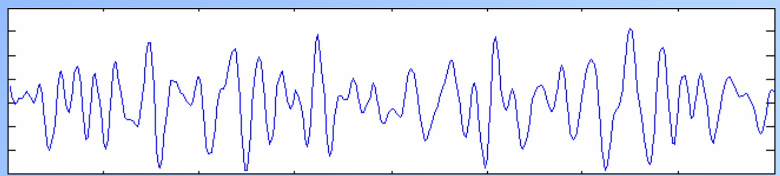
Multichannel Deconvolution of Sparse Reflectivity Images Using Layered Medium Models



Alon Heimer

January, 2008

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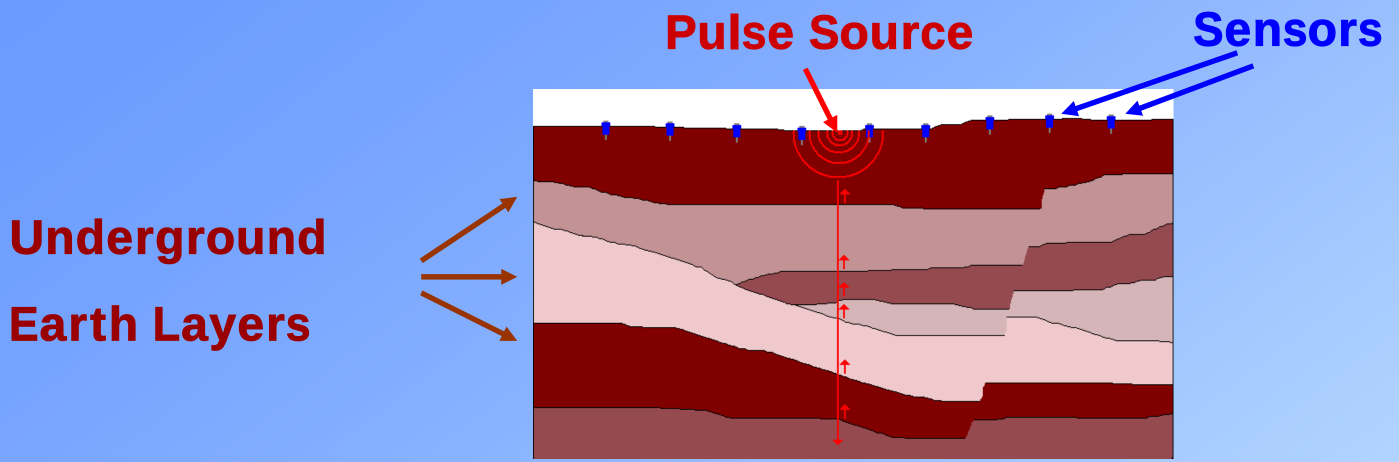
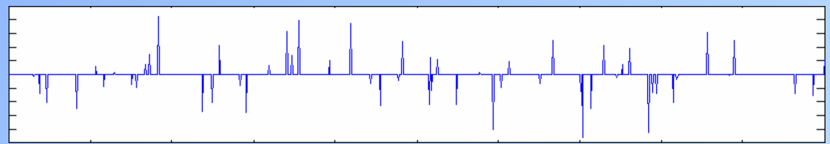
Seismic Exploration - Single Channel Model

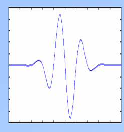


Sparse Reflectivity x[n]:



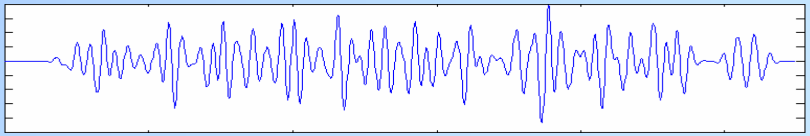
Bernouilli-Gaussian distribution: $x = \{t, a\}: \quad t \sim B(\lambda) \quad (a \mid t) \sim N(0, \sigma_a^2)$

Finite Support Wavelet h[n]:



Seismic Trace z[n]:

$$z[n] = x[n] * h[n] + e[n]$$



Noise e[n]:

$$e \sim N(0, \sigma_e^2)$$

Single Channel Model (2)

$$z^{(m)}[n] = \sum_{k=0}^{K-1} h[k]\, x^{(m)}[n-k] + e^{(m)}[n] \quad \text{for } m = 1, 2, \ldots, M$$

$$z^{(m)}[n] = \sum_{p=0}^{P} h\left[n - n_{m,p}\right] a_{m,p} + e^{(m)}[n]$$

$n_{m,p}$ is the time of reflector *p* in column *m*

$a_{m,p}$ is the amplitude of reflector *p* in column *m*

$$\mathbf{z}^{(m)} = \mathbf{H}^{(m)} \mathbf{a}^{(m)} + \mathbf{e}^{(m)}$$

or

$$\mathbf{z}^{(m)} = \mathbf{X}^{(m)} \mathbf{h} + \mathbf{e}^{(m)}$$

Where:

$$\mathbf{z}^{(m)} = \left[z^{(m)}[0] \;\; z^{(m)}[1] \;\; \ldots \;\; z^{(m)}[N] \right]^T$$

$$\mathbf{H}^{(m)}_{n,p} = h\left[n - n_{m,p}\right]$$

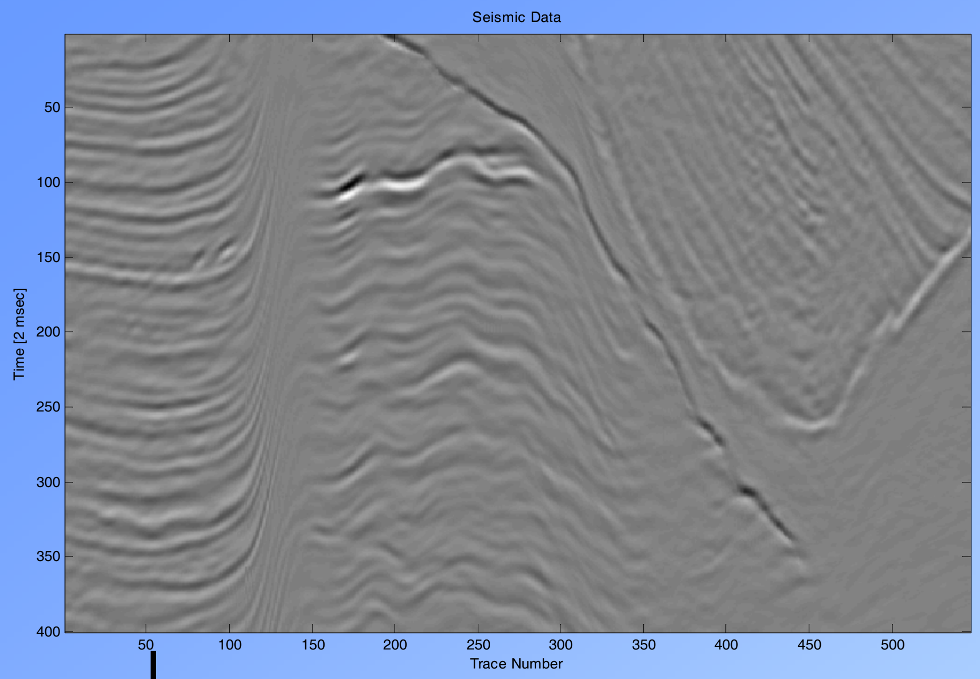
$$\mathbf{a}^{(m)} = \left[a_{m,1} \;\; a_{m,2} \;\; \ldots \;\; a_{m,P} \right]^T$$

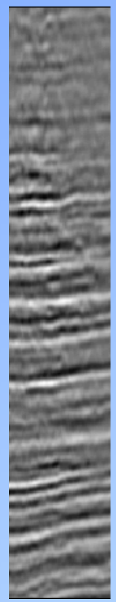
$$\mathbf{e}^{(m)} = \left[e^{(m)}[0] \;\; e^{(m)}[1] \;\; \ldots \;\; e^{(m)}[N] \right]^T$$

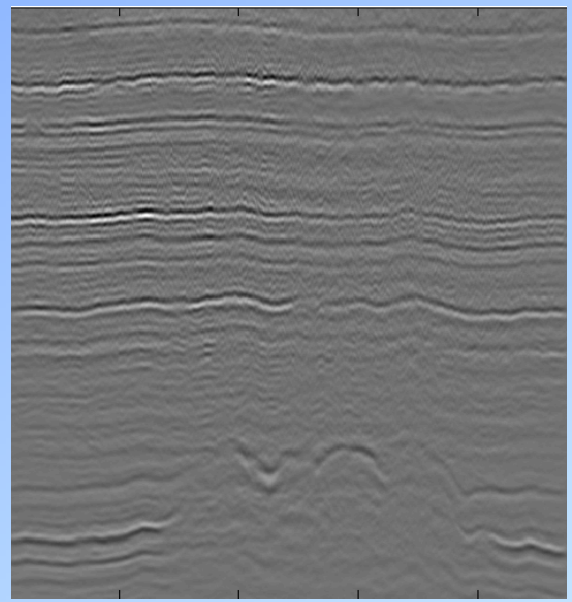
Where:

$$\mathbf{X}^{(m)}_{n,p} = x^{(m)}[n-p]$$

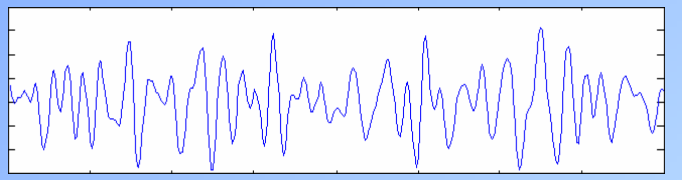
Multichannel Seismic Data



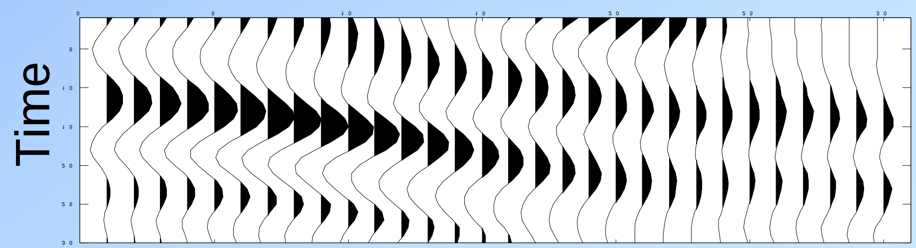




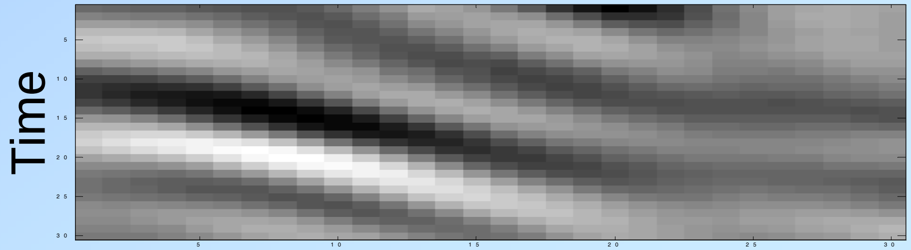
Trace #50



Time [2 msec]



Trace #



Trace #

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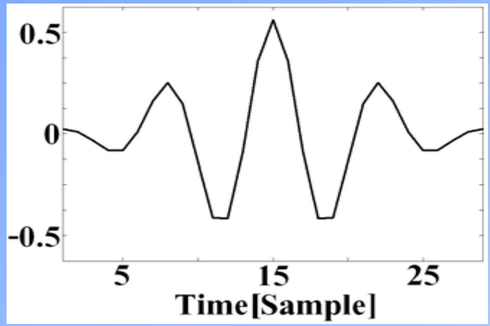
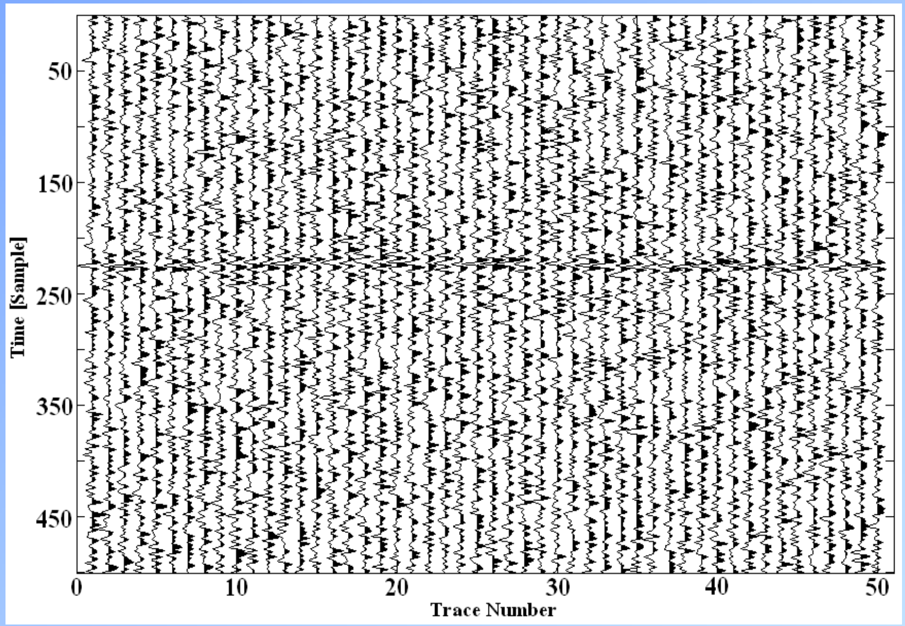
Goal and Motivation
Advantage of Joint Statistics

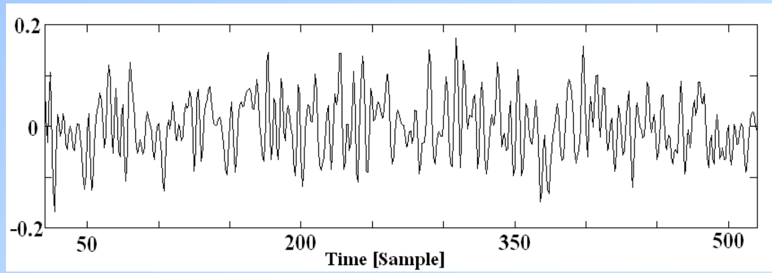
Wavelet



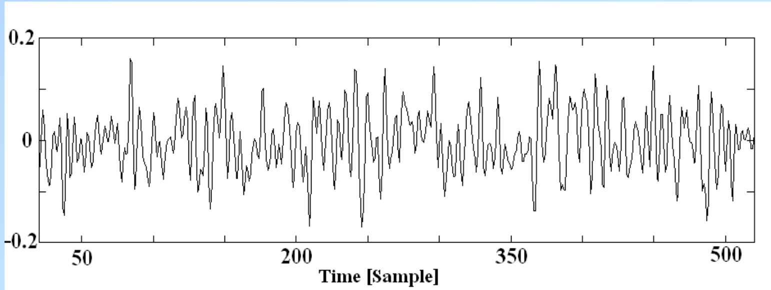
All 50 Traces



Trace #7



Trace #12



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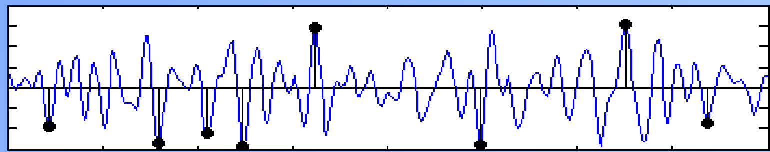
Karresen and Taxt (1998)
Single Channel Blind Deconvolution

Estimation Criterion:

$$[\hat{t},\hat{a},\hat{h}] = \arg\max_{t,a,h} \; p\left(t,a,h \mid z\right)$$

1. Initialization of x[n].



2. Wavelet Estimation.

$$[\hat{h}] = \arg\max_{h} \; p\left(h \mid t, a, z\right)$$

$$\hat{h} = \left(X^T X\right)^{-1} X^T z$$

3. Reflectivity Estimation.

$$[\hat{t},\hat{a}] = \arg\max_{t,a} \; p\left(t, a \mid h, z\right)$$

Alter $\hat{t}$ to get $\hat{t}_i$

$$\hat{a}_i = \arg\max_{a} \; p\left(a \mid t_i, h, z\right)$$

$$\hat{a}_i = \left(H_i^T H_i + \frac{\sigma_e^2}{\sigma_a^2} I \right)^{-1} H_i^T z$$

$$\ell_i = p\left(\hat{t}_i, \hat{a}_i \mid h, z\right)$$

$$\ell_i = \left\| z - H_i a_i \right\|_2^2 - \theta M$$

M is number of reflectors

θ is sparsity measure

$$i = \arg\max_{i} \; \ell_i$$

$$\hat{t} = \hat{t}_i, \quad \hat{a} = \hat{a}_i$$

Karresen and Taxt
Multichannel Blind Deconvolution

Altered quality measure:

$$\ell_i = \|z - H_i a_i\|_2^2 - \theta M + \nu^{-} M^{-} + \nu^{\setminus} M^{\setminus} + \nu^{/} M^{/} - \nu^{|} M^{|} - \nu^{\|} M^{\|}$$

M	Number of reflectors
θ	Sparsity measure
M^{-}	Number of reflectors with neighbors in adjacent columns in same row
$\mathrm{M}^{\setminus}$	Number of reflectors with neighbors in adjacent columns in naxt row
$\mathrm{M}^{/}$	Number of reflectors with neighbors in adjacent columns in previous row
$\mathrm{M}^{|}$	Number of reflector pairs one sample apart in current column
$\mathrm{M}^{\|}$	Number of reflector pairs two samples apart in current column

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- Seismic Exploration - Single Channel Model
- Goal and Motivation
- Previous Works
  – Kaaresen and Taxt 1998
  – Idier and Goussard 1993
- Multichannel Deconvolution Using Dynamic Programming
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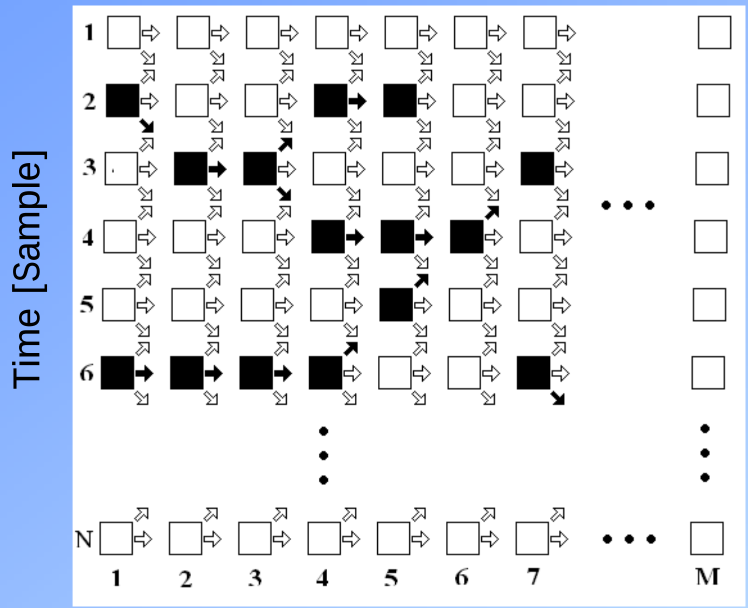
- Conclusion and Future Research

Idier and Goussard (1993)
Markov Bernoulli Reflectivity Model



Notation:

$q_{i,j}$	Trace j, Time i Reflectivity Indicator	$\mathbf{q}_j$	Trace j, Reflectivity Indicator Vector
$t^0_{i,j}$	Trace j, Time i Ascending Transition Variable	$\mathbf{t}^0_j$	Trace j, Ascending Transition Vector
$t^1_{i,j}$	Trace j, Time I Horizontal Transition Variable	$\mathbf{t}^1_j$	Trace j, Horizontal Transition Vector
$t^2_{i,j}$	Trace j, Time i Descending Transition Variable	$\mathbf{t}^2_j$	Trace j, Descending Transition Vector
$r_{i,j}$	Trace j, Time i Reflectivity Sample	$\mathbf{r}_j$	Trace j, Reflectivity Vector

Markov Bernoulli Reflectivity Model (2)

Properties of geometric field:

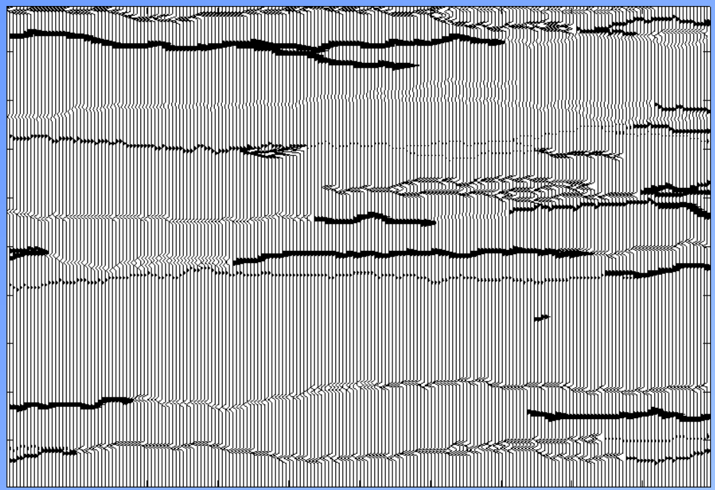
1. $\mathbf{t}_j^0, \mathbf{t}_j^1, \mathbf{t}_j^2$ are white processes
2. $\mathbf{q}_j$ is a white process
3. $p\left(t_{i,j}^0, t_{i,j}^1, t_{i,j}^2\right) = p\left(t_{i,j}^0\right) p\left(t_{i,j}^1\right) p\left(t_{i,j}^2\right)$
4. $p\left(t_{i+1,j}^0, t_{i,j}^1, t_{i-1,j}^2, q_{i,j}\right) = p\left(q_{i,j}, t_{i,j}^0, t_{i,j}^1, t_{i,j}^2\right)$
5. $p\left(t_{i+1,j}^0 = 0, t_{i,j}^1 = 0, t_{i-1,j}^2 = 0 \mid q_{i,j} = 1\right) = 0$
6. $p\left(t_{i+1,j}^0 = 1\right) = \mu^0, p\left(t_{i+1,j}^1 = 1\right) = \mu^1, p\left(t_{i+1,j}^2 = 1\right) = \mu^2, \forall i, j$
7. $p\left(q_{i,j} = 1\right) = \lambda$
8. $p\left(q_{i,j} = 1 \mid t_{i+1,j-1}^0 = 0, t_{i,j-1}^1 = 0, t_{i-1,j-1}^2 = 0\right) = \varepsilon$

***Properties of amplitude field*:**

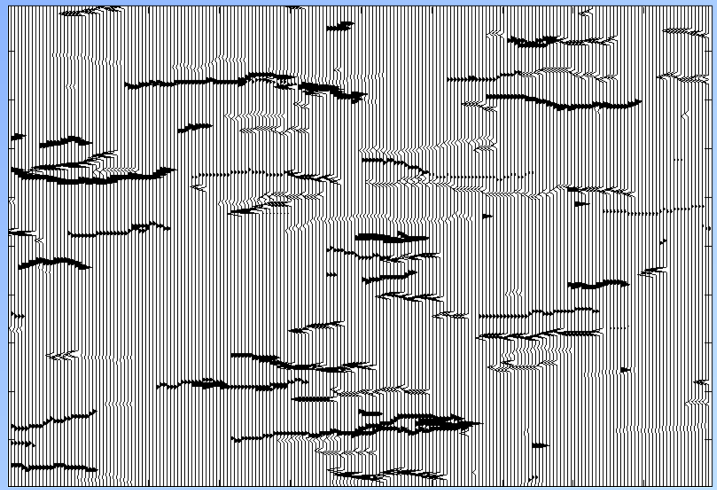
1. $\left(q_{i,j} = 0\right) \Rightarrow \left(r_{i,j} = 0\right)$
2. $\left(q_{i,j} = 1 \text{ , with no predecessors}\right) \Rightarrow \left(r_{i,j} \sim N\left(0, \sigma_a^2\right)\right)$
3. $\left(q_{i,j} = 1 \text{ is a unique successor of unique predecessor } r_{i_1,j-1}\right) \Rightarrow \left(r_{i,j} \sim N\left(c r_{i_1,j-1}, \left(1 - c^2\right)\sigma_a^2\right)\right)$
4. $\left(q_{i,j} = 1 \text{ is not a unique successor or has more than one predecessor}\right) \Rightarrow \left(r_{i,j} \sim N\left(0, \sigma_a^2\right)\right)$

Markov Bernoulli Reflectivity Model (3)

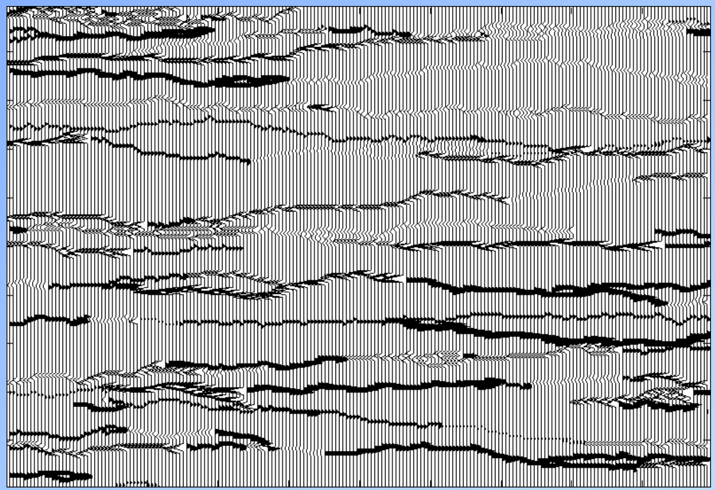
$\lambda = 0.05,\ \varepsilon = 0.0001,\ \mu^0 = 0.0084,\ \mu^1 = 0.0335,\ \mu^2 = 0.0084$



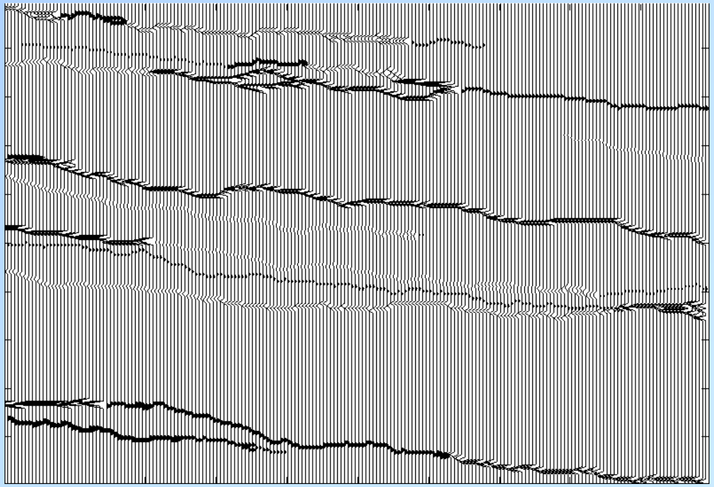
$\lambda = 0.053,\ \varepsilon = 0.0003,\ \mu^0 = 0.0084,\ \mu^1 = 0.0335,\ \mu^2 = 0.0084$



$\lambda = 0.099,\ \varepsilon = 0.0001,\ \mu^0 = 0.0168,\ \mu^1 = 0.067,\ \mu^2 = 0.0168$



$\lambda = 0.05,\ \varepsilon = 0.0001,\ \mu^0 = 0.0044,\ \mu^1 = 0.0335,\ \mu^2 = 0.0124$



Idier and Goussard
The Deconvolution Algorithm

MAP Estimation:

$$P(\mathbf{q},\mathbf{t},\mathbf{r}\mid\mathbf{z}) \propto P(\mathbf{q}_1,\mathbf{r}_1\mid\mathbf{z}_1)\prod_{j=2}^{J}P(\mathbf{t}_{j-1},\mathbf{q}_j,\mathbf{r}_j\mid\mathbf{z}_j,\hat{\mathbf{q}}_{j-1},\hat{\mathbf{r}}_{j-1})$$

Recursive approach for maximization:

1. Initial Step for Column 1:

$$(\hat{\mathbf{q}}_1,\hat{\mathbf{r}}_1) = \arg\max_{\mathbf{q}_1,\mathbf{r}_1} P(\mathbf{q}_1,\mathbf{r}_1\mid\mathbf{z}_1)$$

2. Columns 2,3,..,J

$$(\hat{\mathbf{t}}_{j-1},\hat{\mathbf{q}}_j,\hat{\mathbf{r}}_j) = \arg\max_{\mathbf{t}_{j-1},\mathbf{q}_j,\mathbf{r}_j} P(\mathbf{t}_{j-1},\mathbf{q}_j,\mathbf{r}_j\mid\mathbf{z}_j,\hat{\mathbf{q}}_{j-1},\hat{\mathbf{r}}_{j-1})$$

Idier and Goussard:

Detection and Estimation

Detection:

$$\left(\hat{\mathbf{t}}_{j-1},\hat{\mathbf{q}}_j\right)=\underset{\mathbf{t}_{j-1},\mathbf{q}_j}{\arg\max}\; P\left(\mathbf{t}_{j-1},\mathbf{q}_j \mid \mathbf{z}_j,\hat{\mathbf{q}}_{j-1},\hat{\mathbf{r}}_{j-1}\right)$$

$$\propto P\left(\mathbf{z}_j \mid \mathbf{t}_{j-1},\hat{\mathbf{r}}_{j-1},\mathbf{q}_j\right)P\left(\mathbf{q}_j \mid \mathbf{t}_{j-1}\right)P\left(\mathbf{t}_{j-1} \mid \hat{\mathbf{q}}_{j-1}\right)$$

Extended SMLR Method

Estimation:

$$\mathbf{r}_j \mid \hat{\mathbf{q}}_j,\hat{\mathbf{q}}_{j-1},\hat{\mathbf{t}}_{j-1},\hat{\mathbf{r}}_{j-1} \sim N\left(m_j, M_j\right)$$

$$\hat{\mathbf{r}}_j=\underset{\mathbf{r}_j}{\arg\max}\; P\left(\mathbf{r}_j \mid \mathbf{z}_j,\hat{\mathbf{q}}_j,\hat{\mathbf{q}}_{j-1},\hat{\mathbf{t}}_{j-1},\hat{\mathbf{r}}_{j-1}\right)$$

$$\hat{\mathbf{r}}_j = m_j + M_j\mathbf{H}^T\left(\mathbf{H}M_j\mathbf{H}^T+\sigma_e^2\mathbf{I}\right)^{-1}\left(\mathbf{z}_j-\mathbf{H}m_j\right)$$

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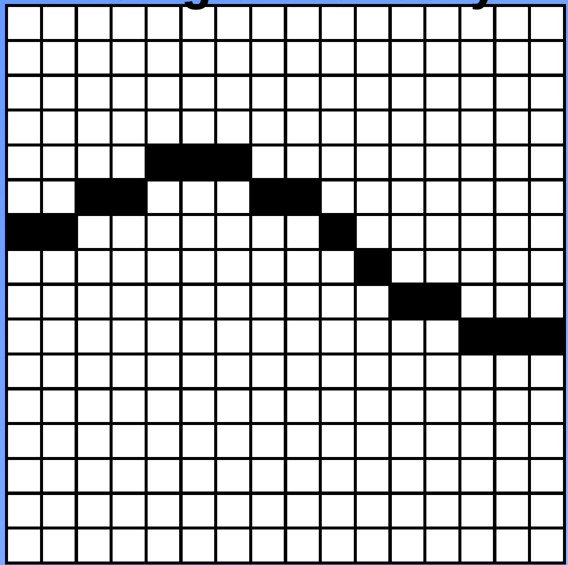
The Continuous Model

Single boundary



$$P=\left(n^{(1)},n^{(2)},\ldots,n^{(M)}\right) \qquad \mathbf{a}(P)=\left(a^{(1)P},a^{(2)P},\ldots,a^{(M)P}\right)$$

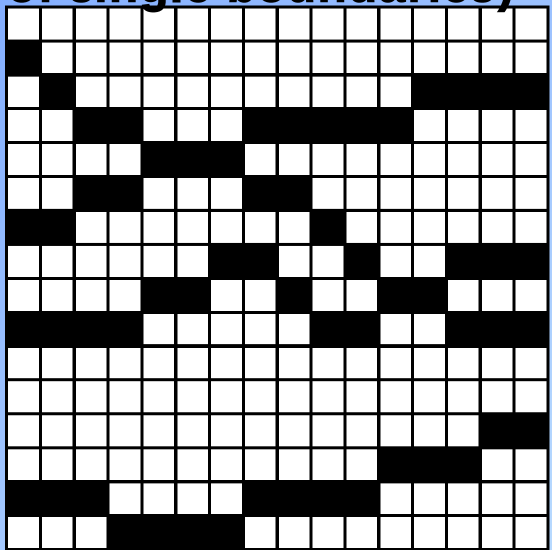
$$n^{(1)}\sim U(1,N)$$

$$n^{(m)}\mid n^{(m-1)}\sim U\left(n^{(m-1)}-1,n^{(m-1)}+1\right)$$

$$a^{(1)P}\sim N\left(0,\sigma_a^2\right)$$

$$a^{(m)P}\mid a^{(m-1)P}\sim N\left(ca^{(m-1)P},\left(1-c^2\right)\sigma_a^2\right)$$

2D reflectivity (Union of single boundaries)



$$X^{(m)}[n]=\begin{cases}0, & \text{No boundary passes through this point}\\ a^{(m)P_i}, & \text{P}_i \text{ is the only boundary that passes through this point}\\ \sim N\left(0,\sigma_a^2\right), & \text{More than one boundary passes through this point}\end{cases}$$

Dynamic Programming (1)

Can be used in case of:

- Overlapping Subproblems
- Optimal Substructure

Example - Fibonacci Sequence:
F(n) = F(n-1)+F(n-2) = F(n-2)+**F(n-3)**+**F(n-3)**+F(n-4)

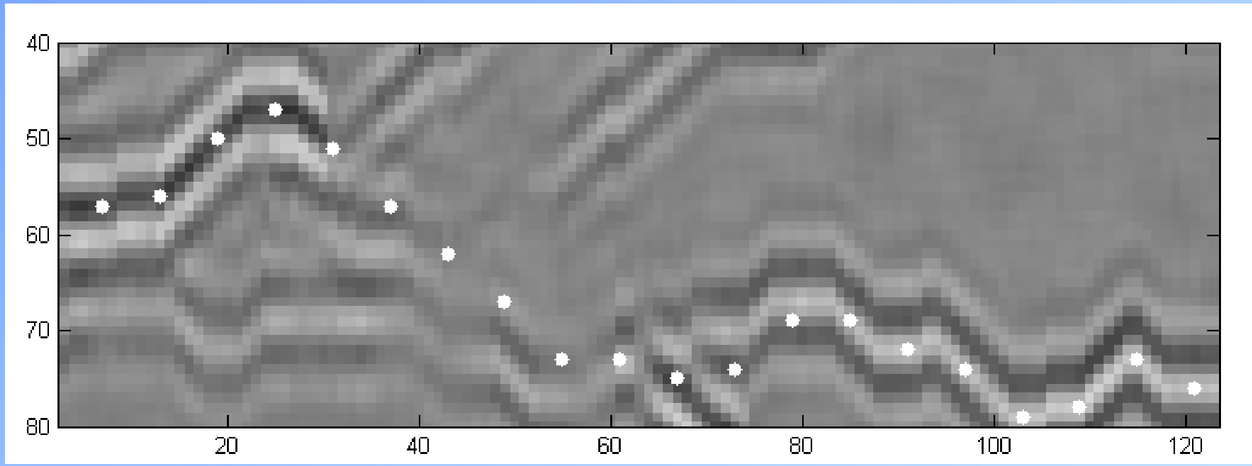
Examples where **Dynamic Programming** can be used and its' benefit:

- Subarray with maximum sum: O(n^2) ⟶ O(n)
- Shortest continuous path computation: O(3^m) ⟶ O(n*m)
- Viterbi Algorithm: O(n^m) ⟶ O(n^2*m)

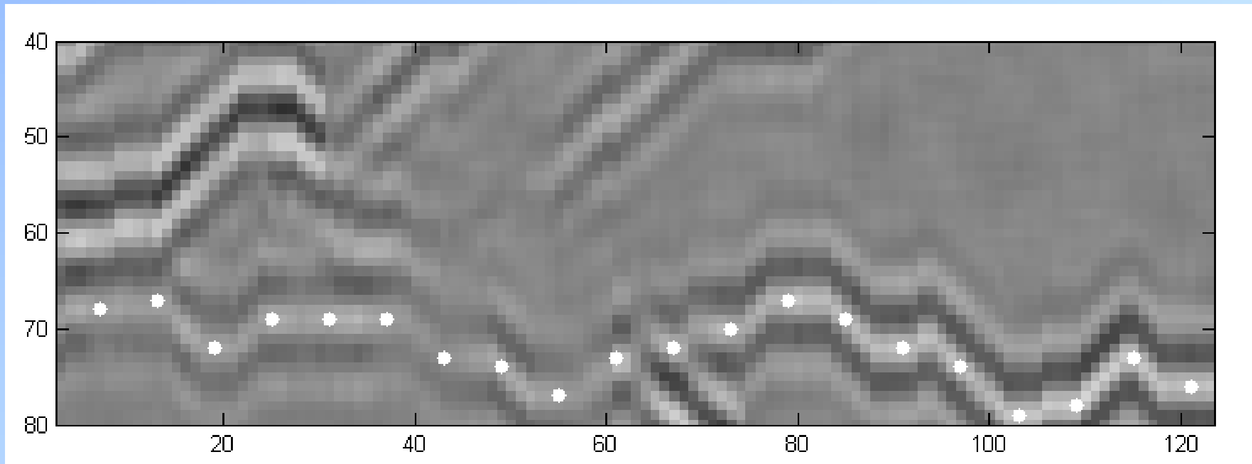
Dynamic Programming (2)

Shortest continuous path

$$Length \triangleq \sum_{m=1}^{M} |p_m|$$



$$Length \triangleq \left| \sum_{m=1}^{M} p_m \right|$$



Dynamic Programming (3)

- $\left\{\left(n^{(m)},m\right)\right\}_{m=1}^{M}$ - Path of the image
- ***Continuous path***: $\left|n^{(m)}-n^{(m-1)}\right|<d$ for all *m,* where *d* is some small positive integer constant.
- ***Shortest path***: a certain length measure is minimized for this path among all the continuous paths.
- S_P - Length of path P
- $s(n,m,P)$ - Contribution of pixel (n,m) to length when concatenated with path P. It depends only on the last pixel in P
- $P_{n,m}$ - Continuous path that starts in column 1 and ends in pixel (n,m)
- $P_{n,m}^{o}$ - Shortest continuous path that starts in column 1 and ends in pixel (n,m)

Extraction of Shortest Path

Initialization:

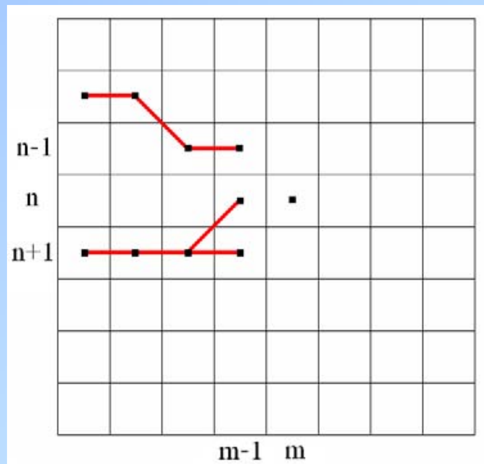
$$P^o_{n,1} = (n,1) \quad \text{for } n = 1, 2, \dots, N$$

$$S_{P^o_{n,1}} = s(n,1,\{\ \}) \quad \text{for } n = 1, 2, \dots, N$$

For each column m=1,2,...,M:

$$S_{P^o_{n,m}} = \min_{n-d \le k \le n+d} S_{P^o_{n,1}} + s\left(n, m, P^o_{k,m-1}\right) \quad \text{for } n = 1, 2, \dots, N$$

$$P^o_{n,m} = \left(P^o_{k^o,m-1} \vdots (n,m)\right) \quad \text{for } n = 1, 2, \dots, N$$



Ending:

$$n^o = \arg\min_n \ S_{P^o_{n,M}}$$

$$P_{opt} = P^o_{n^o,M}$$

Steps of the Algorithm

Reflectivity Initialization: Find a few continuous paths with highest absolute sum of gray levels in the data (using Dynamic Programming)

Wavelet Estimation: Least Squares Wavelet Estimation

Reflectivity Update:

1. Remove a single reflectors path of reflectors from reflectivity estimate

2. For each pixel, find probability of existance of new reflector at that location and estimate its amplitude.

$$\ell_i^{(m)} = \max_{\mathbf{a}_i^{(m)}} \log\left(p\left(\mathbf{z}^{(m)} \mid \mathbf{a}_i^{(m)}, \tilde{\mathbf{n}}_i^{(m)}\right)\right)$$

$$\hat{\mathbf{a}}_i^{(m)} = \arg\max_{\mathbf{a}_i^{(m)}} \log\left(p\left(\mathbf{z}^{(m)} \mid \mathbf{a}_i^{(m)}, \tilde{\mathbf{n}}_i^{(m)}\right)\right)$$

3. Find continuous path of reflectors with highest probability (using Dynamic Programming) and add it to the estimate .

Dynamic Programming and 2D MAP Estimation

MAP criterion for continuous reflectors path:

$$\left(\hat{i}^{(1)},\hat{i}^{(2)},\ldots,\hat{i}^{(M)}\right)=\underset{i^{(1)},i^{(2)},\ldots,i^{(M)}}{\arg\max}\; p\left(\hat{\mathbf{a}}_{i^{(1)}}^{(1)},\hat{\mathbf{a}}_{i^{(2)}}^{(2)},\ldots,\hat{\mathbf{a}}_{i^{(M)}}^{(M)},\hat{\mathbf{n}}_{i^{(1)}}^{(1)},\hat{\mathbf{n}}_{i^{(2)}}^{(2)},\ldots,\hat{\mathbf{n}}_{i^{(M)}}^{(M)}\mid\mathbf{z}^{(1)},\mathbf{z}^{(2)},\ldots,\mathbf{z}^{(M)}\right)$$

$\hat{\mathbf{n}}_{i^{(m)}}^{(m)}$ - vector of reflector indicators in mth column

$\hat{\mathbf{a}}_{i^{(m)}}^{(m)}$ - amplitudes of reflectors

$\mathbf{z}^{(m)}$ - mth trace

$$=\underset{i^{(1)},i^{(2)},\ldots,i^{(M)}}{\arg\max}\; \mathrm{p}\left(\mathbf{z}^{(1)}\mid\hat{\mathbf{a}}_{i^{(1)}}^{(1)},\hat{\mathbf{n}}_{i^{(1)}}^{(1)}\right)p\left(\hat{\mathbf{a}}_{i^{(1)}}^{(1)}\mid\hat{\mathbf{n}}_{i^{(1)}}^{(1)}\right)\prod_{m=2}^{M}\mathrm{p}\left(\mathbf{z}^{(m)}\mid\hat{\mathbf{a}}_{i^{(m)}}^{(m)},\hat{\mathbf{n}}_{i^{(m)}}^{(m)}\right)p\left(\hat{\mathbf{a}}_{i^{(m)}}^{(m)}\mid\hat{\mathbf{n}}_{i^{(m)}}^{(m)},\hat{\mathbf{a}}_{i^{(m-1)}}^{(m-1)}\right)$$

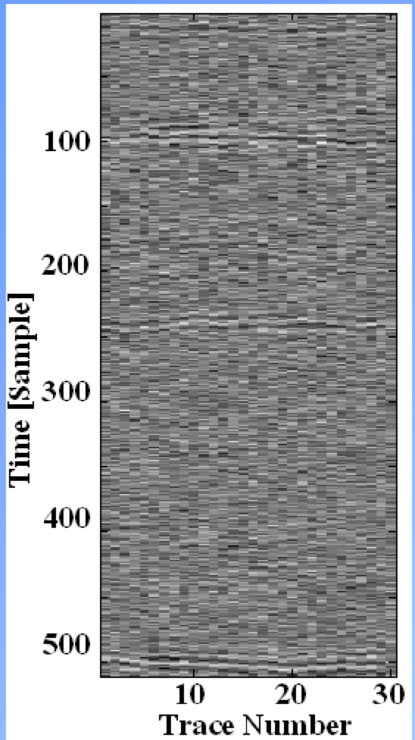
$$\left(\hat{i}^{(1)},\hat{i}^{(2)},\ldots,\hat{i}^{(M)}\right)=\underset{i^{(1)},i^{(2)},\ldots,i^{(M)}}{\arg\min}\left[-\ell_{i^{(1)}}^{(1)}+\frac{\left(\hat{a}_{i^{(1)}}^{(1)}\right)^2}{2\sigma_a^2}\right]+\sum_{m=2}^{M}\left[-\ell_{i^{(m)}}^{(m)}+\frac{\left(\hat{a}_{i^{(m)}}^{(m)}-c\hat{a}_{i^{(m-1)}}^{(m-1)}\right)^2}{2\left(1-c^2\right)\sigma_a^2}\right]$$

Path with highest probability can be found using Dynamic Programming by defining:

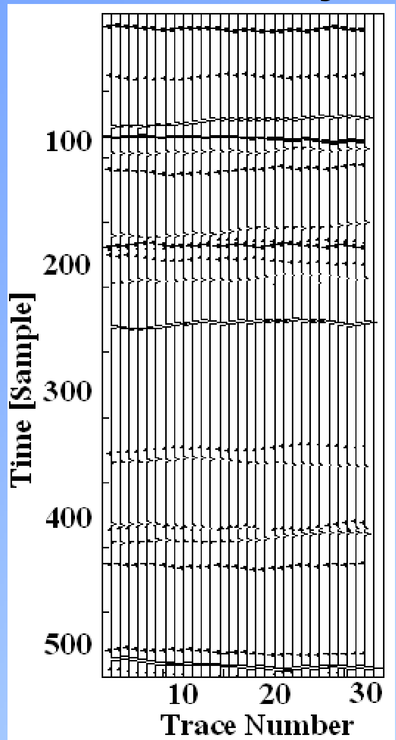
$$s\left(n,1,\{\ \}\right)\triangleq\left[-\ell_{i^{(1)}}^{(1)}+\frac{\left(\hat{a}_{i^{(1)}}^{(1)}\right)^2}{2\sigma_a^2}\right]\qquad s\left(n,m,P\right)\triangleq\left[-\ell_{i^{(m)}}^{(m)}+\frac{\left(\hat{a}_{i^{(m)}}^{(m)}-c\hat{a}_{last}\left(P\right)\right)^2}{2\left(1-c^2\right)\sigma_a^2}\right]$$

Low SNR Results (-15 dB)

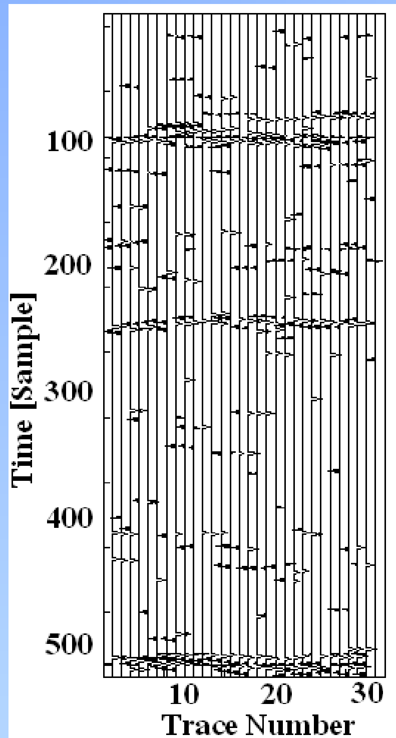
Data

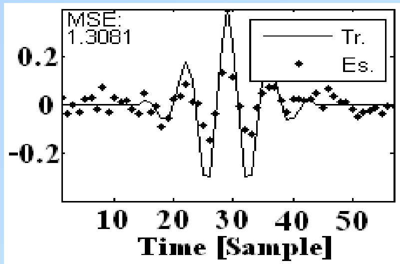


True Reflectivity

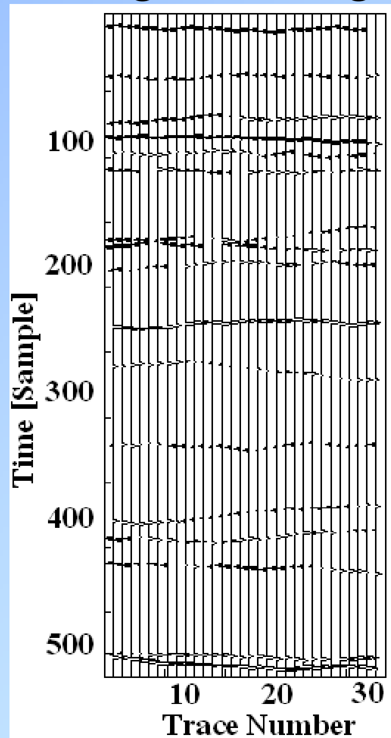


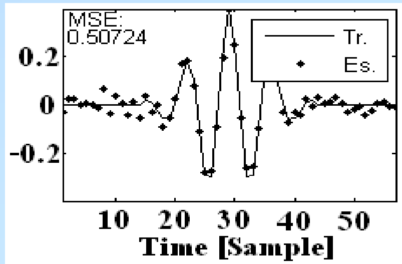
Karresen and Taxt





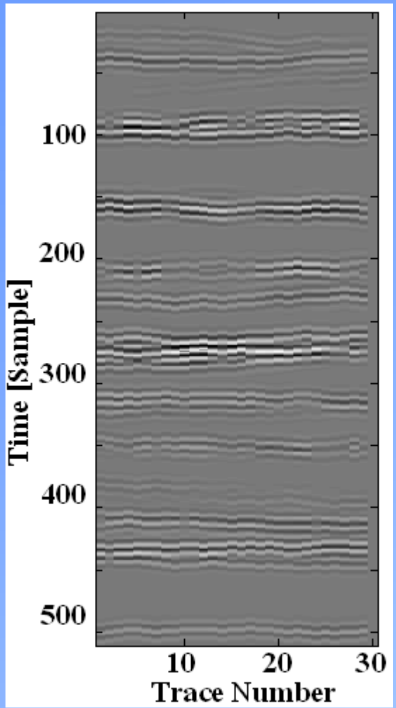
Dynamic Programming



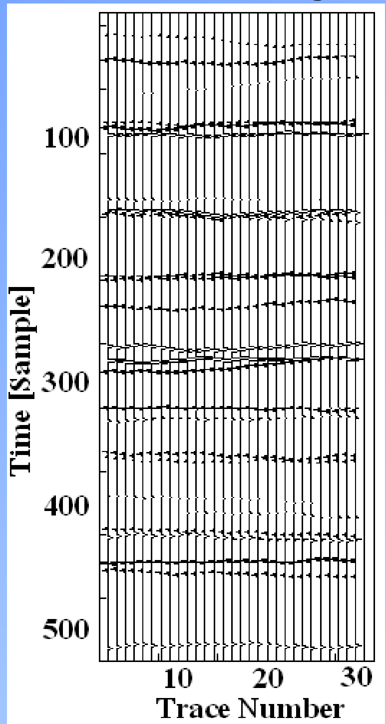


High SNR Results (100 dB)

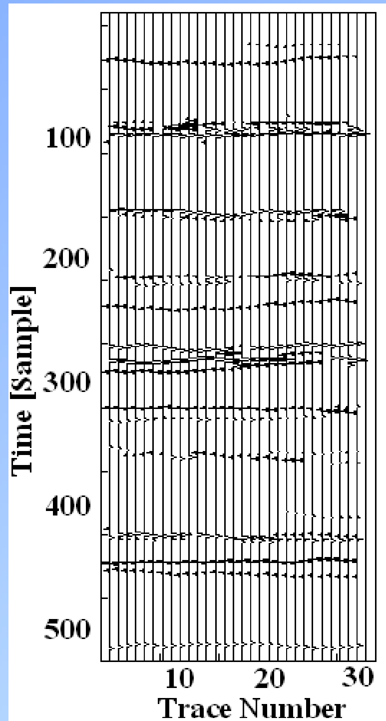
Data



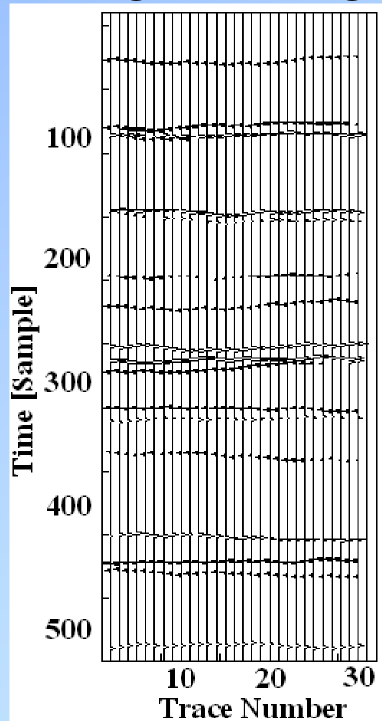
True Reflectivity

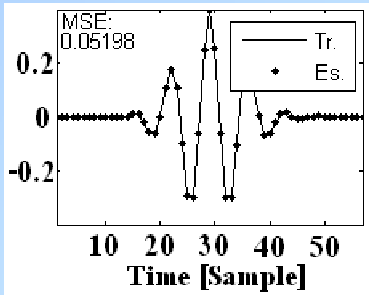


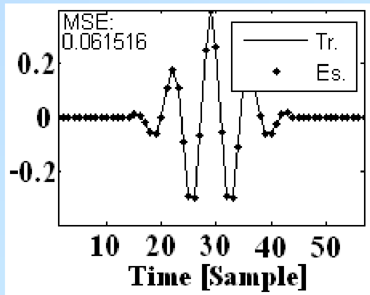
Karresen and Taxt



Dynamic Programming

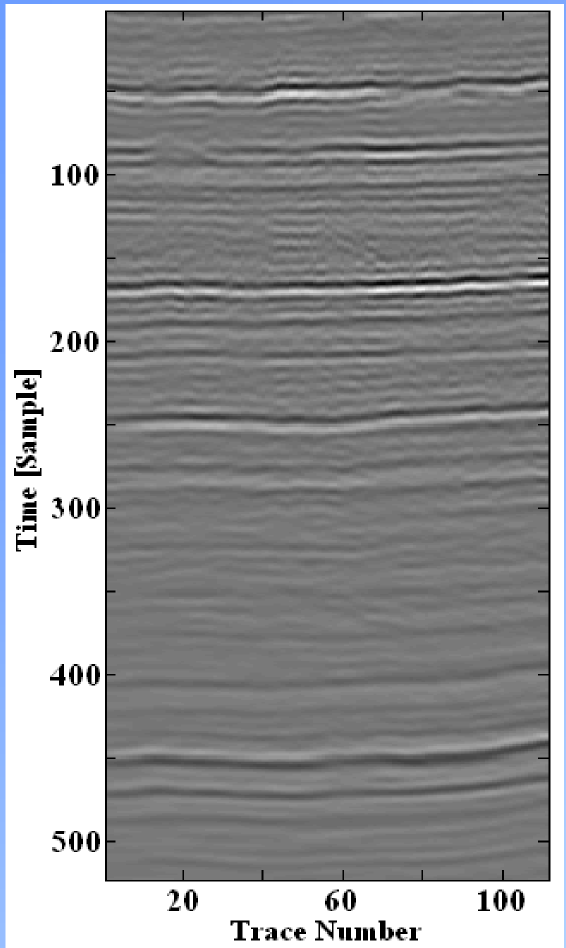




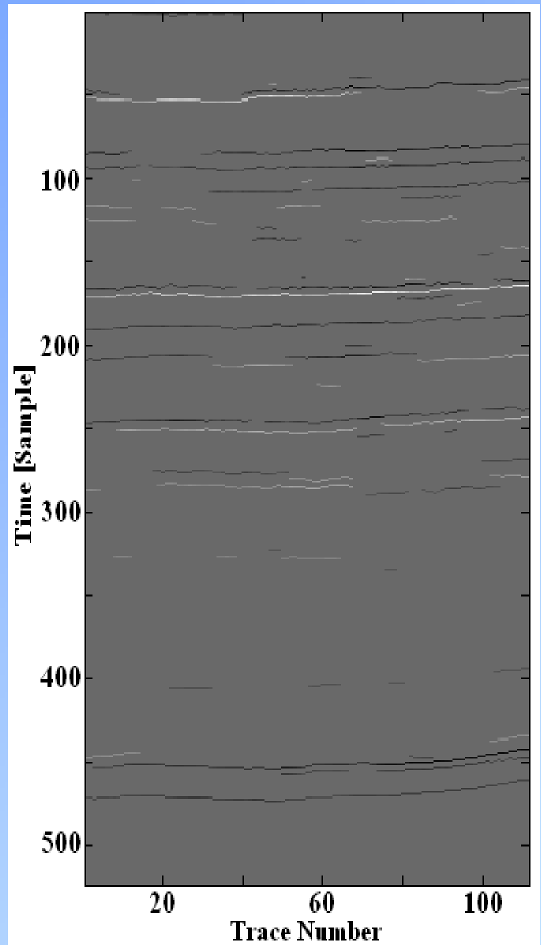


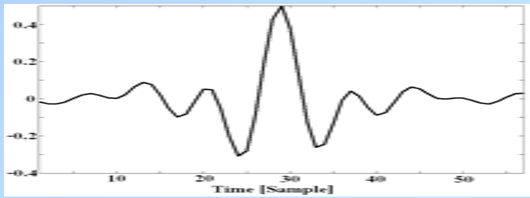
Real Data Result

Data

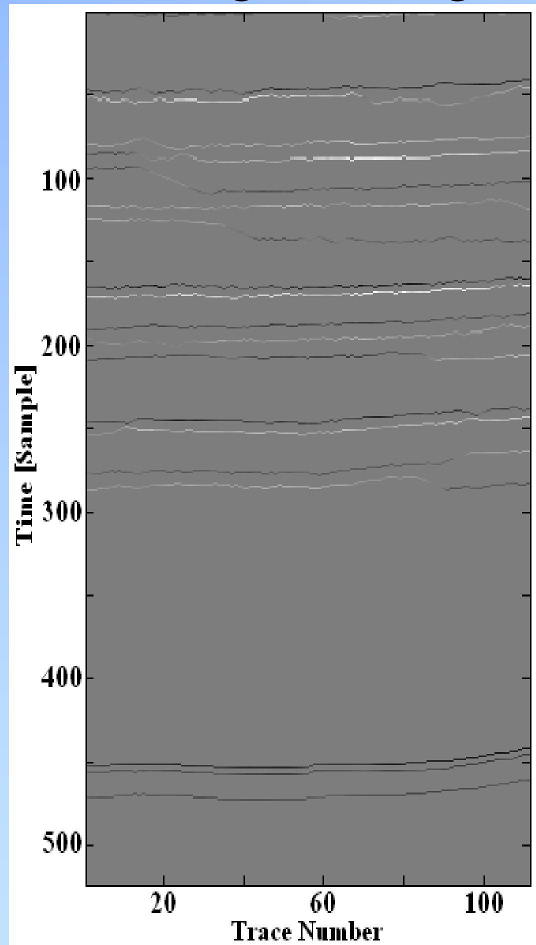


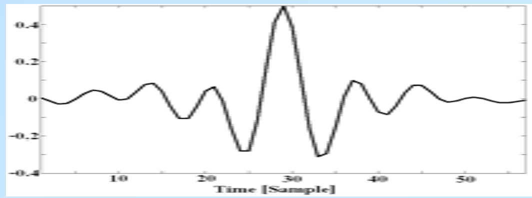
Karresen and Taxt





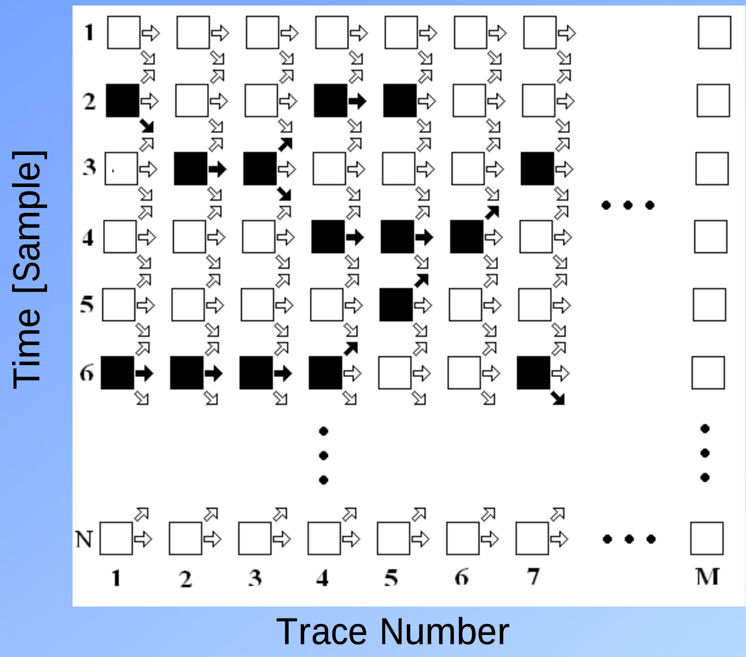
Dynamic Programming





Contents

Idier and Goussard (1993)
Markov Bernoulli Reflectivity Model



Notation:

$q_{i,j}$	Trace j, Time i Reflectivity Indicator	$\mathbf{q}_j$	Trace j, Reflectivity Indicator Vector
$t^0_{i,j}$	Trace j, Time i Ascending Transition Variable	$\mathbf{t}^0_j$	Trace j, Ascending Transition Vector
$t^1_{i,j}$	Trace j, Time I Horizontal Transition Variable	$\mathbf{t}^1_j$	Trace j, Horizontal Transition Vector
$t^2_{i,j}$	Trace j, Time i Descending Transition Variable	$\mathbf{t}^2_j$	Trace j, Descending Transition Vector
$r_{i,j}$	Trace j, Time i Reflectivity Sample	r_j	Trace j, Reflectivity Vector

Optimal Optimization Criterion

$$\left\{\hat{\mathbf{q}},\hat{\mathbf{a}}\right\} = \arg\max_{\mathbf{q},\mathbf{a}} \; p\left(\mathbf{q},\mathbf{a} \mid \mathbf{z}\right)$$

$\hat{\mathbf{a}} \triangleq$ group of reflectors amplitudes

$\mathbf{z} \triangleq$ The 2D Recieved Data

$$p\left(\mathbf{q},\mathbf{a} \mid \mathbf{z}\right) \propto p\left(\mathbf{z} \mid \mathbf{q},\mathbf{a}\right) p\left(\mathbf{a} \mid \mathbf{q}\right) p\left(\mathbf{q}\right)$$

$$p\left(\mathbf{z} \mid \mathbf{q},\mathbf{a}\right) = \prod_{m=1}^{M} p\left(\mathbf{z}_m \mid \mathbf{q}_m,\mathbf{a}_m\right)$$

$$p\left(\mathbf{a} \mid \mathbf{q}\right) = p\left(\mathbf{a}_1,\mathbf{a}_2,\dots,\mathbf{a}_M \mid \mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M\right) =$$
$$p\left(\mathbf{a}_1 \mid \mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M\right) p\left(\mathbf{a}_2 \mid \mathbf{a}_1,\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M\right) p\left(\mathbf{a}_3 \mid \mathbf{a}_1,\mathbf{a}_2,\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M\right),\dots$$
$$\dots, p\left(\mathbf{a}_M \mid \mathbf{a}_1,\mathbf{a}_2,\dots,\mathbf{a}_M,\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M\right) = p\left(\mathbf{a}_1 \mid \mathbf{q}_1\right) p\left(\mathbf{a}_2 \mid \mathbf{a}_1,\mathbf{q}_1,\mathbf{q}_2\right) p\left(\mathbf{a}_3 \mid \mathbf{a}_2,\mathbf{q}_2,\mathbf{q}_3\right)\dots$$
$$\dots p\left(\mathbf{a}_M \mid \mathbf{a}_{M-1},\mathbf{q}_{M-1},\mathbf{q}_M\right)$$

$$p\left(\mathbf{q}\right) = p\left(\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M\right) = p\left(\mathbf{q}_1\right) p\left(\mathbf{q}_2 \mid \mathbf{q}_1\right) p\left(\mathbf{q}_3 \mid \mathbf{q}_1,\mathbf{q}_2\right),\dots$$
$$\dots, p\left(\mathbf{q}_M \mid \mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_{M-1}\right) = p\left(\mathbf{q}_1\right) \prod_{m=2}^{M} p\left(\mathbf{q}_m \mid \mathbf{q}_{m-1}\right)$$

$$\left\{\hat{\mathbf{q}}_1,\hat{\mathbf{q}}_2,\dots,\hat{\mathbf{q}}_M,\hat{\mathbf{a}}_1,\hat{\mathbf{a}}_2,\dots,\hat{\mathbf{a}}_M\right\} =$$

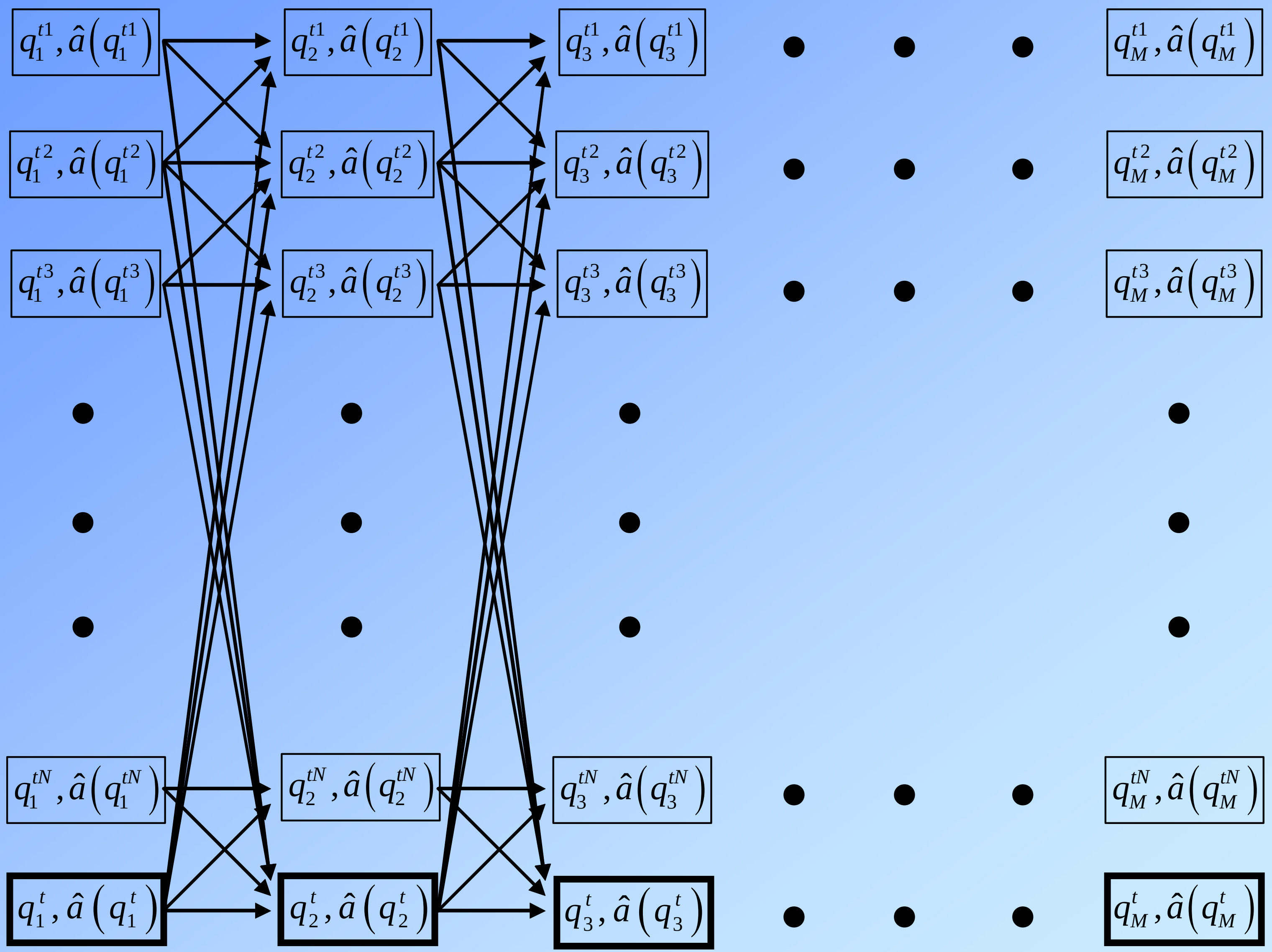
$$= \arg\max_{\mathbf{q}_1,\mathbf{q}_2,\dots,\mathbf{q}_M,\mathbf{a}_1,\mathbf{a}_2,\dots,\mathbf{a}_M} p\left(\mathbf{z}_1 \mid \mathbf{q}_1,\mathbf{a}_1\right) p\left(\mathbf{a}_1 \mid \mathbf{q}_1\right) p\left(\mathbf{q}_1\right) \prod_{m=2}^{M} p\left(\mathbf{z}_m \mid \mathbf{q}_m,\mathbf{a}_m\right) p\left(\mathbf{a}_m \mid \mathbf{q}_m,\mathbf{q}_{m-1},\mathbf{a}_{m-1}\right) p\left(\mathbf{q}_m \mid \mathbf{q}_{m-1}\right)$$

Maximization Tactics

$$\left\{\hat{\mathbf{q}}_1,\hat{\mathbf{q}}_2,\ldots,\hat{\mathbf{q}}_M,\hat{\mathbf{a}}_1,\hat{\mathbf{a}}_2,\ldots,\hat{\mathbf{a}}_M\right\}=\underset{\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_M,\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_M}{\arg\max}\; p\left(\mathbf{z}_1\,|\,\mathbf{q}_1,\mathbf{a}_1\right)p\left(\mathbf{a}_1\,|\,\mathbf{q}_1\right)p\left(\mathbf{q}_1\right)$$
$$\prod_{m=2}^{M}p\left(\mathbf{z}_m\,|\,\mathbf{q}_m,\mathbf{a}_m\right)p\left(\mathbf{a}_m\,|\,\mathbf{q}_m,\mathbf{q}_{m-1},\mathbf{a}_{m-1}\right)p\left(\mathbf{q}_m\,|\,\mathbf{q}_{m-1}\right)$$

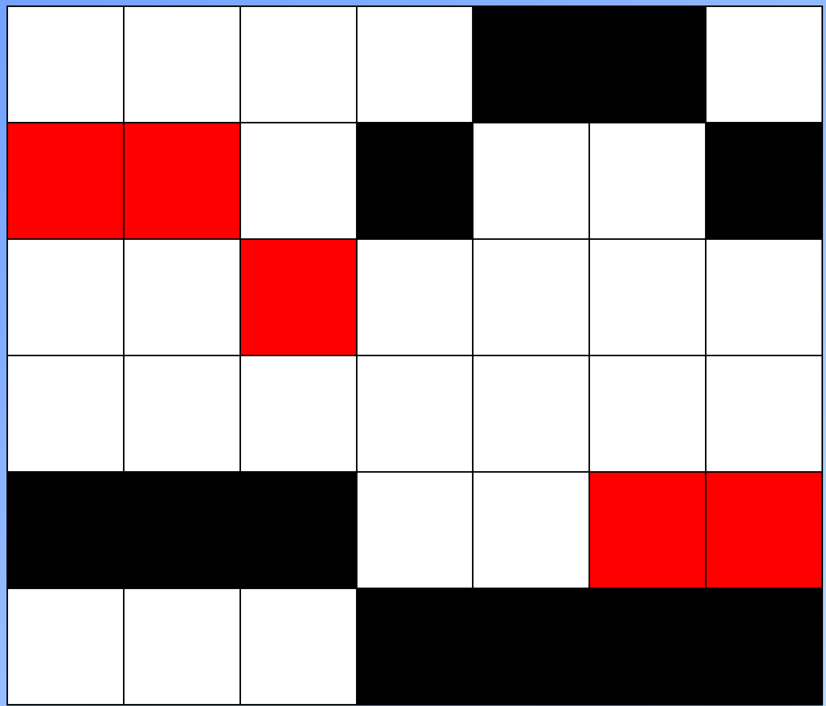
1. Maximize for all columns simultaneously
2. Search only among model fitted configurations
3. Choose from examined configurations using Viterbi algorithm

State Diagram of Reflectivity Estimate

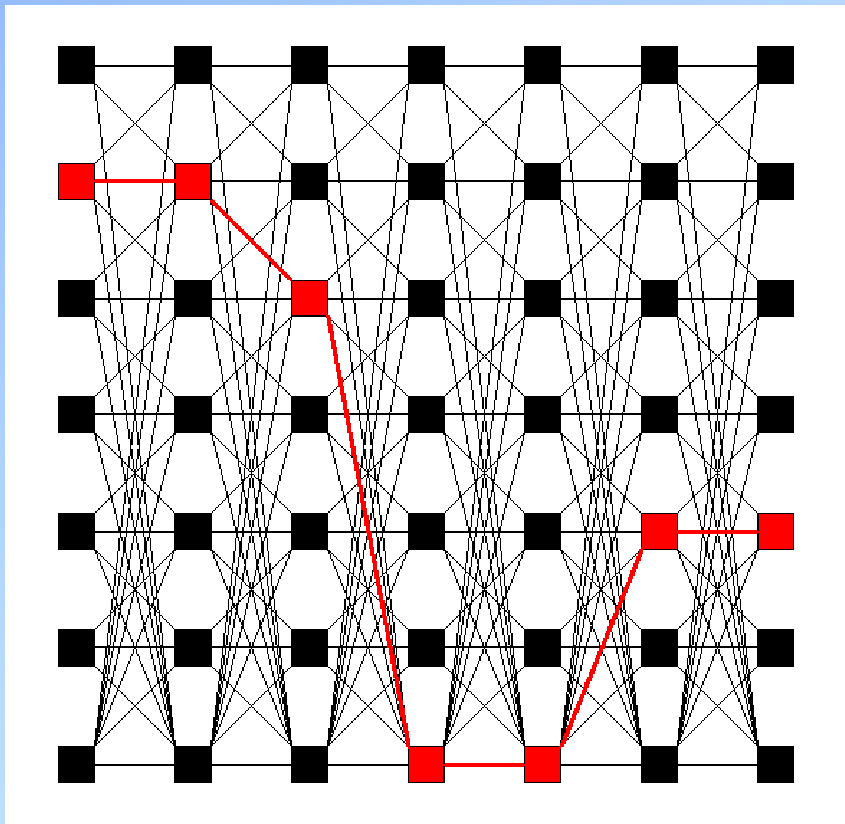


Connection Between States Set and Reflectivity Estimate

Added Reflectors In Estimate



Chosen Path In States Set



Connection Between Chosen State Path and Optimal Criterion

Notation: s_m is the state from column m of the chosen path.

$$\left(\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_M\right) = \arg\max_{s_1, s_2, \ldots, s_M} p\left(s_1, s_2, \ldots, s_M \mid \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_M\right)$$

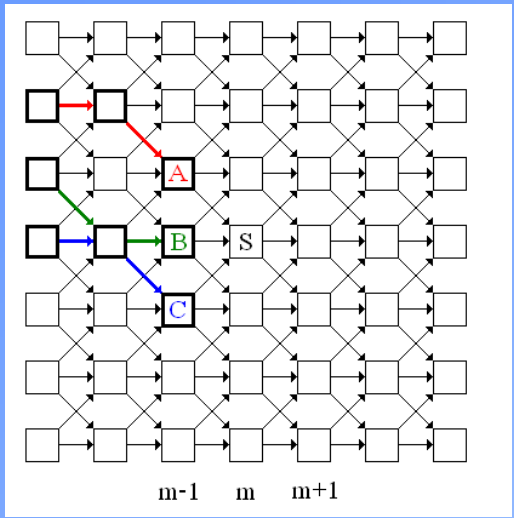
Suboptimal Criterion For Optimization

$$\left\{\hat{n}_1, \hat{n}_2, \ldots, \hat{n}_M\right\} = \arg\max_{n_1, n_2, \ldots, n_M} p\left(\mathbf{z}_1 \mid \mathbf{q}_1^{t,n_1}, \hat{\mathbf{a}}\left(\mathbf{q}_1^{t,n_1}\right)\right) p\left(\hat{\mathbf{a}}\left(\mathbf{q}_1^{t,n_1}\right) \mid \mathbf{q}_1^{t,n_1}\right) p\left(\mathbf{q}_1^{t,n_1}\right) \cdot$$
$$\cdot \prod_{m=2}^{M} p\left(\mathbf{z}_m \mid \mathbf{q}_m^{t,n_m}, \hat{\mathbf{a}}\left(\mathbf{q}_m^{t,n_m}\right)\right) p\left(\hat{\mathbf{a}}\left(\mathbf{q}_m^{t,n_m}\right) \mid \mathbf{q}_m^{t,n_m}, \mathbf{q}_{m-1}^{t,n_{m-1}}, \hat{\mathbf{a}}\left(\mathbf{q}_{m-1}^{t,n_{m-1}}\right)\right) p\left(\mathbf{q}_m^{t,n_m} \mid \mathbf{q}_{m-1}^{t,n_{m-1}}\right)$$

Optimal Criterion For Optimization

$$\left\{\hat{\mathbf{q}}_1, \hat{\mathbf{q}}_2, \ldots, \hat{\mathbf{q}}_M, \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \ldots, \hat{\mathbf{a}}_M\right\} = \arg\max_{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_M, \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_M} p\left(\mathbf{z}_1 \mid \mathbf{q}_1, \mathbf{a}_1\right) p\left(\mathbf{a}_1 \mid \mathbf{q}_1\right) p\left(\mathbf{q}_1\right)$$
$$\prod_{m=2}^{M} p\left(\mathbf{z}_m \mid \mathbf{q}_m, \mathbf{a}_m\right) p\left(\mathbf{a}_m \mid \mathbf{q}_m, \mathbf{q}_{m-1}, \mathbf{a}_{m-1}\right) p\left(\mathbf{q}_m \mid \mathbf{q}_{m-1}\right)$$

Viterbi Algorithm



Notation:

P_s is a "Path" of states starting in column 1 and ending in state S

P_S^o is a "Path" of states starting in column 1 and ending in state S whose probability given the observations is maximal

$$p\left(P_S \text{ that is an extention of some } \mathrm{P}_A \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_m\right) = Q\cdot p\left(P_A \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_{m-1}\right) p\left(S \mid A\right)$$

$$p\left(P_S \text{ that is an extention of some } \mathrm{P}_B \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_m\right) = Q\cdot p\left(P_B \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_{m-1}\right) p\left(S \mid B\right)$$

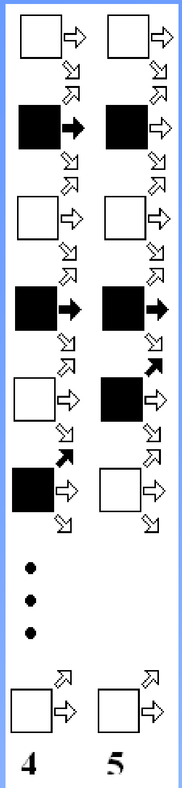
$$p\left(P_S \text{ that is an extention of some } \mathrm{P}_C \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_m\right) = Q\cdot p\left(P_C \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_{m-1}\right) p\left(S \mid C\right)$$

$$Q = p\left(\mathbf{z}_m \mid S\right)\frac{p\left(\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_{m-1}\right)}{p\left(\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_m\right)}$$

$$\Downarrow$$

$$P_S^o = \left\{ \underset{x\in\{A,B,C\}}{\arg\max}\; p\left(P_x^o \mid \mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_m\right) p\left(S \mid x\right) \;,\; S \right\}$$

Calculations



$$p\left(\mathbf{q}_m \mid \mathbf{q}_{m-1}\right) = \prod_{n=1}^{N} p\left(t^0_{n,m-1}, t^1_{n,m-1}, t^2_{n,m-1} \mid q_{n,m-1}\right) p\left(q_{n,m} \mid t^0_{n+1,m-1}, t^1_{n,m-1}, t^2_{n-1,m-1}\right)$$

We Need:

$$p\left(\hat{\mathbf{a}}(\mathbf{q}_m) \mid \mathbf{q}_m, \mathbf{q}_{m-1}, \hat{\mathbf{a}}(\mathbf{q}_m)\right)$$

But The Original 2D Reflectivity Model Determines:

$$p\left(\mathbf{a}(\mathbf{q}_m) \mid \mathbf{q}_m, \mathbf{q}_{m-1}, \mathbf{a}(\mathbf{q}_m)\right)$$

$$\mathbf{a}(\mathbf{q}_{m-1}) \mid \hat{\mathbf{a}}(\mathbf{q}_{m-1}) \sim N\left(B_1 \hat{\mathbf{a}}(\mathbf{q}_{m-1}), \Sigma_1\right)$$

$$\mathbf{a}(\mathbf{q}_m) \mid \mathbf{a}(\mathbf{q}_{m-1}) \sim N\left(B_2 \mathbf{a}(\mathbf{q}_{m-1}), \Sigma_2\right)$$

$$\hat{\mathbf{a}}(\mathbf{q}_m) \mid \mathbf{a}(\mathbf{q}_m) \sim N\left(B_3 \mathbf{a}(\mathbf{q}_m), \Sigma_3\right)$$

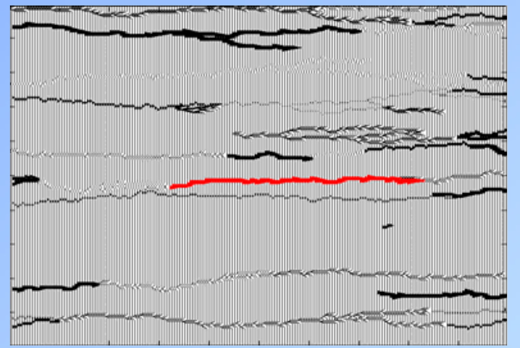
$$p\left(\hat{\mathbf{a}}(\mathbf{q}_m) \mid \hat{\mathbf{a}}(\mathbf{q}_{m-1})\right) =$$

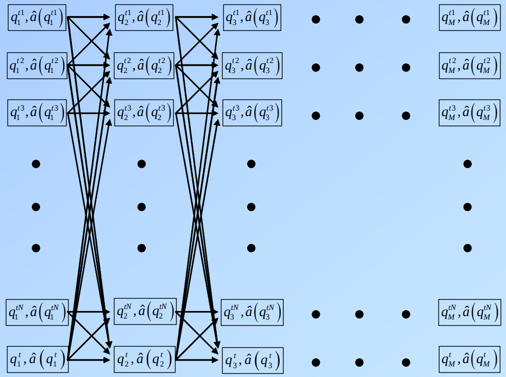
$$= \int_{\mathbf{a}(\mathbf{q}_m)} \int_{\mathbf{a}(\mathbf{q}_{m-1})} p\left(\hat{\mathbf{a}}(\mathbf{q}_m) \mid \mathbf{a}(\mathbf{q}_m)\right) p\left(\mathbf{a}(\mathbf{q}_m) \mid \mathbf{a}(\mathbf{q}_{m-1})\right) p\left(\mathbf{a}(\mathbf{q}_{m-1}) \mid \hat{\mathbf{a}}(\mathbf{q}_{m-1})\right) d\mathbf{a}(\mathbf{q}_{m-1})\, d\mathbf{a}(\mathbf{q}_m)$$

$$\hat{\mathbf{a}}(\mathbf{q}_m) \mid \hat{\mathbf{a}}(\mathbf{q}_{m-1}) \sim N\left(B_1 B_2 B_3 \hat{\mathbf{a}}(\mathbf{q}_{m-1}), \Sigma_3 + B_3 \Sigma_2 B_3^T + B_3 B_2 \Sigma_1 B_2^T B_3^T\right)$$

Steps of Algorithm

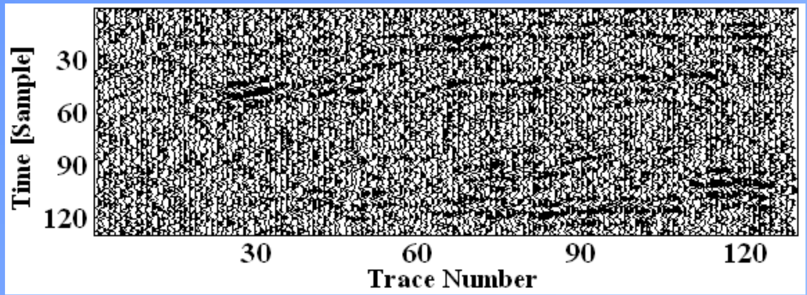
1. Start with empty reflectivity estimate.
2. If there are any reflectors in the estimate, remove a section of reflectors from it.
3. Define the state set, and estimate amplitudes for each state.
4. Using Viterbi algorithm, find path of states with highest probability.
5. Update estimate according to reflectors in the best path.
6. Return to step 2.



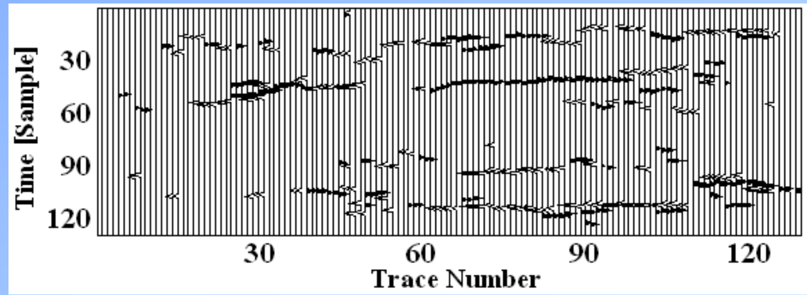


Non Blind Deconvolution Results (SNR = -5 dB)

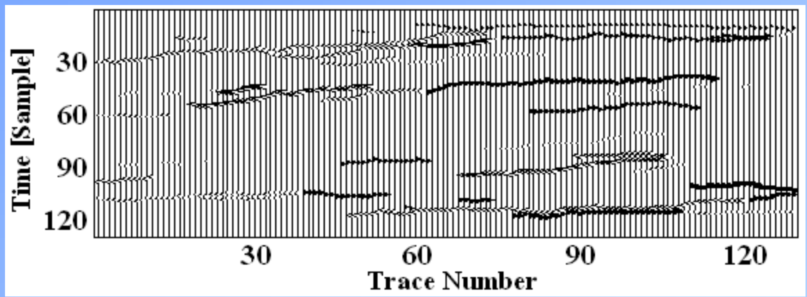
Data



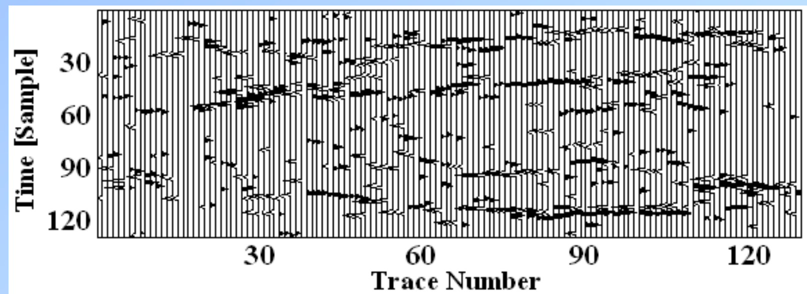
Karresen and Taxt (High sparsity)



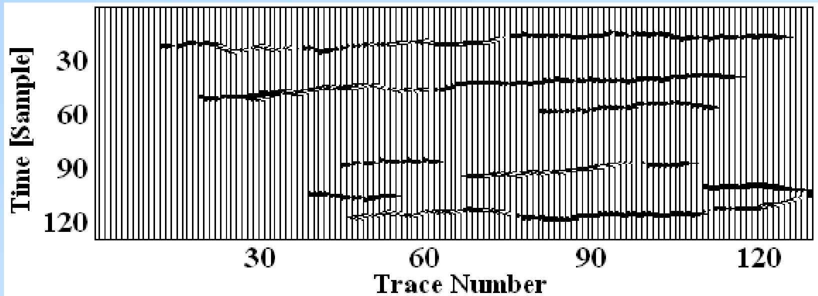
True Reflectivity



Karresen and Taxt (Low sparsity)

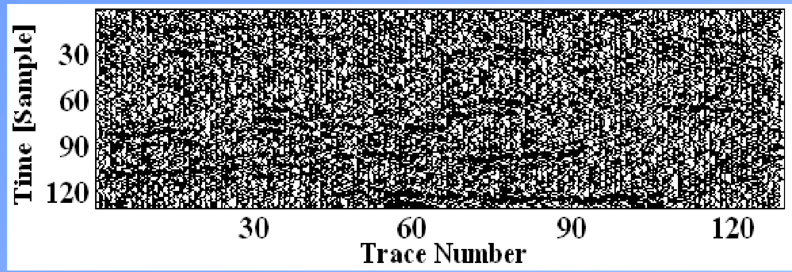


Viterbi

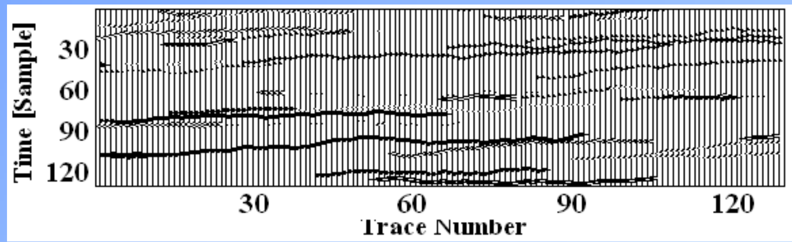


Blind Deconvolution Results (SNR = -5 dB)

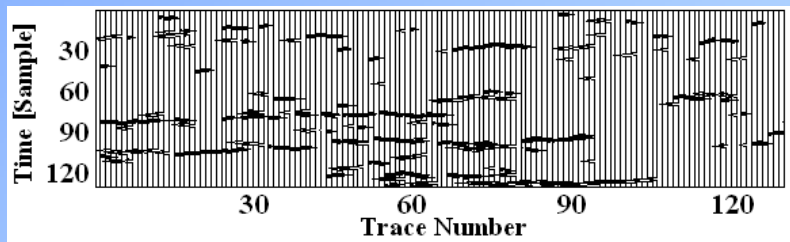
Data

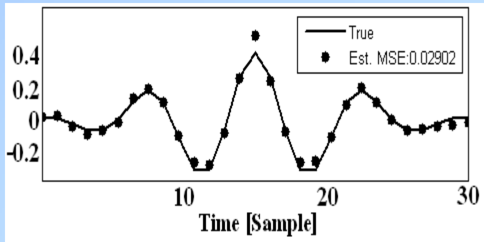


True Reflectivity

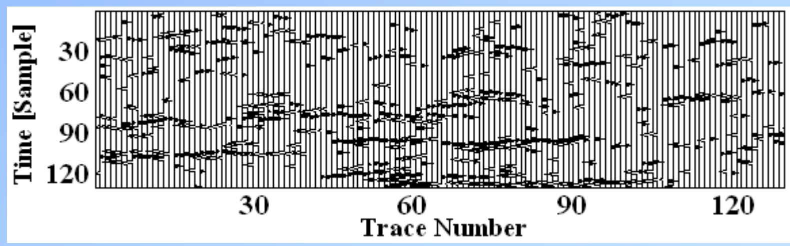


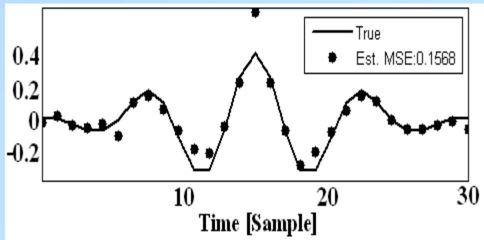
Karresen and Taxt (High Sparsity)



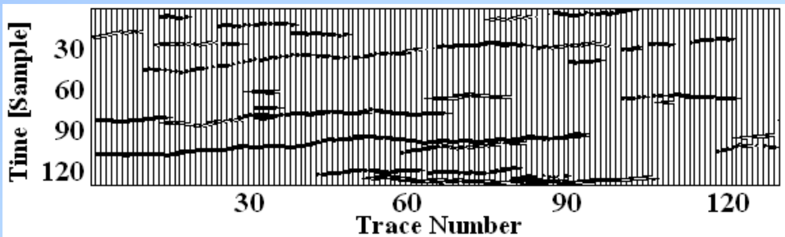


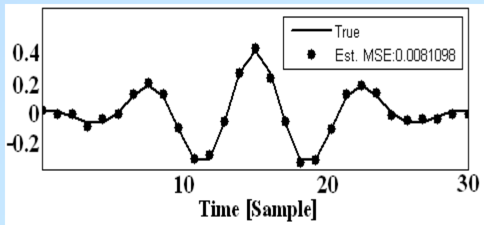
Karresen and Taxt (Low Sparsity)





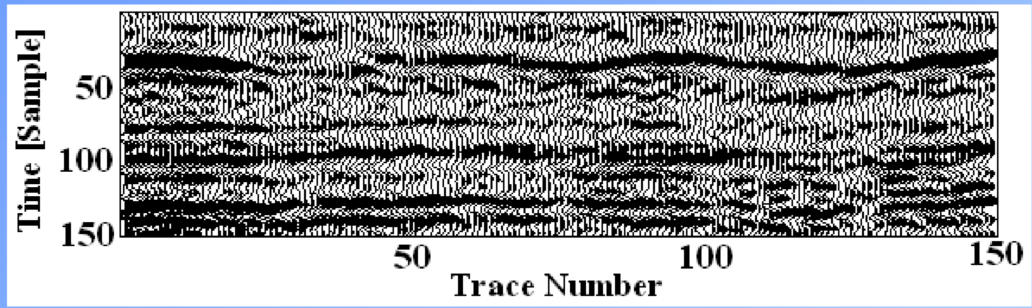
Viterbi



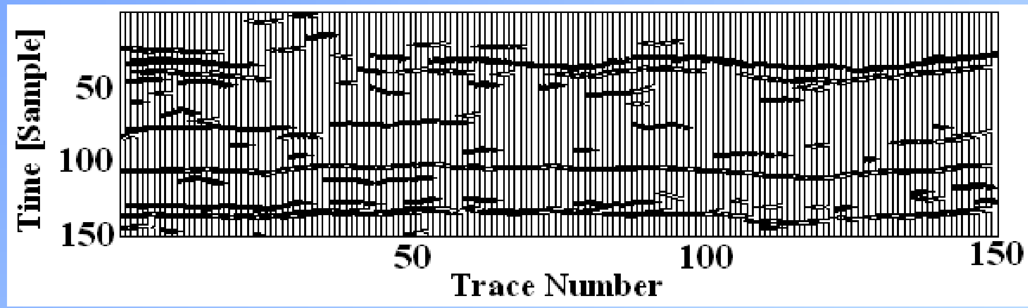


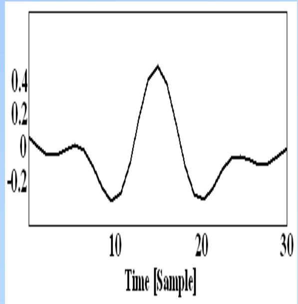
Real Data Results

Data

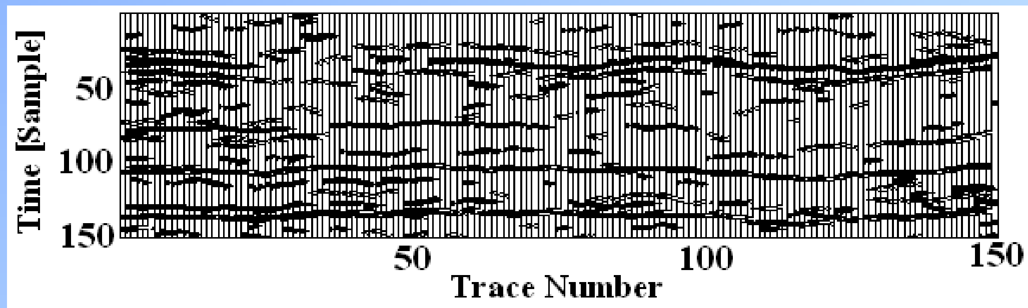


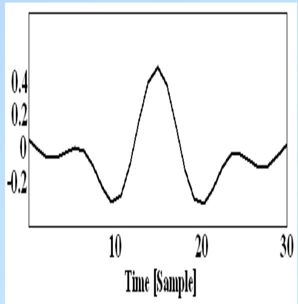
Karresen and Taxt (High Sparsity)



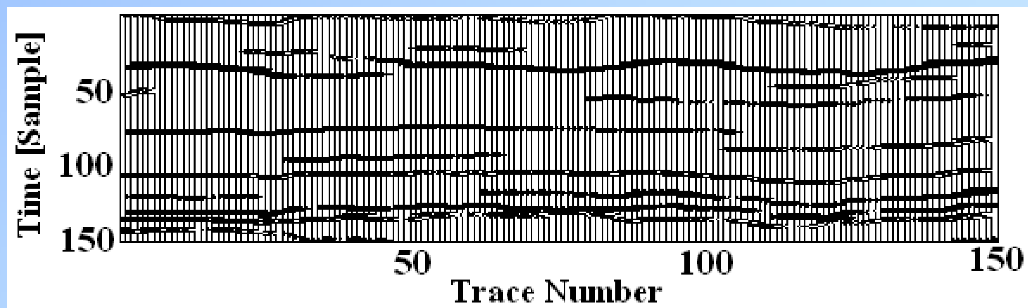


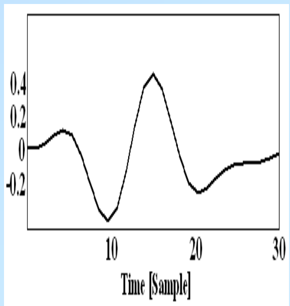
Karresen and Taxt (Low Sparsity)





Viterbi





Contents

- Seismic Exploration - Single Channel Model
- Goal and Motivation
- Previous Works
  - Kaaresen and Taxt 1998
  - Idier and Goussard 1993
- Multichannel Deconvolution Using Dynamic Programming
- Multichannel Deconvolution Using Viterbi Algorithm
- Conclusion and Future Research

Conclusion

1. Optimization Criteria based on continuity of reflectors locations and amplitude.
2. Two algorithms, one accounts for possible discontinuities and one does not.
3. Improved recovery of 2D reflectivity pattern.
4. Proper management of reflectors can accelerate algorithms.

Future Research

1. Combination of concept with other multichannel deconvolution algorithms.
2. Varying wavelet, or model parameters.
3. Extention of methods to 3D data.
4. Deconvolution or direction of arrival estimation of Multipath signals received in array of sensors.