}{T}$

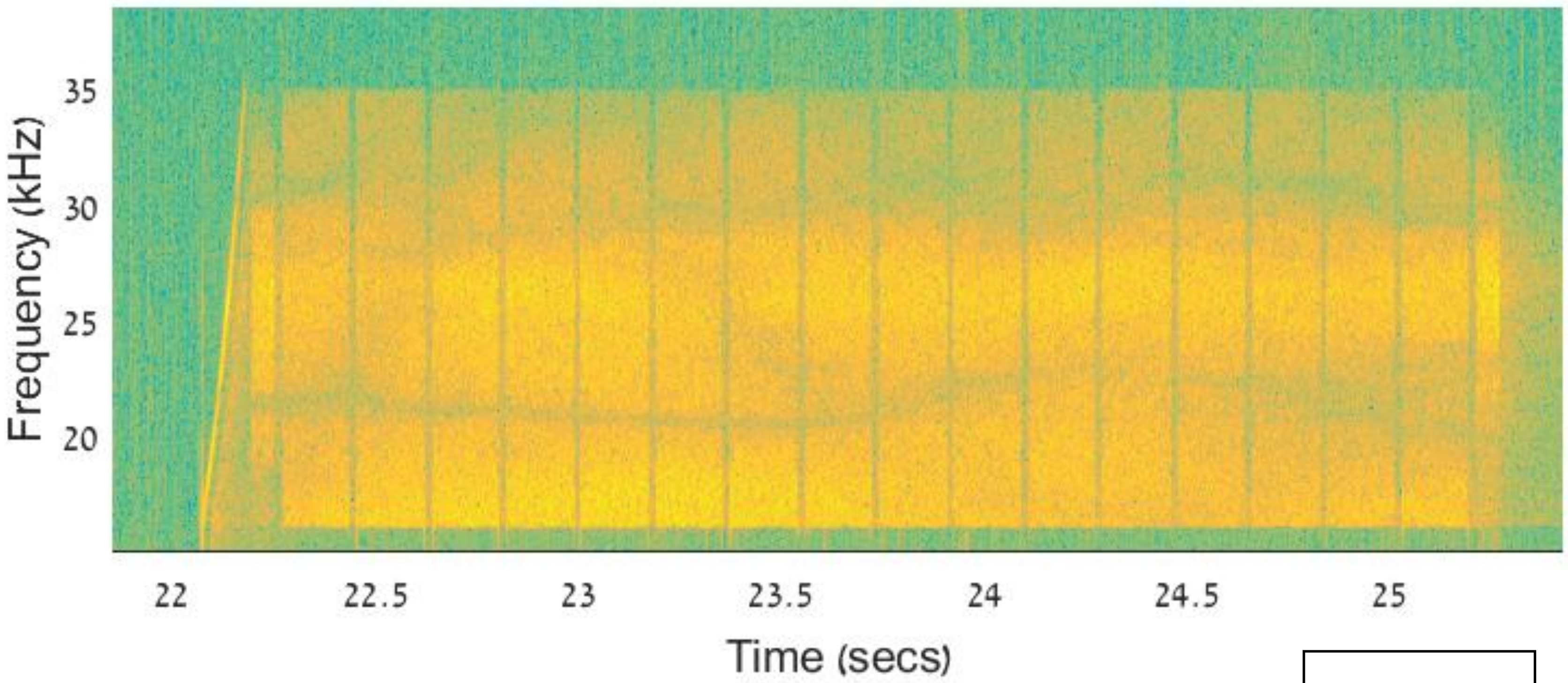
OFDM modulation

The baseband OFDM signal:

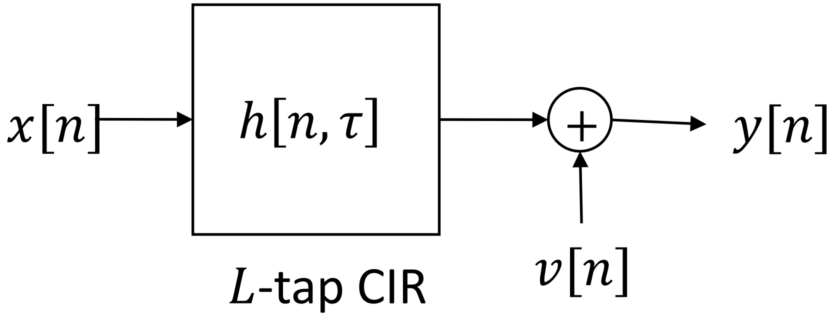
$$x[n] = g[n]\sum_{k=0}^{K-1} s[k]e^{j2\pi\left(\frac{n}{W}\right)f_k} = g[n]\sum_{k=0}^{K-1} s[k]e^{j2\pi\left(\frac{n}{W}\right)k\Delta f} = g[n]\sum_{k=0}^{K-1} s[k]e^{\frac{j2\pi nk}{K}} = \frac{g[n]}{\sqrt{K}}\mathrm{IDFT}\{s[k]\}$$



Sea Trial OFDM Signal



Recording from sea trial in the Mediterranean, December 2016

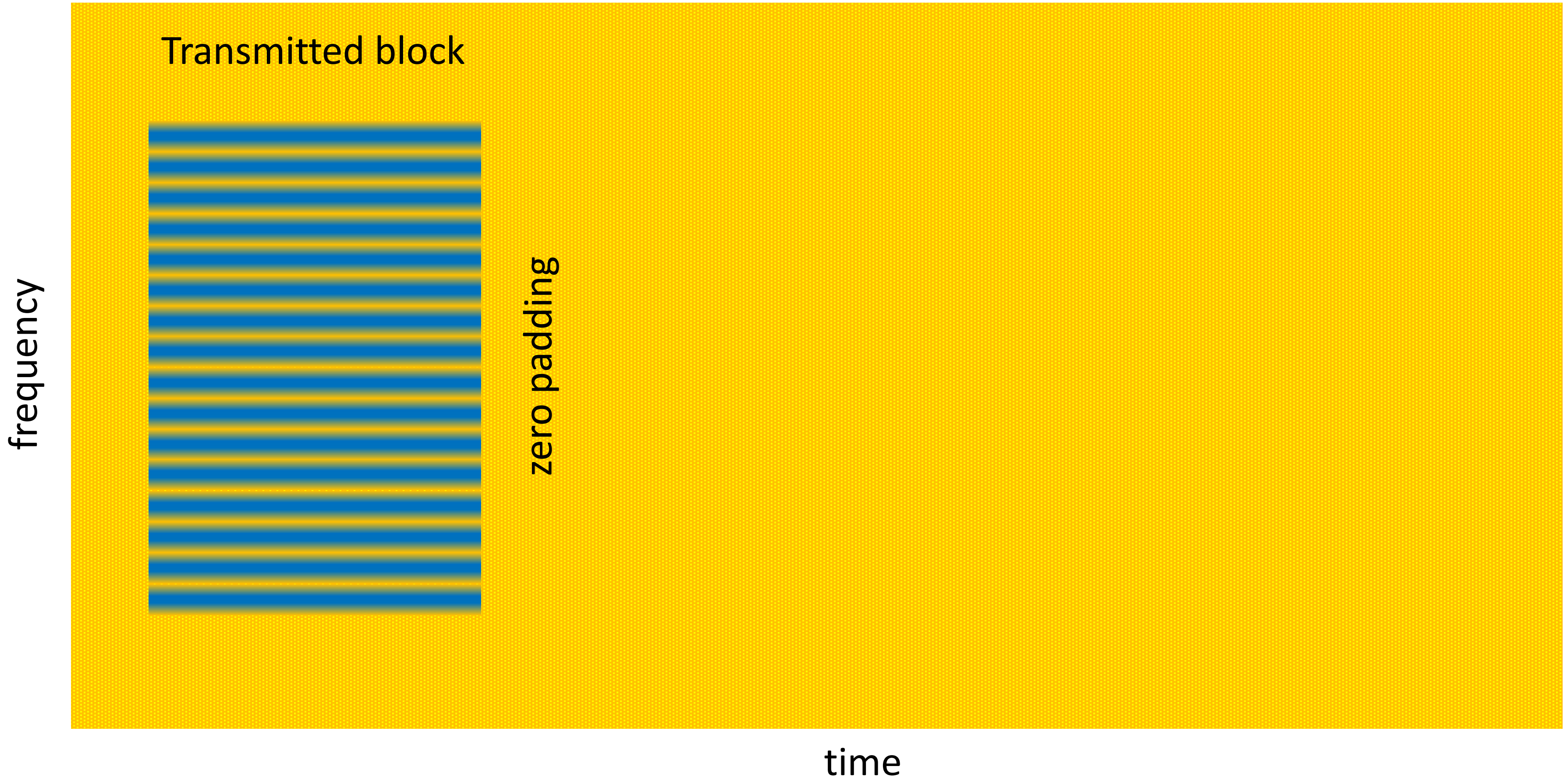


Research Goal

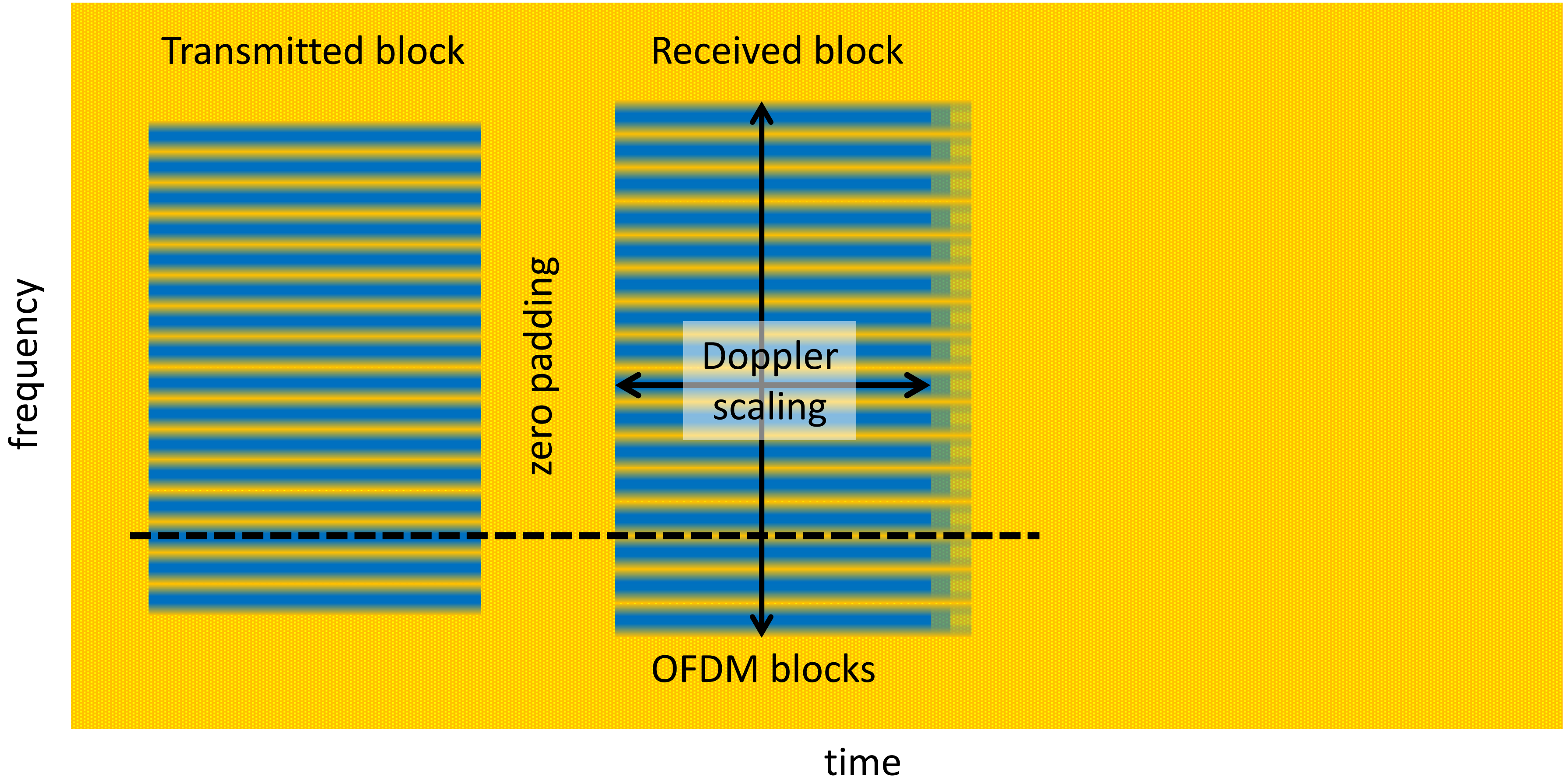
Develop a carrier frequency offset estimator for underwater acoustic OFDM modems.

The solution is required to be computationally efficient and practical for the underwater acoustic channel.

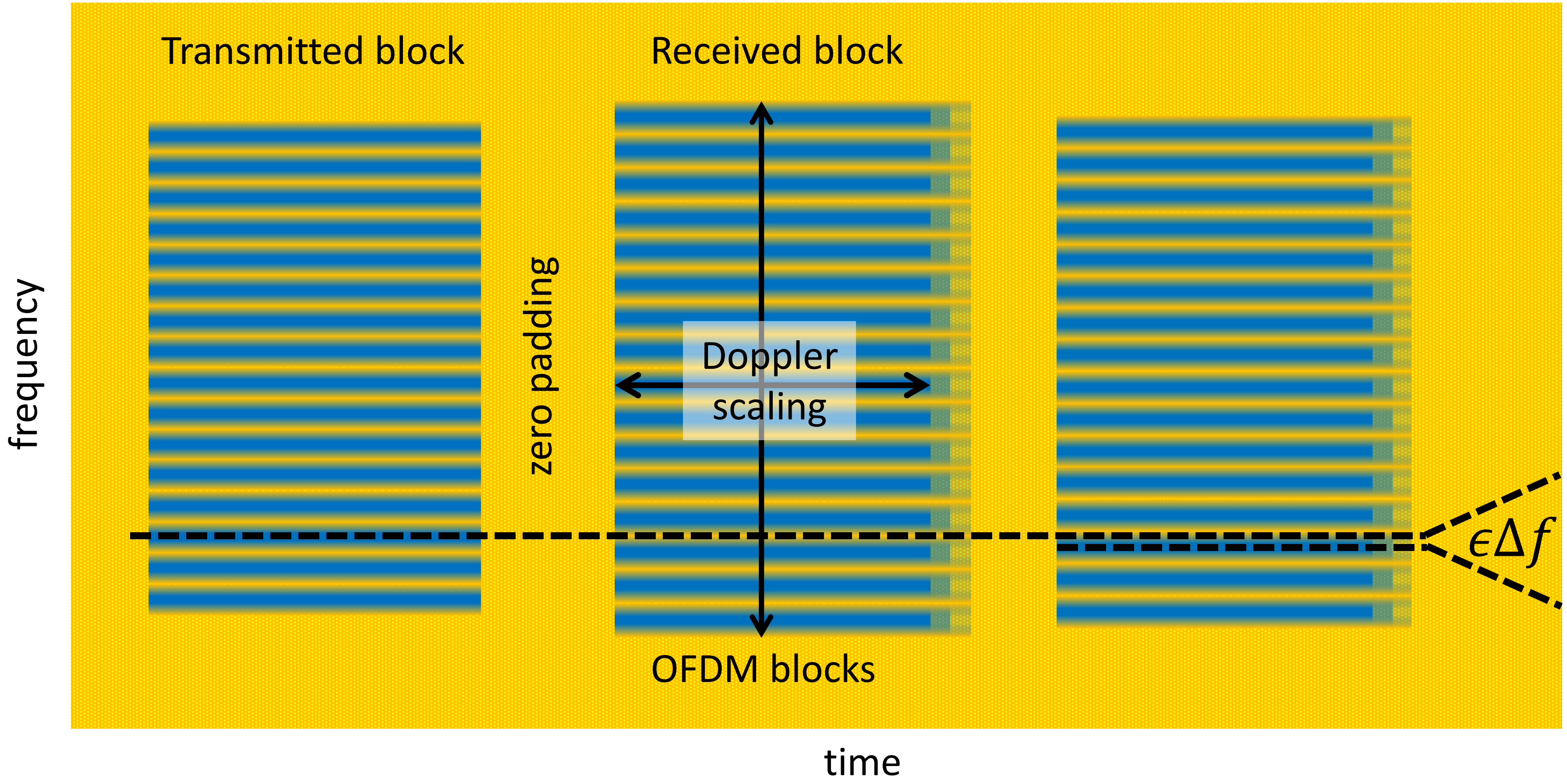
Multicarrier UAC Effects



Multicarrier UAC Effects



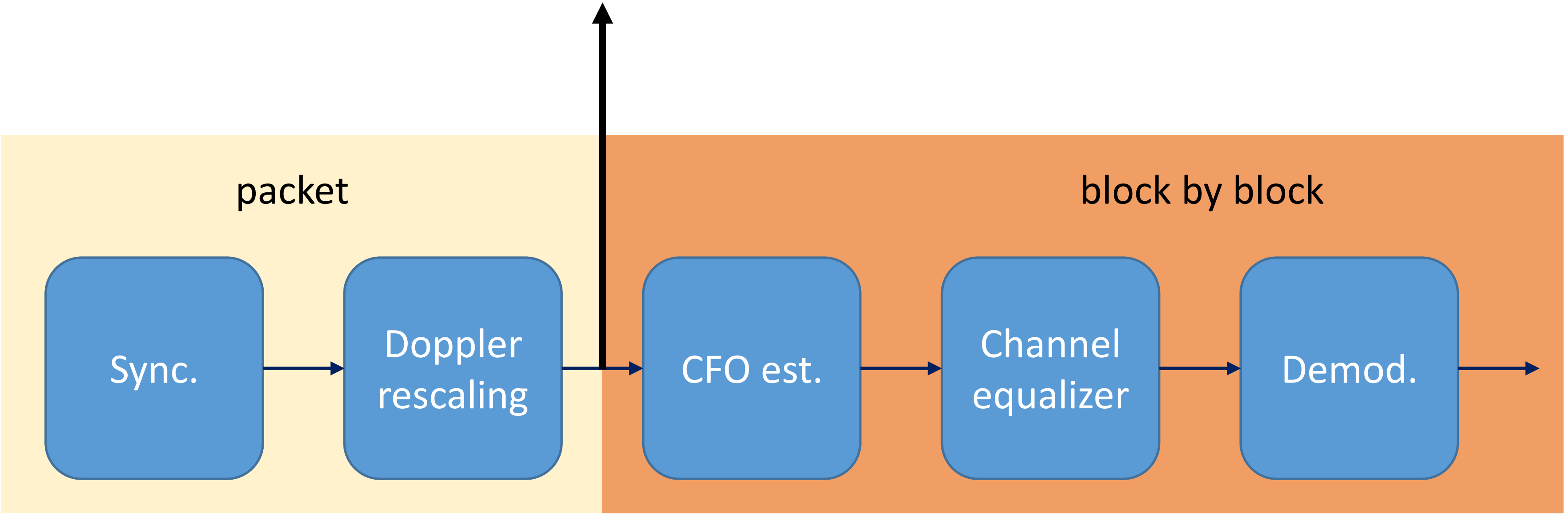
Multicarrier UAC Effects



Received Signal Model

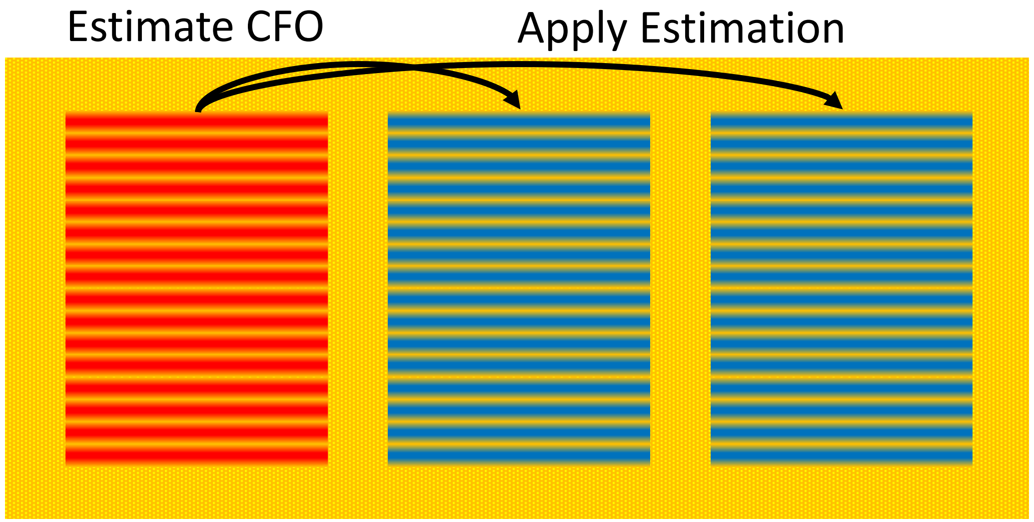
Li et al. '08

$$y[n] = e^{\frac{j2\pi\epsilon_0 n}{K}} h[n] * x[n] + v[n] \quad \text{or in matrix form} \quad \mathbf{y} = \boldsymbol{\Gamma}_K(\epsilon_0)\underbrace{\mathbf{Hx}}_{\mathbf{z}} + \mathbf{v}$$

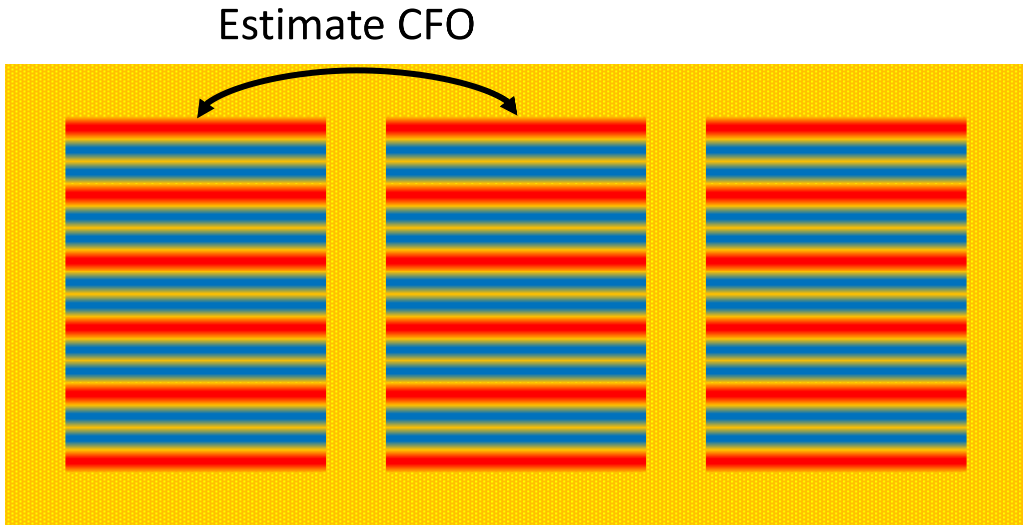


Radio Frequency Approaches

Training blocks with periodic characteristics
(Classen & Meyer ‘94)



Block to block pilot signal cross-correlation
(Schmidl & Cox ‘97)

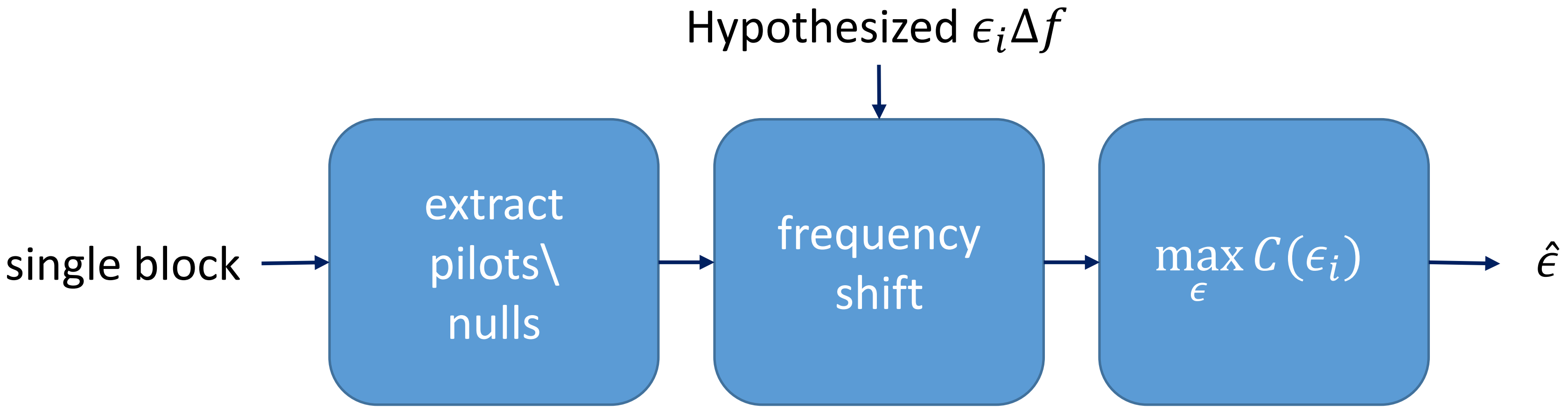


In UAC – CFO varies between adjacent blocks

UAC Approaches

Null Carriers
Minimum variance
(Li et al. '08)

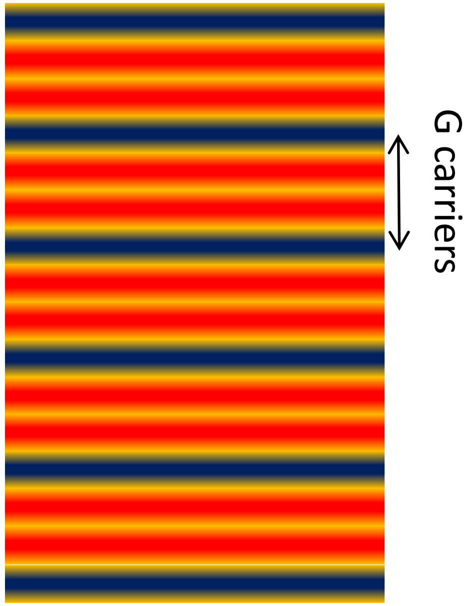
Pilot aided
Maximum power
(Li et al. '06)



Requires exhaustive grid search

Pilot Based Estimation

$$x[n] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} s(k) e^{\frac{j2\pi nk}{K}}$$

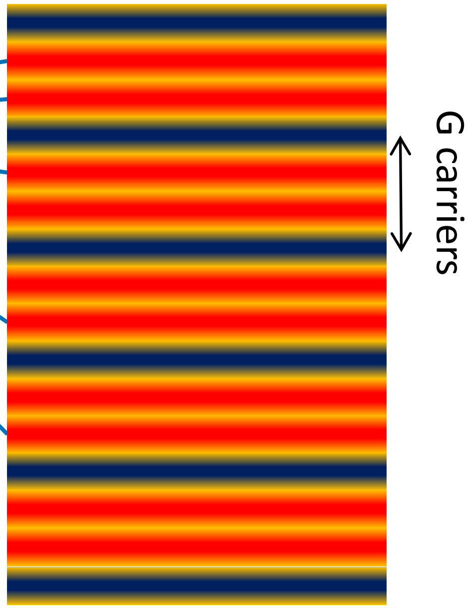


Pilot Based Estimation

data signal $\sim\mathcal{N}\left(0,\frac{K-Q}{Q}\right)$

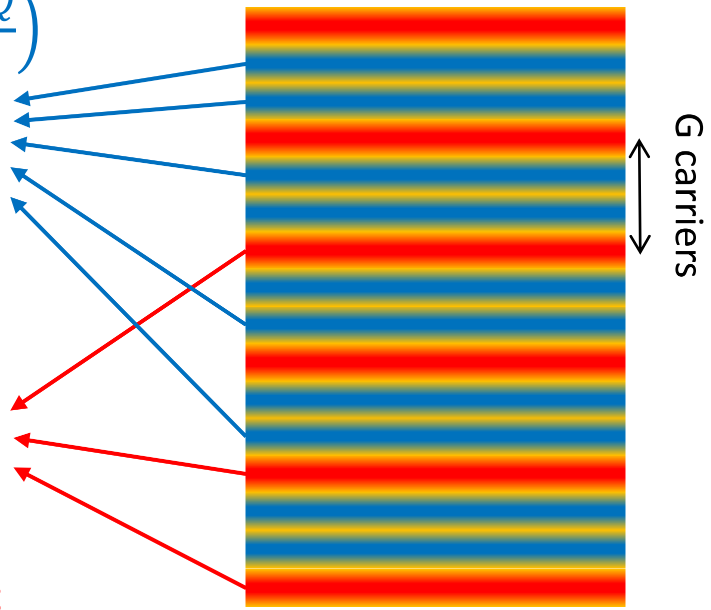
$$\overbrace{\frac{1}{\sqrt{K}}\sum_{k\in S_D} s(k)e^{\frac{j2\pi nk}{K}}}$$

$$x[n]=\frac{1}{\sqrt{K}}\sum_{k=0}^{K-1} s(k)e^{\frac{j2\pi nk}{K}} \qquad =$$



Pilot Based Estimation

$$x[n] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} s(k) e^{\frac{j2\pi nk}{K}} = \underbrace{\frac{1}{\sqrt{K}} \sum_{k \in S_D} s(k) e^{\frac{j2\pi nk}{K}}}_{\text{data signal } \sim \mathcal{N}\left(0, \frac{K-Q}{Q}\right)} + \underbrace{\frac{1}{\sqrt{K}} \sum_{q=0}^{Q-1} s(qG) e^{\frac{j2\pi nq}{Q}}}_{\text{pilot signal } Q\text{-periodic}}$$



Idea: Use correlation between periods of the pilot signal

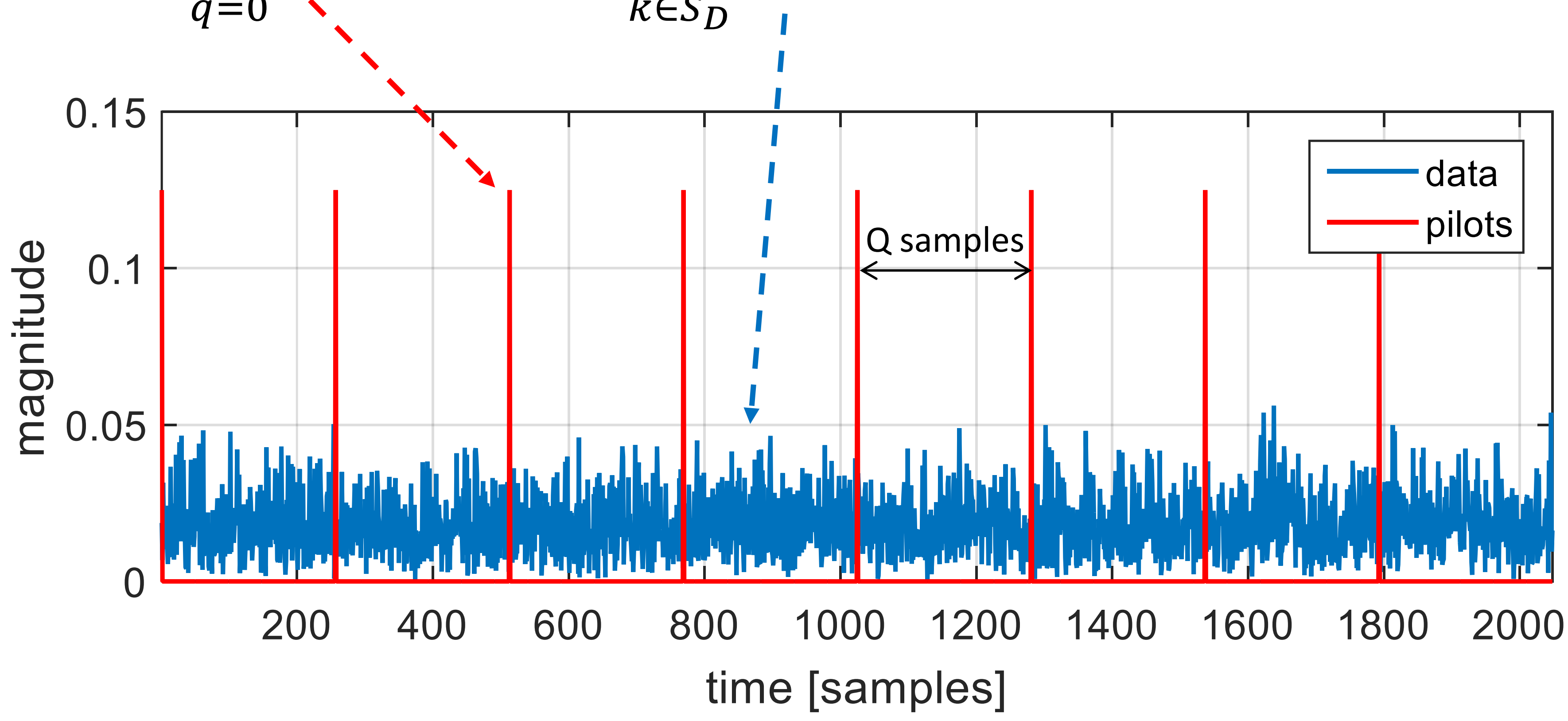
Problem: Low “SNR” – Pilot to Data Ratio (PDR)

Solution: Design pilot signal with “Good” auto-correlation

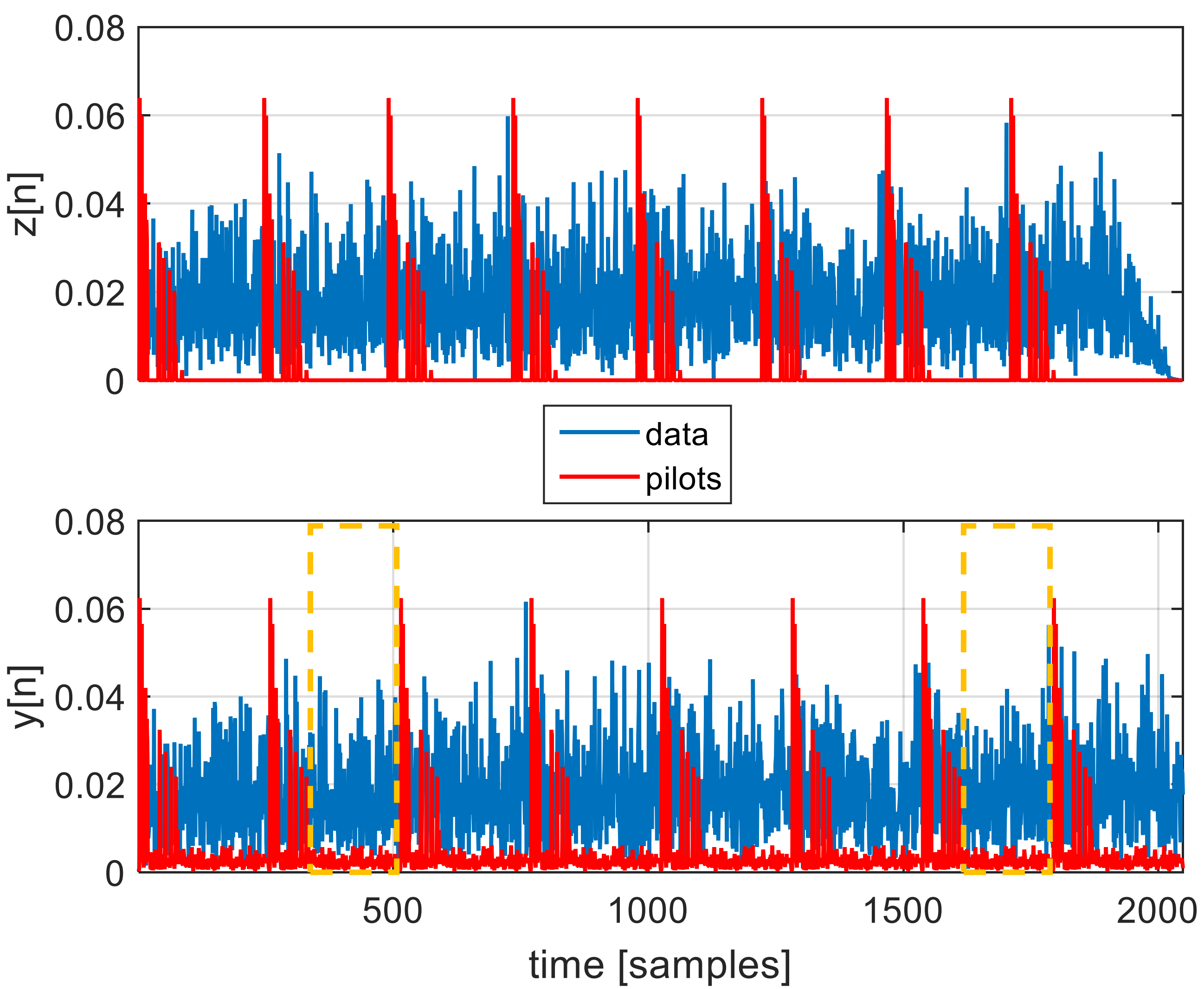
Best auto correlation: identical pilots

Amar, Avrashi, Stojanovic '16

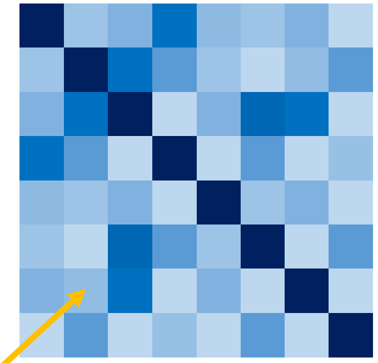
$$\tilde{s}[n] = \frac{1}{\sqrt{K}} \sum_{q=0}^{Q-1} {\color{red}u} e^{\frac{j2\pi nq}{Q}} + \frac{1}{\sqrt{K}} \sum_{k \in S_D} s(k) e^{\frac{j2\pi nk}{K}}$$



Exploiting Inter-Segment Correlations



$$y[n] = e^{\frac{j2\pi\epsilon_0 n}{K}} \underbrace{h[n] * x[n]}_{z[n]}$$



$$\mathbf{y}_g^H \mathbf{y}_{g'} = \underbrace{\left(e^{-j\frac{2\pi}{G}\epsilon_0}\right)^{g'-g}}_{\alpha(\epsilon_0)} \mathbf{z}_g^H \mathbf{z}_{g'}$$

Eigen Value Decomposition

The cost function in matrix formulation

We look for $\hat{\epsilon}$ that minimizes (maximizes)

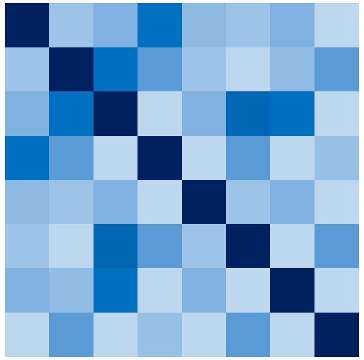
$$l = \boldsymbol{\alpha}(\epsilon)^H \mathbf{R} \boldsymbol{\alpha}(\epsilon)$$

Under two constraints:

- $\|\boldsymbol{\alpha}\| = 1$
- $\arg(\boldsymbol{\alpha}) \propto \epsilon$

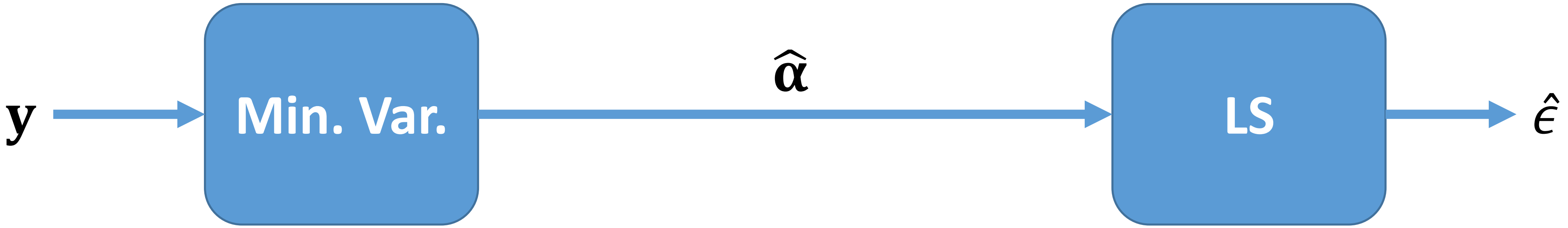
R

$$\frac{1}{G}\begin{bmatrix} 1 & \alpha^{-1} & \cdots & \alpha^{-(G-1)} \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ \alpha^{1} \\ \vdots \\ \alpha^{G-1} \end{bmatrix}$$

EVD estimator



$$\min_{\boldsymbol{\alpha}} \boldsymbol{\alpha}^H \mathbf{R} \boldsymbol{\alpha} \quad \text{s.t.} \quad \boldsymbol{\alpha}^H \boldsymbol{\alpha} = 1$$

$$\arg[\hat{\boldsymbol{\alpha}}] \approx -\frac{2\pi}{G} \mathbf{g} \epsilon$$

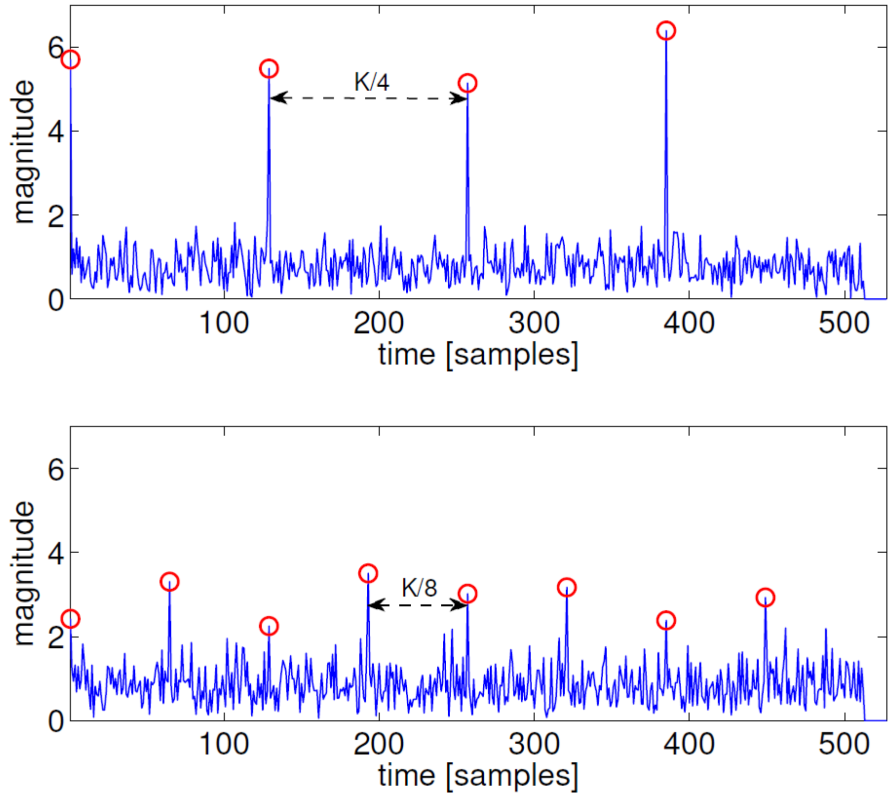
$$\rightarrow \hat{\boldsymbol{\alpha}} = \mathbf{V}_{\min}(\mathbf{R})$$

$$\rightarrow \hat{\epsilon} = -\frac{G}{2\pi} \frac{\sum_{g=0}^{G-1} g \left[\arg[\hat{\boldsymbol{\alpha}}]\right]_g}{\sum_{g=0}^{G-1} g^2}$$

Decompose R → find the eigenvector of the smallest EV → extract $\hat{\epsilon}$

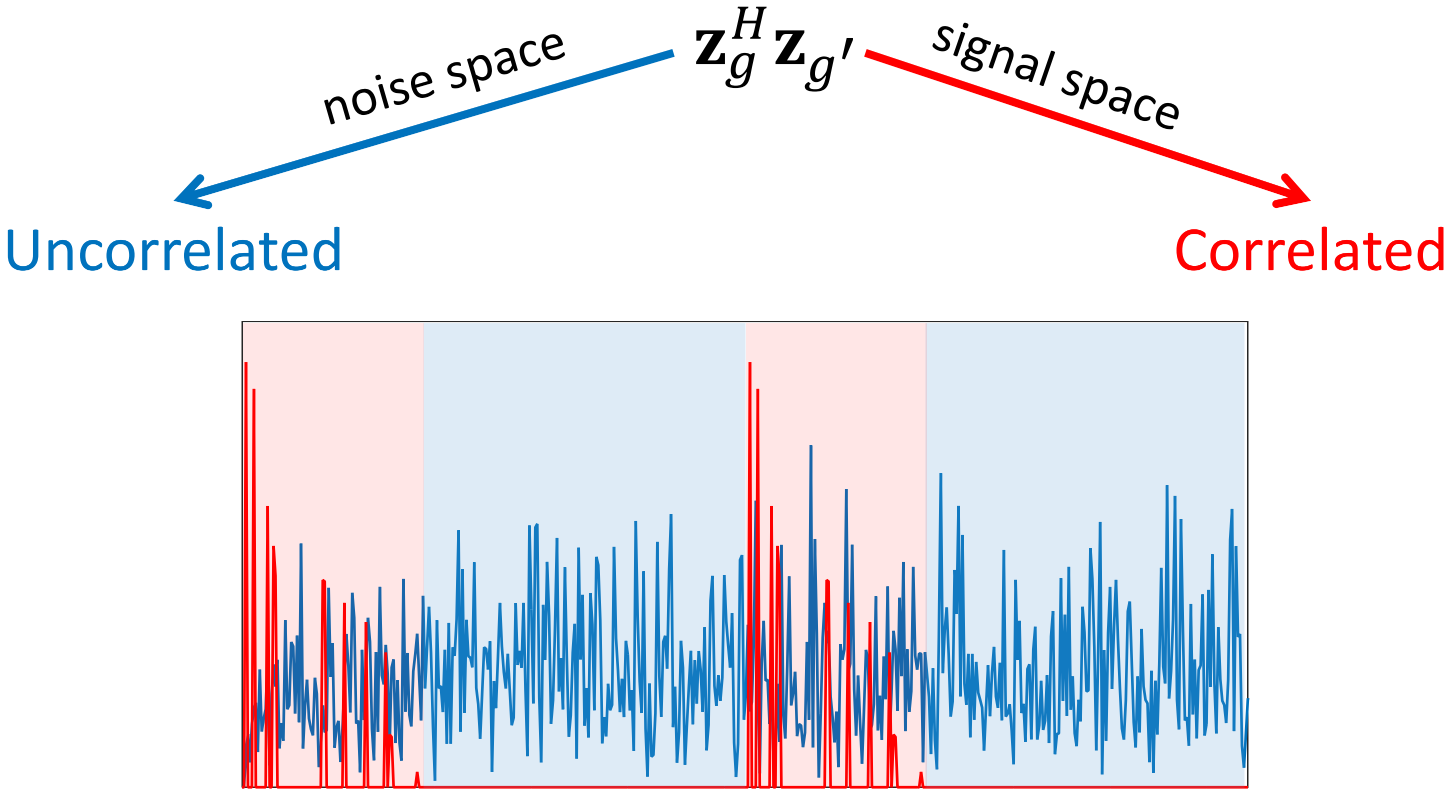
Research objectives

- The EVD-based estimator has two drawbacks:
  - High PAPR
  - Requires constant CFO during the block
- **Our goal**: Propose a CFO estimator for UAC with the following characteristics:
  - Low complexity
  - Negligible PAPR
  - Adjustable for time-varying channels



Signal Space Estimation

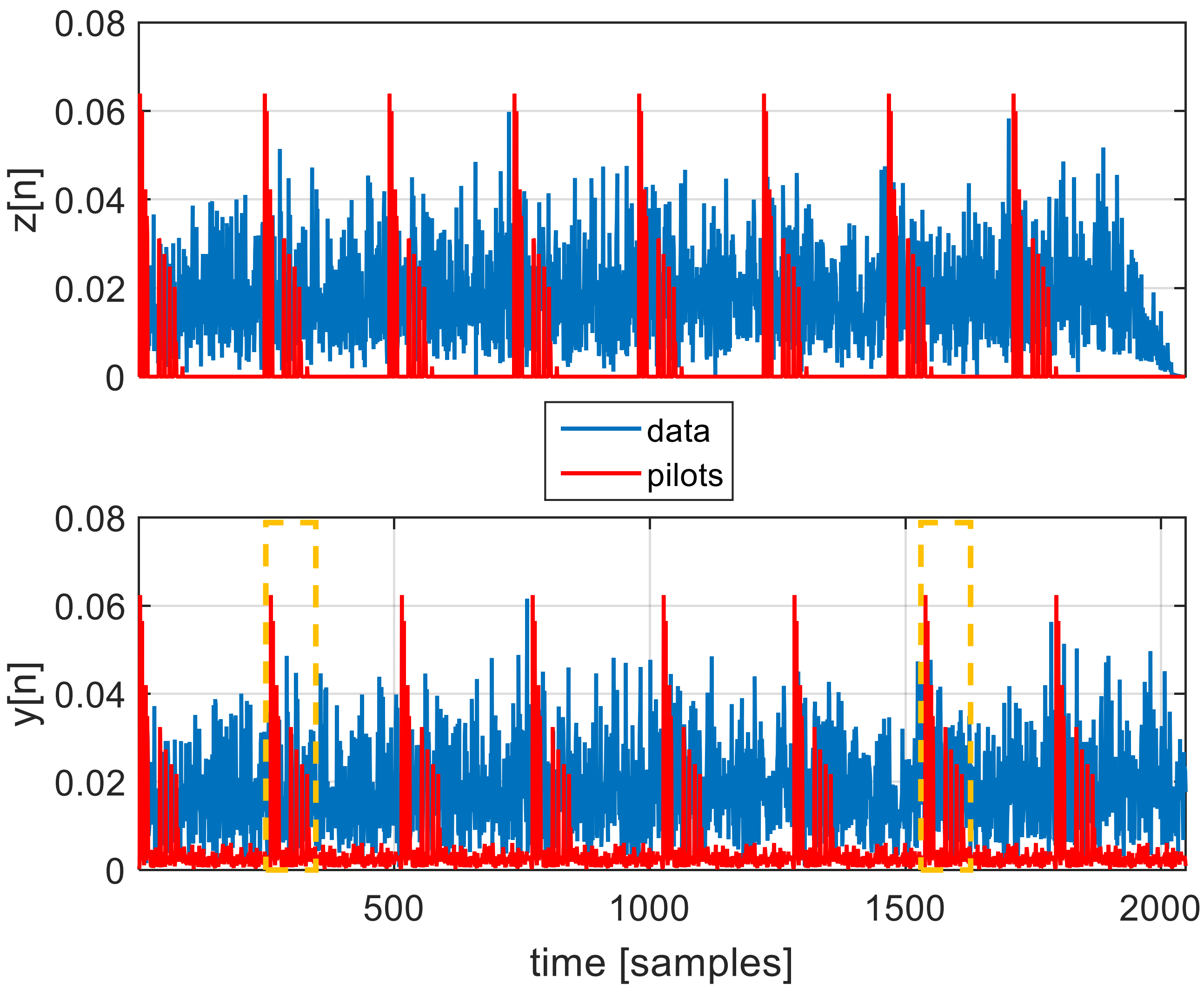
Two sides of the same coin



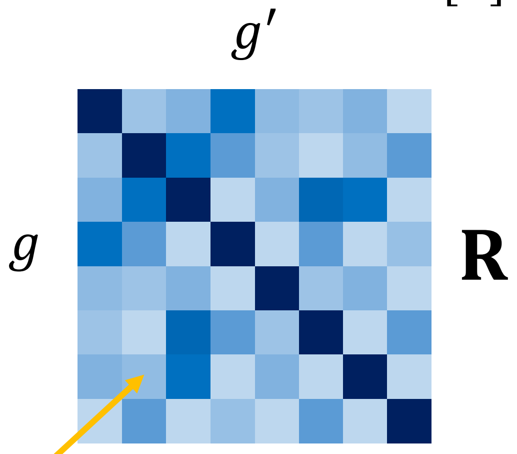
array processing interpretation

Steering ϵ in the **noise****signal** space to achieve **lowest****highest** SNR

Exploiting Inter-Segment Correlations

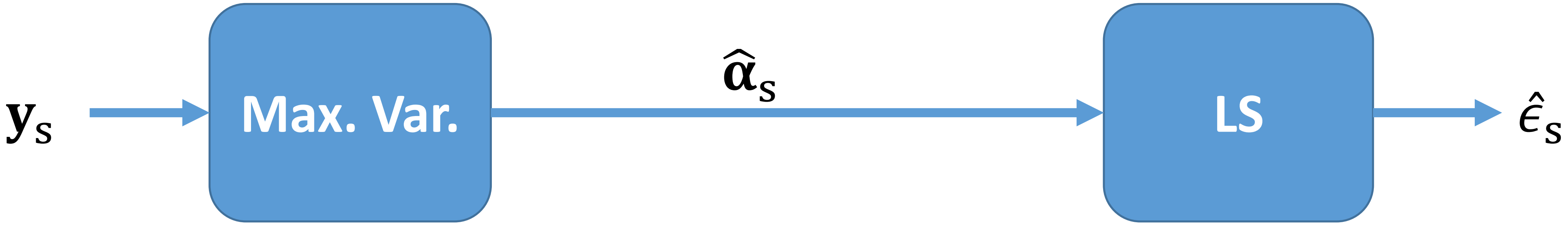


$$y[n] = e^{\frac{j2\pi\epsilon_0 n}{K}} \underbrace{h[n] * x[n]}_{z[n]}$$



$$\mathbf{y}_g^H \mathbf{y}_{g'} = \underbrace{\left(e^{-j\frac{2\pi}{G}\epsilon_0}\right)^{g'-g}}_{\alpha(\epsilon_0)} \mathbf{z}_g^H \mathbf{z}_{g'}$$

EVD in Signal Space



$$\max_{\boldsymbol{\alpha}} \boldsymbol{\alpha}^H \mathbf{R}_s \boldsymbol{\alpha} \quad \text{s.t.} \quad \boldsymbol{\alpha}^H \boldsymbol{\alpha} = 1$$

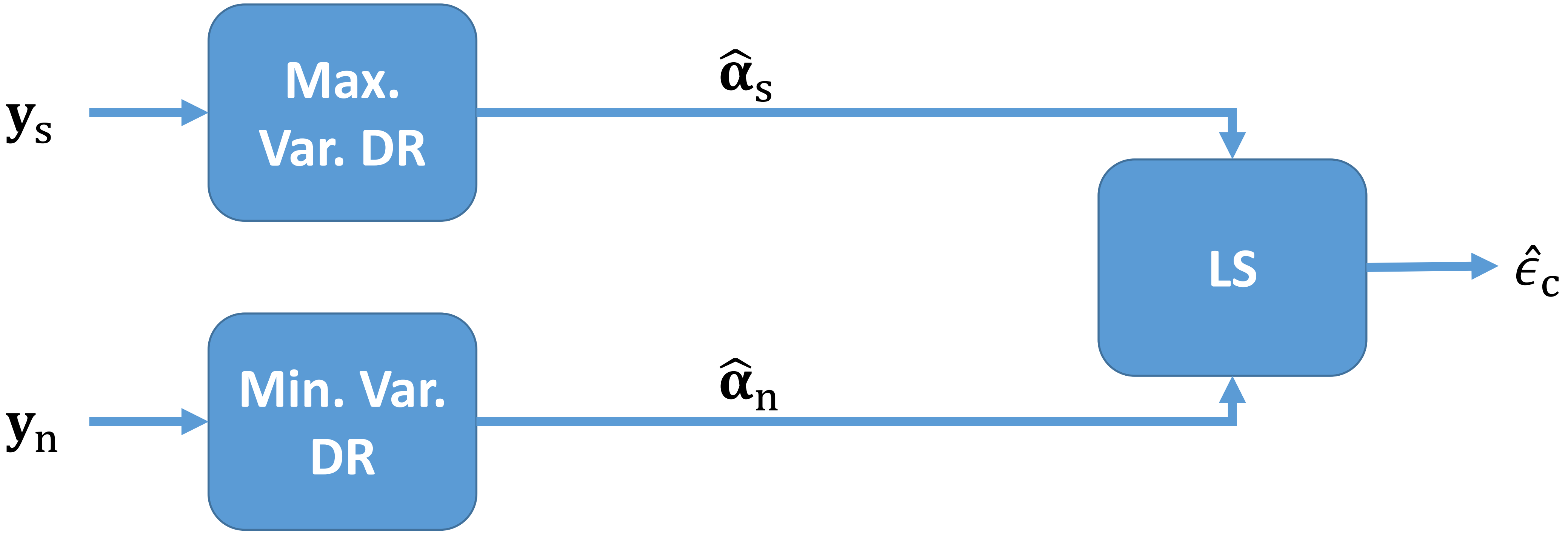
$$\arg[\hat{\boldsymbol{\alpha}}] \approx -\frac{2\pi}{G} \mathbf{g}\epsilon$$

$$\rightarrow \; \hat{\boldsymbol{\alpha}}_s = \mathbf{V}_{\max}(\mathbf{R}_s)$$

$$\rightarrow \hat{\epsilon} = -\frac{G}{2\pi} \frac{\sum_{g=0}^{G-1} g \left[\arg[\hat{\boldsymbol{\alpha}}]\right]_g}{\sum_{g=0}^{G-1} g^2}$$

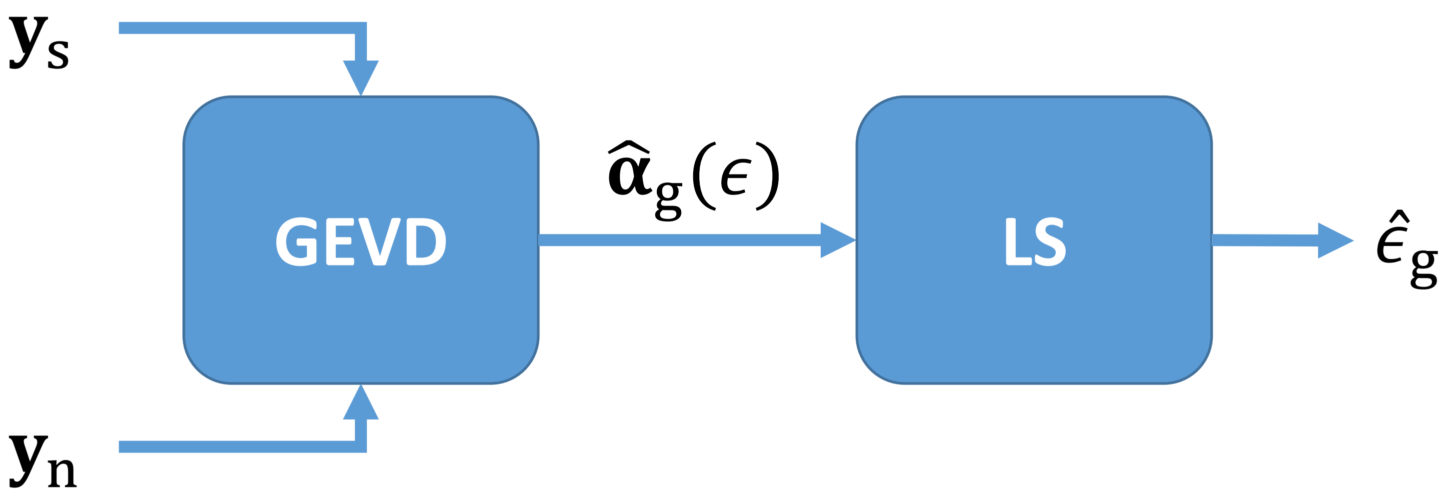
Decompose $\mathbf{R}_s$ → find the eigenvector of the largest EV → extract $\hat{\epsilon}$

Combined LS estimate



$$\hat{\epsilon}_\mathrm{c} = \beta\hat{\epsilon}_\mathrm{n} + (1-\beta)\hat{\epsilon}_\mathrm{s} \;, \qquad 0 \le \beta \le 1$$

Generalized EVD



$$\max_{\boldsymbol{\alpha}} \frac{\boldsymbol{\alpha}^H \mathbf{R}_s \boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \mathbf{R}_n \boldsymbol{\alpha}} \quad \text{s.t. } \boldsymbol{\alpha}^H \boldsymbol{\alpha} = 1$$

$$\rightarrow \hat{\boldsymbol{\alpha}}_g = \mathbf{V}_{max}(\mathbf{R}_n^{-1}\mathbf{R}_s)$$

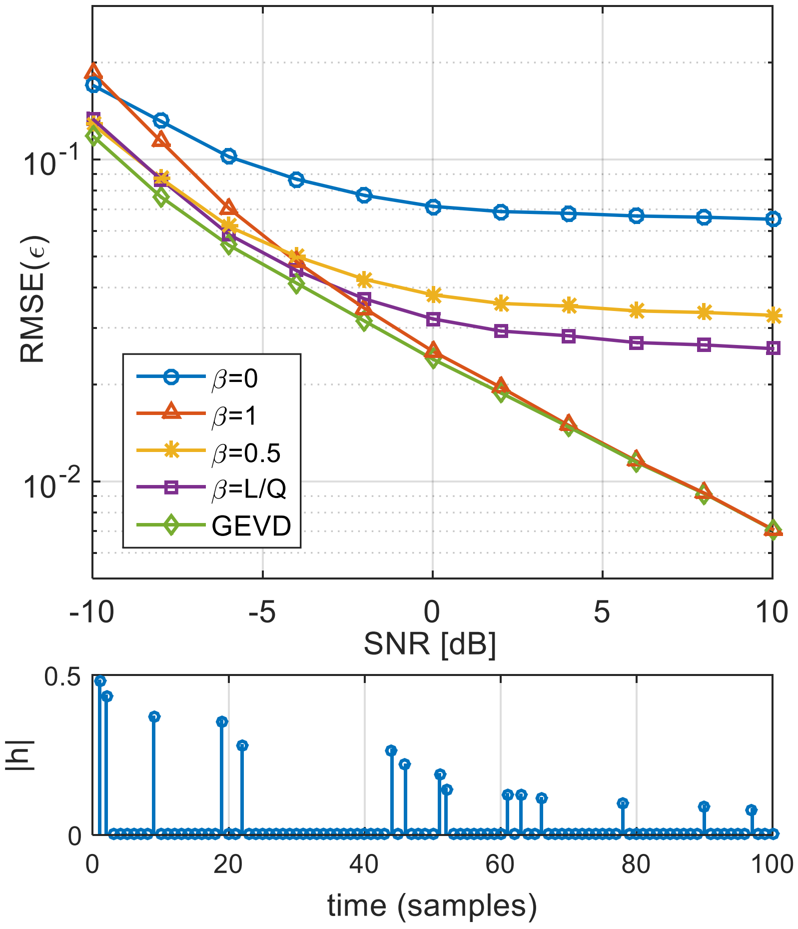
Decompose $\mathbf{R}_n^{-1}\mathbf{R}_s$ → find the eigenvector of the largest EV → extract $\hat{\epsilon}$

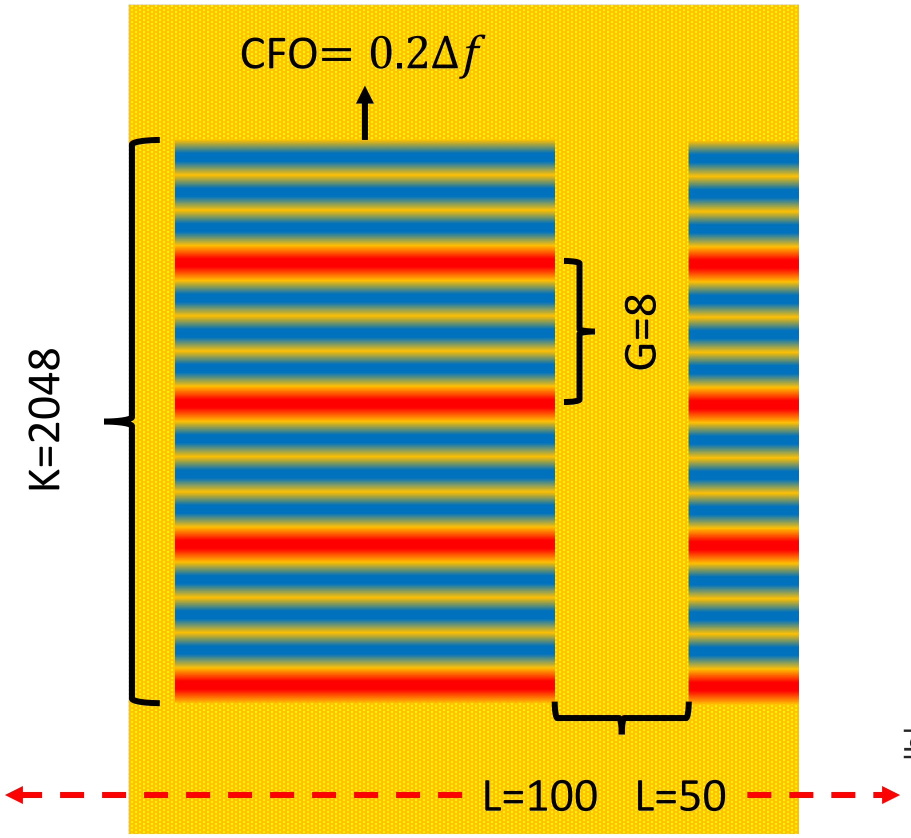
Computational Complexity

Method	Complexity
Grid Search	$\mathcal{O}(K\sqrt{K})$
Noise Space EVD	$\mathcal{O}(G^2(Q-L)) = \mathcal{O}(KG)$
Signal Space EVD	$\mathcal{O}(G^2L)$
Combined LS	$\mathcal{O}(G^2 \max\{Q-L, L\})$
Generalized EVD	$\mathcal{O}(G^2 \max\{Q-L, L\})$

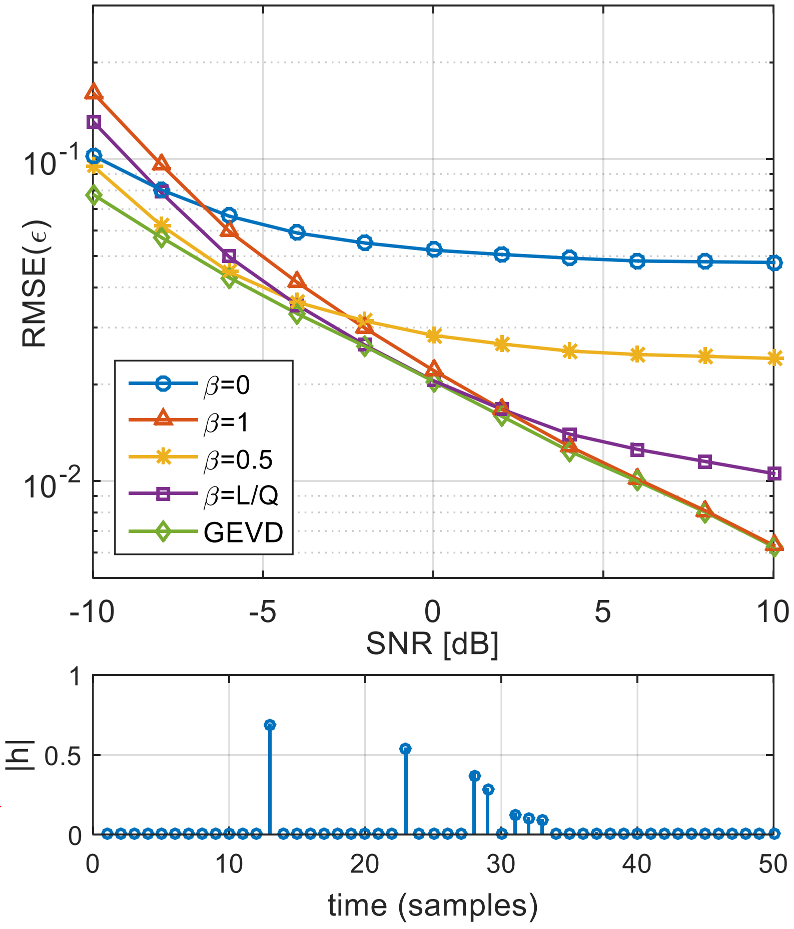
RMSE vs SNR

Long Delay Spread

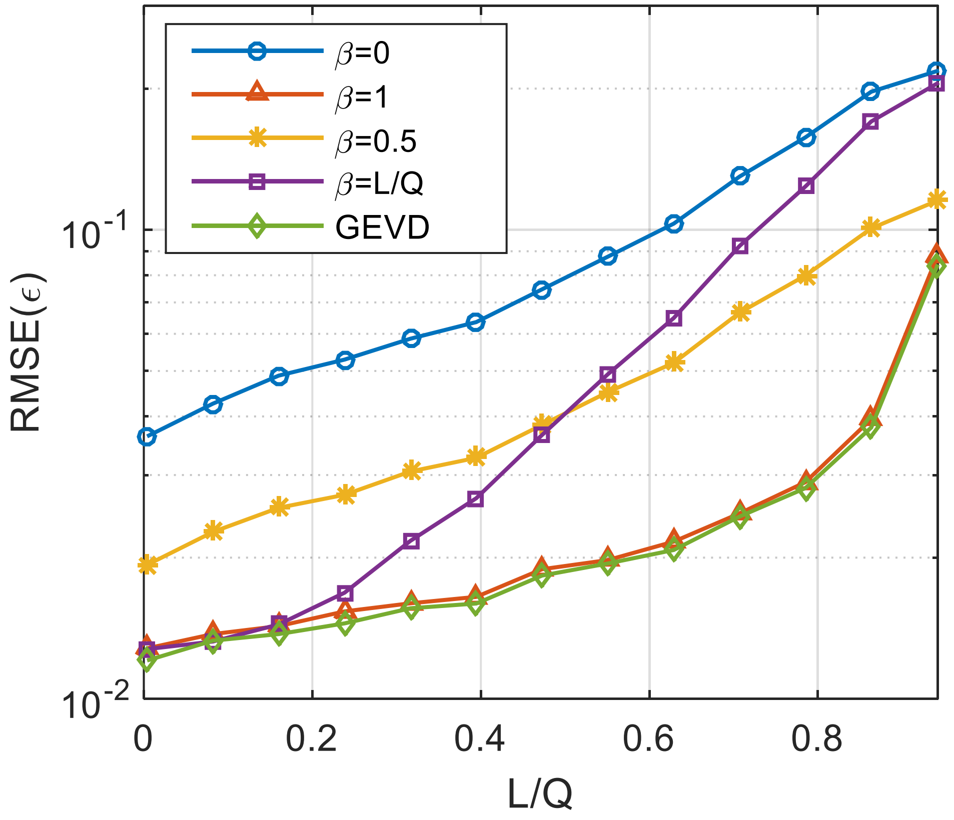


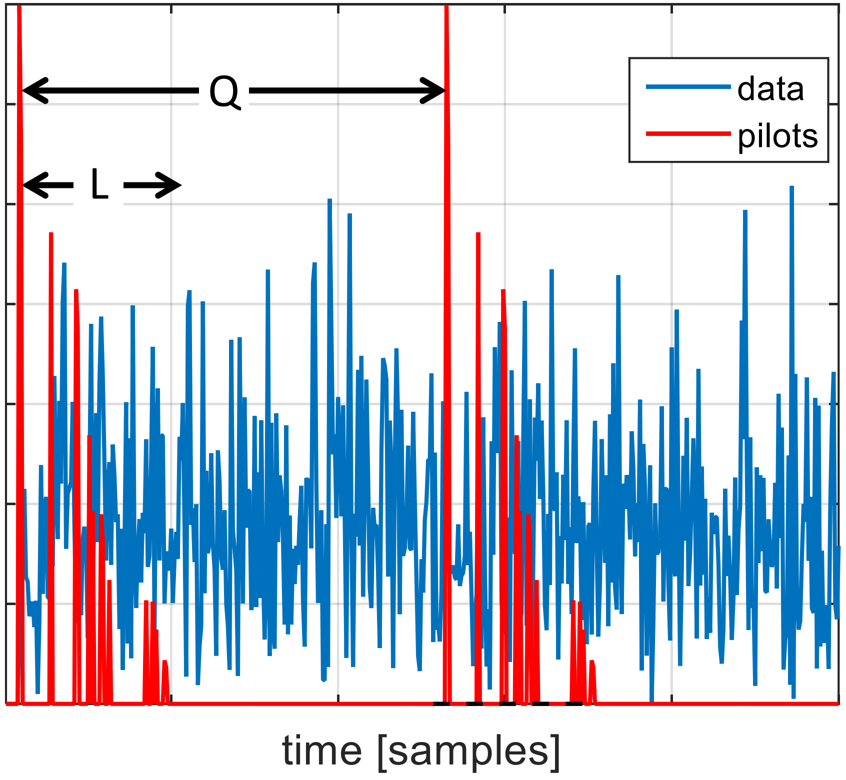


Short Delay Spread

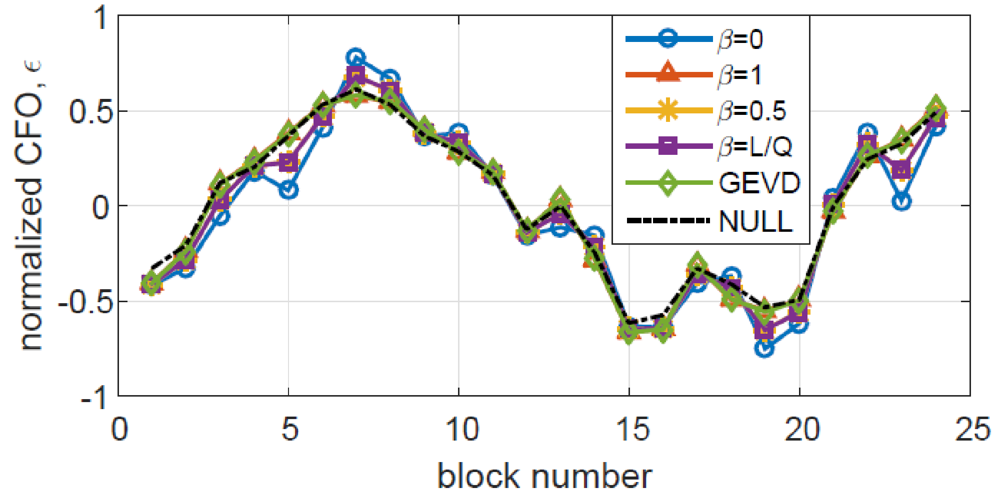


Effect of Delay Spread

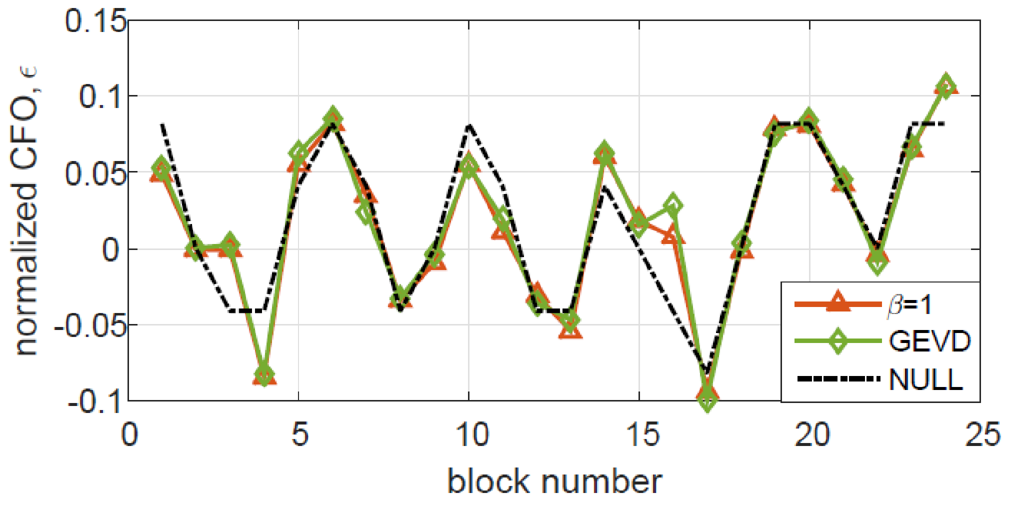




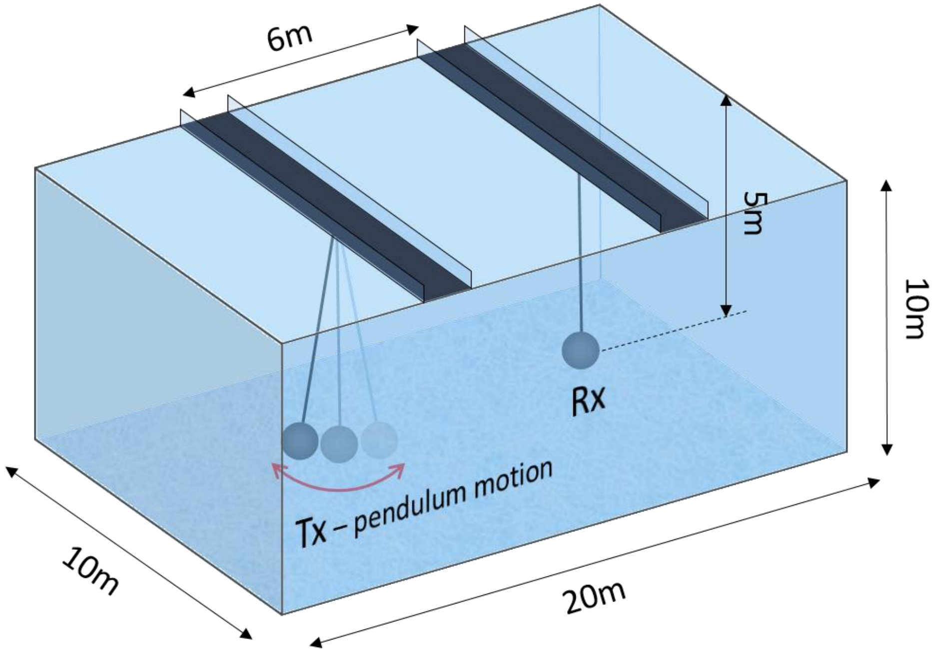
Pool Trial



normalized CFO, ϵ
block number
β=0
β=1
β=0.5
β=L/Q
GEVD
NULL



normalized CFO, ϵ
block number
β=1
GEVD
NULL

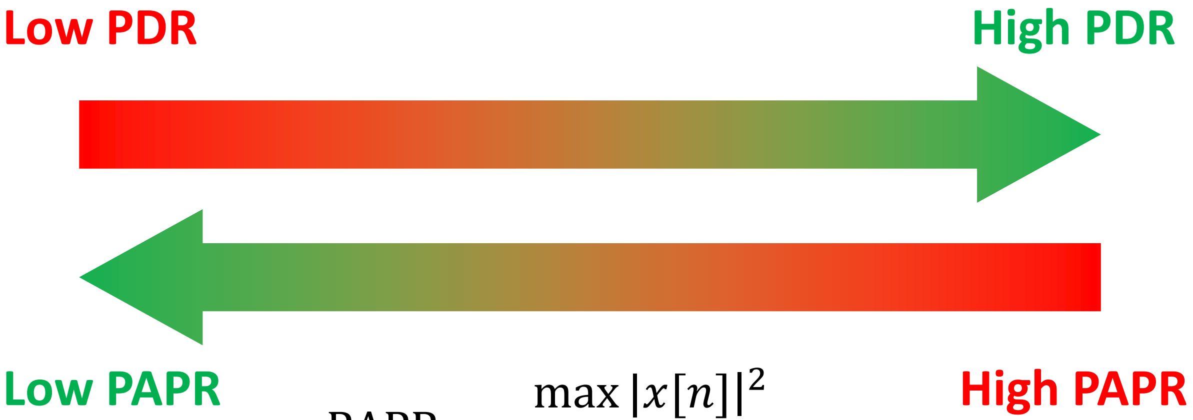


6m
5m
10m
Rx
Tx – pendulum motion
10m
20m

Pilot Design Optimization

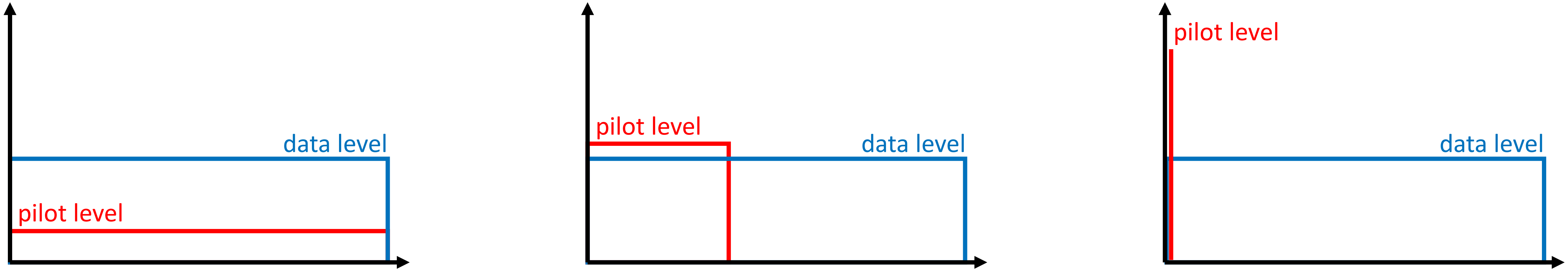
PDR and PAPR tradeoff

Random Pilots



Identical Pilots

$$\mathrm{PAPR} = \frac{\max |x[n]|^2}{\frac{1}{K}\sum_{n=1}^{K} |x[n]|^2}$$



Proposed pilot design formulation

Pilot tones: $\phi[k] = s[qG] = e^{j\phi_k}$

Pilot signal (one period): $\psi[n] = \frac{1}{\sqrt{Q}}\mathrm{IDFT}\{\phi[k]\}$

Unit amplitude

Known envelope

$$w_{L_p}[n] = \begin{cases} \frac{1}{\sqrt{L_p}}, & n < L_p \\ 0, & n \geq L_p \end{cases}$$

Find the phases ϕ_k

Phase retrieval problem: $\min_{\boldsymbol{\psi},\boldsymbol{\phi}} \left\| \mathbf{W}_{L_p}\boldsymbol{\psi} - \mathbf{F}_Q^H\boldsymbol{\phi} \right\|^2$

Pilot design – generalized GSA algorithm

Initialize $\boldsymbol{\phi}_0 = \text{rand}(Q,1), \quad J = \infty$

while $J > \eta$ do

$\boldsymbol{\psi}_i = e^{j\measuredangle[\text{IFFT}\{\boldsymbol{\phi}_{i-1}\}]}$ — time domain pilot signal

$\boldsymbol{\phi}_i = e^{j\measuredangle[\text{FFT}\{\mathbf{W}\boldsymbol{\psi}_i\}]}$ — pilot tones

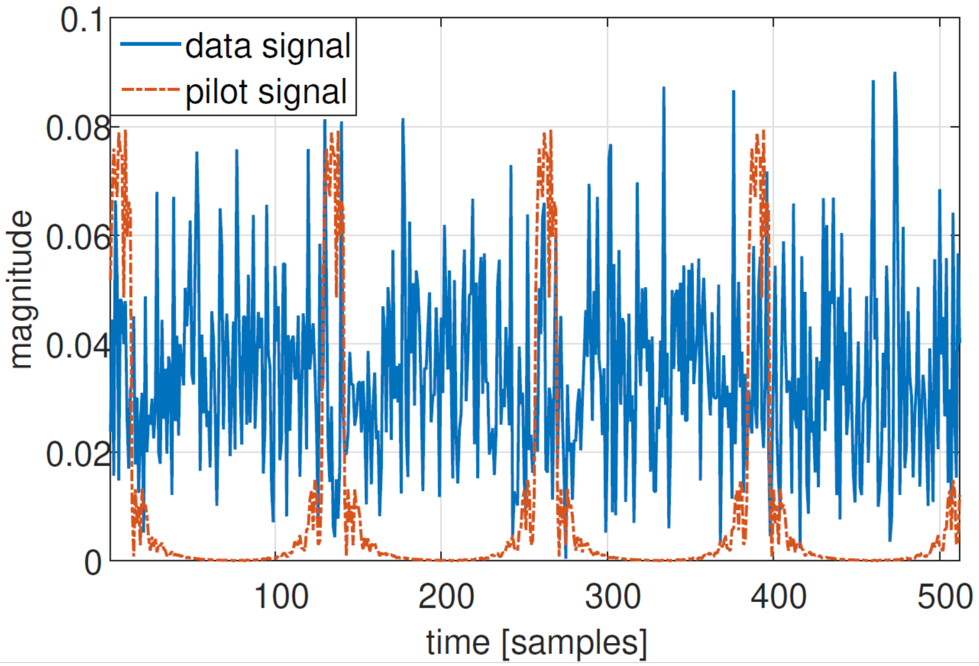
$\varepsilon = \| |\text{IFFT}\{\boldsymbol{\phi}_i\}| - \mathbf{w} \|^2$ — envelope error norm

$p = \dfrac{\max |\text{IFFT}\{\boldsymbol{\phi}_i\}|^2}{\frac{1}{Q}\sum_{n=1}^{Q} |\text{IFFT}\{\boldsymbol{\phi}_i\}|^2}$ — PAPR

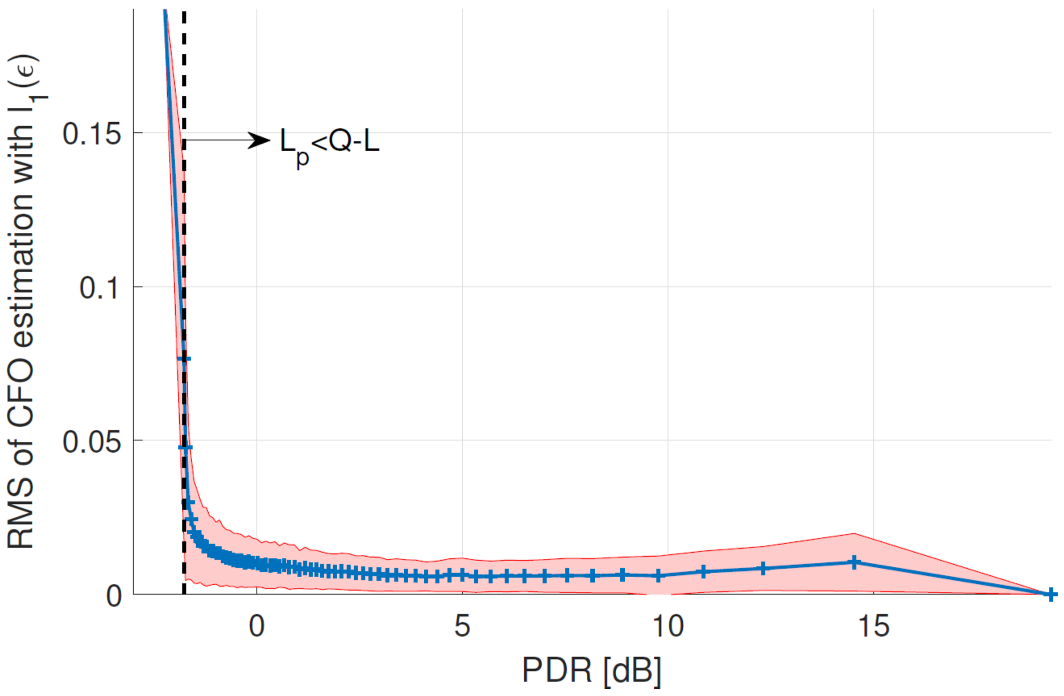
$J = \alpha\varepsilon + \beta p$

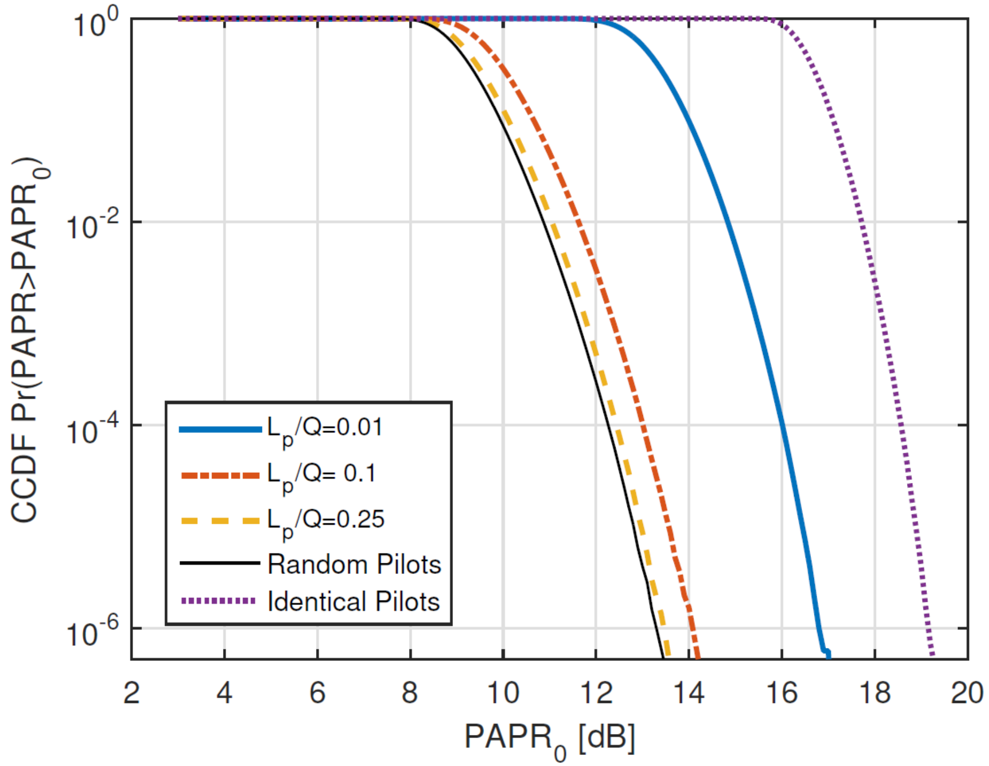
end while

return $\boldsymbol{\phi}_i$



Simulation results





Time-Varying CFO Estimation

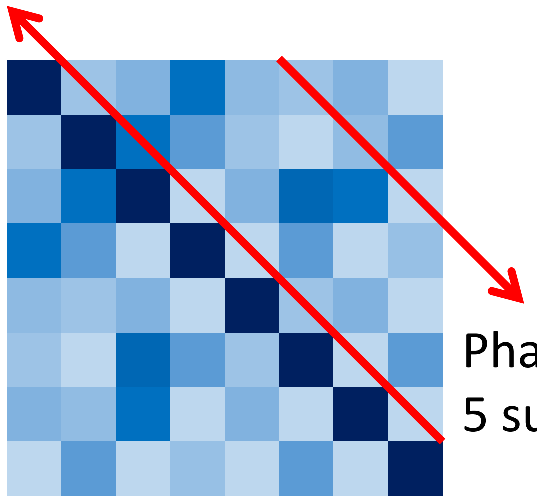
How can we capture TV-CFO?

$\mathbf{R}(\epsilon, g)$ becomes $\mathbf{R}(\epsilon, g, n)$:

$\mathbf{R}_{g,g'} = \mathbf{y}_g^H \mathbf{y}_{g'} = \mathbf{z}_g^H \mathbf{\Gamma}_g^H(\epsilon, n) \mathbf{\Gamma}_{g'}(\epsilon, n) \mathbf{z}_{g'}$

$\mathbf{\Gamma}_g^H(\epsilon, n) \mathbf{\Gamma}_{g'}(\epsilon, n)$ is a diagonal matrix

For constant CFO the diagonal is $d_{g,g'} = \alpha_g^* \alpha_{g'}$



Polynomial Model

- Time variations are decomposed to its Taylor series:

$$\epsilon[n] = \sum_{l=0}^{\infty} \epsilon_l n^l \quad , \qquad 0 \le n \le K-1$$

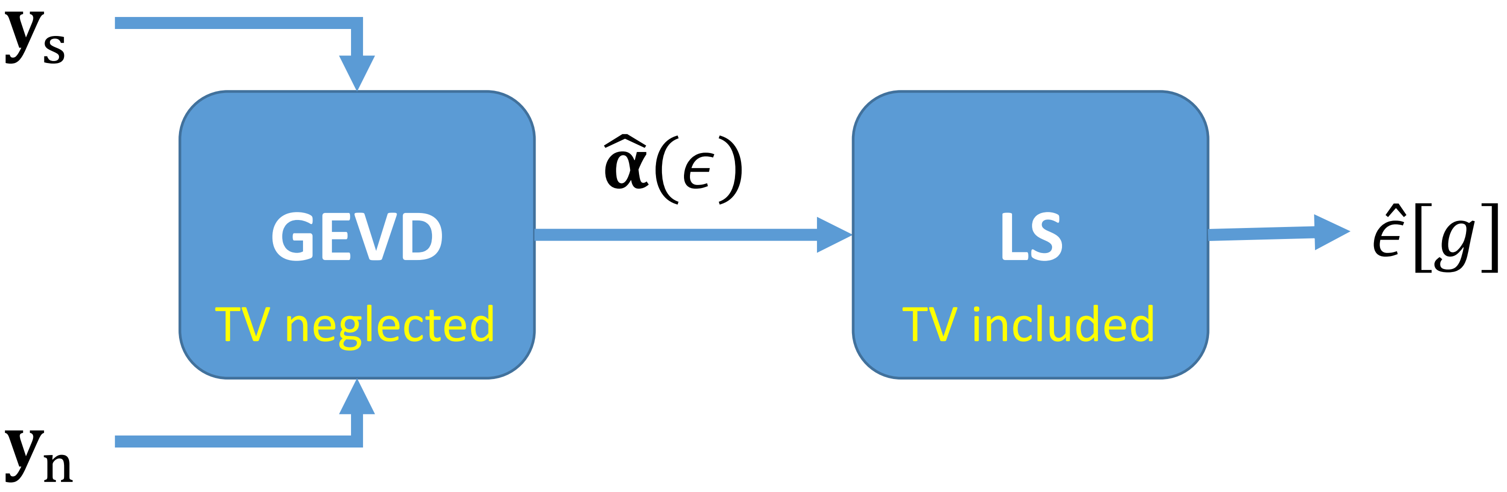
- The diagonal of $\mathbf{\Gamma}_g^H(\epsilon, n)\mathbf{\Gamma}_{g'}(\epsilon, n)$ becomes

$$d_{g,g'} = \alpha_g^* \exp\left\{\frac{j2\pi}{K}\sum_{l=1}^{\infty} \epsilon_l [r_l(n,g') - r_l(n,g)]\right\} \alpha_{g'}$$

$$\alpha_g = \exp\left\{\frac{2\pi}{G}\sum_{l=1}^{G-1} g^l Q^{l-1} \epsilon_{l-1}\right\}$$

$$r_{l(n,g)} = \sum_{k=1}^{l-1} \binom{l}{k} n^{l-k} (gQ)^k$$

Approximated solution



$$\max_{\boldsymbol{\alpha}} \frac{\boldsymbol{\alpha}^H \mathbf{R}_{\mathrm{s}} \boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \mathbf{R}_{\mathrm{n}} \boldsymbol{\alpha}} \quad \mathrm{s.t.}\ \boldsymbol{\alpha}^H \boldsymbol{\alpha} = 1$$

$$\arg[\hat{\boldsymbol{\alpha}}] \approx -\frac{2\pi}{G} \mathbf{A} \mathbf{Q} \boldsymbol{\epsilon}$$

$$\Rightarrow \hat{\boldsymbol{\alpha}} = \mathbf{V}_{\max}(\mathbf{R}_{\mathrm{n}}^{-1} \mathbf{R}_{\mathrm{s}})$$

$$\Rightarrow \hat{\epsilon} = -\frac{G}{2\pi} \mathbf{Q}^{-1} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \arg[\hat{\boldsymbol{\alpha}}]$$

Piecewise-Constant Model

- Time variations are represented as piecewise-constant:

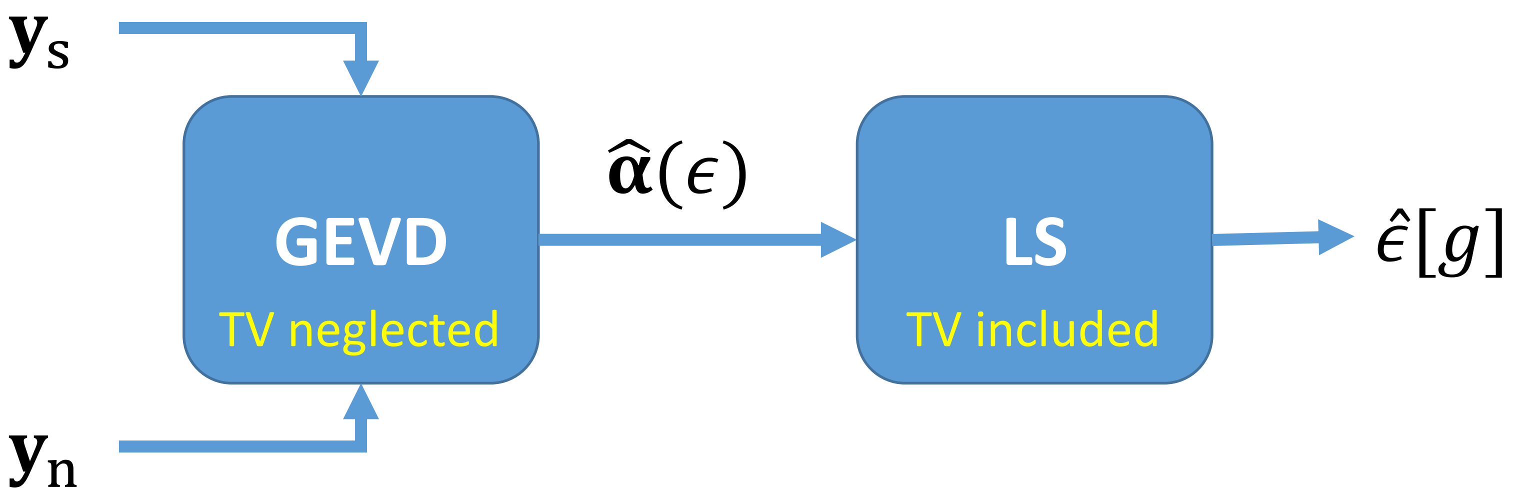
$$\epsilon[n] = \sum_{g=0}^{G-1} \epsilon_g u_g[n] \quad , \qquad 0 \le n \le K-1$$

- The diagonal of $\mathbf{\Gamma}_g^H(\epsilon, n)\mathbf{\Gamma}_{g'}(\epsilon, n)$ becomes

$$d_{g,g'} = \alpha_g^* \exp\left\{\frac{j2\pi}{K}\sum_{l=1}^{\infty}(\epsilon_{g'} - \epsilon_g)n\right\}\alpha_{g'} \quad , \qquad \alpha_g = e^{\frac{2\pi}{G}\epsilon_g g}$$

The time varying component is bounded by $\exp\left\{\frac{j2\pi}{K}|\epsilon_{g'} - \epsilon_g|Q\right\}$

Approximated solution



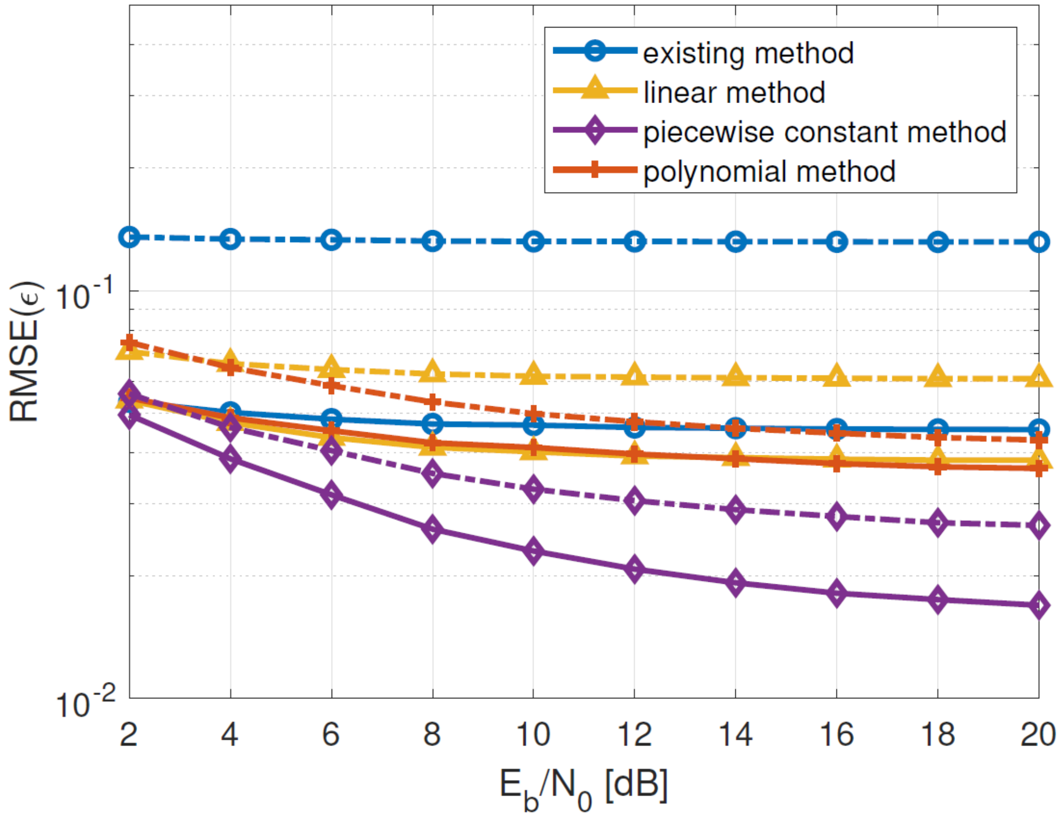
$$\max_{\boldsymbol{\alpha}} \frac{\boldsymbol{\alpha}^H \mathbf{R}_{\mathrm{s}} \boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \mathbf{R}_{\mathrm{n}} \boldsymbol{\alpha}} \quad \mathrm{s.t.}\ \boldsymbol{\alpha}^H \boldsymbol{\alpha} = 1$$

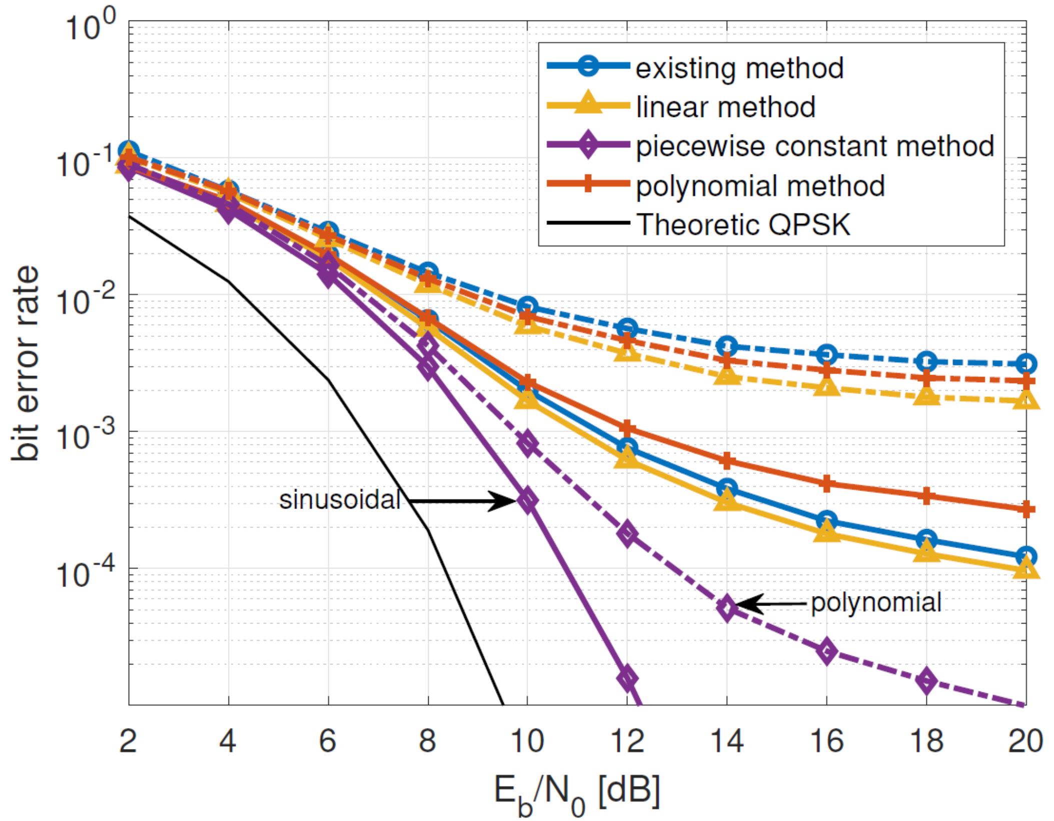
$$\mathrm{arg}[\hat{\boldsymbol{\alpha}}] \approx -\frac{2\pi}{G} \mathbf{G} \boldsymbol{\epsilon}$$

$$\Rightarrow \hat{\boldsymbol{\alpha}} = \mathbf{V}_{\mathrm{max}}(\mathbf{R}_{\mathrm{n}}^{-1} \mathbf{R}_{\mathrm{s}})$$

$$\Rightarrow \hat{\epsilon} = -\frac{G}{2\pi} \mathbf{G}^{-1} \mathrm{arg}[\hat{\boldsymbol{\alpha}}]$$

Simulation results





Polynomial model: $\epsilon[n] = \sum_{l=0}^{4} \frac{b_l}{K^l} n^l \quad , \quad b_l \sim U[-0.25, 0.25]$

Sinusoidal model: $\epsilon[n] = \Delta f \left[A_0 + A \sin\left(2\pi n \frac{f_{\sin}}{K} \right) \right] \quad , \quad A_0, A \sim U[-0.25, 0.25] \ , f_{\sin} \sim U[0.25, 2]$

Conclusions and Future Research

- A complete Tx-Rx scheme was suggested:
  - Reduced complexity closed form CFO estimation
  - Pilot design resolves the PAPR problem and makes the solution practical
  - Time-varying model allows deployment in harsh environments
- Future Research
  - Time Varying CIR
  - Combined pilots-data PAPR reduction
  - Proving the solution in sea trials
  - Model order estimation and channel sensing for the TV estimator